

# AN ADAPTIVE BEAMFORMER UTILIZING BINARY BAT ALGORITHM FOR ANTENNA ARRAY PATTERN NULLING

BỘ ĐỊNH DẠNG VÀ ĐIỀU KHIỂN BÚP SÓNG THÍCH NGHI SỬ DỤNG THUẬT TOÁN ĐÀN DƠI NHỊ PHẦN ĐỂ ĐẶT ĐIỂM “KHÔNG” TRÊN GIẢN ĐỒ BỨC XẠ CỦA MẢNG ANTEN

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## ABSTRACT

In this paper, we propose an adaptive beamformer based on a binary bat algorithm (BBA) for pattern nulling of uniformly linear array (ULA) antennas. The optimized array pattern is obtained by controlling the phase of each array excitation weight. Several scenarios have been conducted to evaluate the performance of the proposal including convergence speed and pattern nulling ability. The simulation results show that proposed beamformer is able to precisely impose nulls at arbitrary directions of interferences while suppressing sidelobes and maintaining the main lobe. Furthermore, the proposal is faster and more efficient than binary particle swarm optimization (BPSO)-based one with respect to pattern nulling in array pattern synthesis.

**Keywords:** Beamforming, ULA antennas, binary bat algorithm, pattern nulling, interference suppression, array pattern synthesis.

## TÓM TẮT

Trong bài báo này, chúng tôi đề xuất bộ định dạng và điều khiển búp sóng thích nghi dựa trên thuật toán đàn dơi nhị phân (BBA: Binary Bat Algorithm) để đặt điểm “không” trên giản đồ bức xạ của mảng anten tuyến tính cách đều (ULA: Uniformly linear array). Bức xạ tối ưu của mảng thu được bằng cách điều khiển pha của các trọng số tác động vào từng phần tử trong mảng. Một số kịch bản được thực hiện để đánh giá hiệu năng của đề xuất bao gồm tốc độ hội tụ và khả năng đặt điểm “không” trên giản đồ bức xạ. Các kết quả mô phỏng cho thấy bộ định dạng và điều khiển búp sóng được đề xuất có khả năng đặt điểm “không” chính xác tại các hướng nhiễu bất kỳ đồng thời nén búp sóng phụ và duy trì hướng và độ rộng búp sóng chính. Hơn nữa, đề xuất này nhanh hơn và hiệu quả hơn bộ định dạng và điều khiển búp sóng dựa trên thuật toán tối ưu bầy đàn nhị phân (BPSO: Binary Particle Swarm Optimization) về khả năng đặt điểm “không” trên giản đồ bức xạ trong quá trình tổng hợp giản đồ bức xạ của mảng anten.

**Từ khóa:** Định dạng tia, ăng-ten ULA, thuật toán đàn dơi nhị phân, vô hiệu hóa nhiễu, tổng hợp mẫu mảng.

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## 1. INTRODUCTION

Adaptive beamformers are being widely applied in radar, sonar, and wireless communication systems to

enhance performance by improving radio signal spectrum efficiency, suppressing interferences, and saving utilization power. Moreover, adaptive beamformers are capable of producing appropriate weights for antenna arrays to obtain desired patterns [1]. It is apparent that the gradual increase of wireless devices is causing serious pollution in the electromagnetic propagation environment, which leads to the emergence of null-steering capabilities in smart antenna systems as a promising solution for interference suppression in radar, sonar, and wireless communications applications.

Several adaptive pattern nulling approaches have been widely researched and implemented, such as position-only control, array thinning, excitation weights control [1, 2]. Each method has its advantages and limitations where the position-only method [3] needs a mechanical driving system as servomotors for adjusting the position of antenna elements, which leads to more complicated and difficult to accurately control. The array thinning method does not require digital beamformers due to leveraging the robustness of adaptive algorithms to turn array elements on (active) or off (inactive) but this is not an appropriate solution for small antenna arrays [4].

The amplitude-only control [5-7] unique adjusts the amplitude excited at each array element; also, this approach is less flexible in placing various kinds of nulls, and the main lobe cannot be steered. The complex-weight control has been acknowledged as the most effective and flexible one; however, this approach is required to have controllers, phase shifters, and attenuators for each array element, so it is the most complicated and costly [8-10]. The phase-only control is attractive for the phased array systems despite low complication and no extra cost [7, 11, 12]; moreover, the main lobe direction can be steered by adjusting the phase of excitation weights.

Recently, the optimal pattern has been achieved by applying various optimization techniques where nature-inspired optimization approaches have been proved as promising global optimization solutions in terms of flexibility and efficiency [8, 13-28]. Specifically, particle swarm optimization (PSO)-based solutions were introduced for array pattern synthesis [17, 20], adaptive interference

suppression in continuous optimization [21], and in the discrete optimization of complex communication scenarios [22]. Moreover, other pattern nulling solutions were conducted such as ant colony optimization [10], backtracking search[24], and bat algorithm (BA) [8, 13, 14, 15, 25].

BA, which was proposed by Xin-She Yang in 2010, is a metaheuristic algorithm for global optimization techniques. It has been developed on the natural behavior of microbats manipulating echolocation to detect prey, avoid obstacles, and locate their roosting crevices in the dark. So far, this algorithm has been successfully utilized to deal with various types of engineering problems and proved to be more powerful than other methods like the genetic algorithm (GA) and PSO [26, 27]. Q. Yao and Y. Lu first employed BA for adaptive beamforming [28], and BA-based design of a double-sided printed dipole antenna array with a low first sidelobe level has been performed in [15]; also, adaptive BA-based beamformers for the pattern nulling have been demonstrated and successfully implemented for ULAs in [8, 13, 14].

The authors of [8, 13, 14] proved that BA-based beamformers are more effective than GA and accelerated particle swarm optimization (APSO)-based one about pattern nulling but optimized weight vectors are real number while the phase of each array element is generally adjusted by digital phase shifters. So as to conveniently apply in phased array systems, a basic BBA [29], is used to determine the phase of weights and is leveraged to suppress interference in the sidelobe region while maintaining the main lobe and keeping the sidelobes at low levels. Five scenarios will be performed to verify the proposal where the beamformer based on BBA will be compared to the beamformer based on a basic BPSO [30]. The simulation results will show that the BBA-based beamformer performs pattern nulling more powerful than the BPSO-based one.

The remains of this work are organized as follows: In Section 2, the formulation of the problem is depicted, and the proposed beamformers employing BBA are presented in Section 3. Section 4 demonstrates the numerical results of the proposal before concluding the work in Section 5.

**2. PROBLEM FORMULATION**

In this study, ULA antennas of 2N isotropic elements have been employed and illustrated in Figure 1. The elements are symmetrically located across the center of the array, so the array factor can be expressed as [31]:

$$AF(\theta) = \sum_{n=-N}^N \omega_n e^{jndksin(\theta)} \tag{1}$$

where:  $\omega_n = \omega_n^{re} + j\omega_n^{im} = a_n e^{j\delta_n}$  is the excitation weight (complex-weight) of the  $n^{th}$  element;  $\lambda$  is wavelength;  $k = \frac{2\pi}{\lambda}$  is the wavenumber;  $d$  is the distance between adjacent elements.

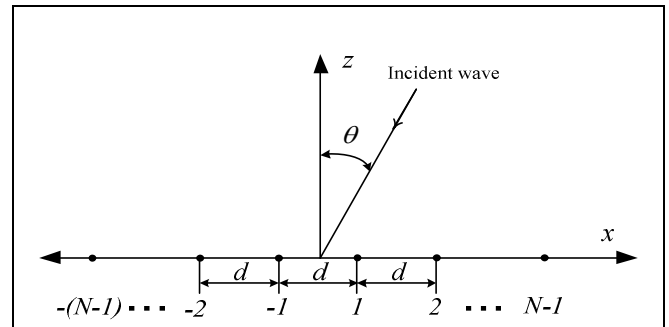


Figure 1. The geometry of the 2N elements ULA antennas

In order to obtain faster convergence, the minimum weight perturbation phase-only synthesis requires odd phase shift ( $\delta_{-n} = -\delta_n$ ) [32]. Thus, an asymmetrical pattern through the main lobe direction ( $\theta = 0^\circ$ ) is obtained. When  $a_{-n} = a_n$  and  $\delta_{-n} = -\delta_n$ , the array factor in (1) can be expressed as follows:

$$AF(\theta) = 2 \sum_{n=1}^N a_n \cos(ndksin(\theta) + \delta_n) \tag{2}$$

According to (2), the number of phase shifters is equal to the number of elements, yet the number of controllers, attenuators, and computational time will be reduced by half. Moreover, this approach can be implemented for the actual phased array systems without extra cost, which is the highlight of phase-only control compared to amplitude-only control and complex-weight control.

The objective function  $O$  has been built in [8, 13, 14] as follows:

$$O = \begin{cases} 10,000 \sum_{i=1}^I [ |AF_o(\theta_i)|^2 ], & \text{for } \theta = \theta_i \\ \sum_{\theta=-90^\circ}^{90^\circ} [ |AF_o(\theta) - AF_d(\theta)|^2 ], & \text{elsewhere} \end{cases} \tag{3}$$

where:  $AF_o$  and  $AF_d$  are the optimized array factor achieved by using optimization algorithms, which will be BBA and the desired array factor (Chebyshev pattern) in this work, respectively;  $\theta_i$  and  $I$  correspond to the angles and the total number of null points. (3) is used for setting null point, and (4) is to suppress the sidelobe level (SLL) and to keep the beamwidth of the main lobe.

**3. PROPOSAL OF THE BEAMFORMER**

*Initialize* the parameters of arrays; termination condition; objective function  $O$ ; bat population {frequency ( $f_i$ ), velocity ( $v_i$ ), pulse emission rate ( $r_i$ ), loudness ( $A_i$ ), and location/solution ( $x_i$ )}; Define an initial location vector ( $x_i$ ) based on the weight vector of Chebyshev array.

*While* (the termination condition is not satisfied) Update positions.

$i\Phi$ (rand >  $r_i$ )

Select a solution ( $G_{best}$ ) among the best solutions.

Change some of the dimensions of location vector with some of the dimensions of  $G_{best}$ .

$\epsilon\nu\delta i\Phi$

Generate a new solution by flying randomly.

$i\Phi$ (rand <  $A_i$  &  $O(x_i) < O(G_{best})$ )

Accept the new solutions.

$\epsilon\nu\delta i\Phi$

Rank the bats and find the final  $G_{best}$ .

$\epsilon\nu\delta\omega\eta\lambda\epsilon$

Build array element weights from the final  $G_{best}$  and conduct pattern nulling.

Algorithm 1. Pseudocode of the proposed adaptive pattern nulling approach

Based on studies in [8, 13, 14, 29], the BBA-based beamformer utilizing phase-only control for interference suppression applications have been deployed step-by-step in Algorithm 1, in which the termination condition is chosen as the max number of iterations in all simulation scenarios except for the computational time in the first scenario.

**4. NUMERICAL RESULTS**

Table 1. Parameters and common items for the proposal

Parameters, common items	Details
Applied arrays	ULAs: inter-element spacing: $\lambda/2$ ; 20 elements (2N)
Array elements	Isotropic elements (Ideal)
Array factor (AF)	AF in (2)
Control techniques	Phase-only
The objective function (O)	0 in (3), (4)
Global parameters	<ul style="list-style-type: none"> <li>– The step size of theta angle (<math>\theta</math>) is <math>0.5^\circ</math>;</li> <li>– Population size (pop) is 100;</li> <li>– The number of iterations (ite) is 2 (except for simulation results presented in Figure (ite 200); in Figure (ite 50))</li> <li>– Search value of variables in the range of <math>[-5^\circ, 5^\circ]</math>;</li> <li>– One bat (locations) in the population is initialized by weights of the Chebyshev array with the SLL of -30dB. Others are randomly initialized.</li> <li>– Variables: 8-bit numbers.</li> <li>– Transfer function: V-shaped [20, 21].</li> </ul>
Optimization algorithms	
BBA	The step size of random walk is 0.01; boundary frequency values: $f_{min} = 0$ and $f_{max} = 2$ ; $A = 0.25$ ; $r = 0.1$ [20]
BPSO	$C_1 = C_2 = 2$ ; $W$ is linearly decreased from 0.9 to 0.4; max velocity: 6 [21].

In this section, five scenarios will be investigated to evaluate the performance of the proposals for pattern nulling. It is well known that the Chebyshev array weights distribution produces optimized patterns in terms of a trade-off between the sidelobe level (SLL) and the first null beamwidth (FNBW) of the main beam for equally spaced arrays [33]. In this work, the parameters of the Chebyshev array (the desired pattern  $AF_d$  is to control SLL and the beamwidth of the main beam) have been chosen as follows: SLL is -30dB;  $\lambda/2$  is adjacent elements spacing; isotropic elements are 20. Other parameters and common items for the proposal are introduced in Table 1.

To show the capability of BBA utilization in our proposed beamformer for interference suppression, five scenarios have been built. Scenario1 named Convergence characteristics is the first step to evaluate the operation of the proposal beamformer by comparing the convergence rate of the objective functions based on BBA and BPSO. Scenarios 2 - 5 are for investigating and comparing the ability of null-steering of the proposed beamformer to the BPSO-based one. All simulation scenarios have been presented in Figure 2 - 7, where the results are the averaged values of Monte Carlo simulations with 1000 times for the first scenario, and 100 times for the others.

**4.1. Convergence characteristics**

In the first scenario, the computational capability of the proposed BBA-based beamformer has been evaluated and compared to the BPSO-based one in the case of a single null placed at the second sidelobe peak ( $14.5^\circ$ ) in the Dolph-Chebyshev array pattern. To do that, at the initial step, the location vector of one bat in the population has been initialized as Chebyshev array weights with SLL of -30dB; pop is 100; ite is 200. The simulation results of the objective function are displayed in Figure 2, and the computational time of two beamformers has been investigated in case of getting the same value of the objective function ( $O < 10$ ). The results show that the BBA-based beamformer and BPSO-based one take 0.52 seconds and 14.2 seconds, respectively on Desktop PC (CPU Intel i7-8700, RAM 8GB, and MATLAB 2020a). It is apparent that the BBA-based approach converges much faster than the BPSO-based one.

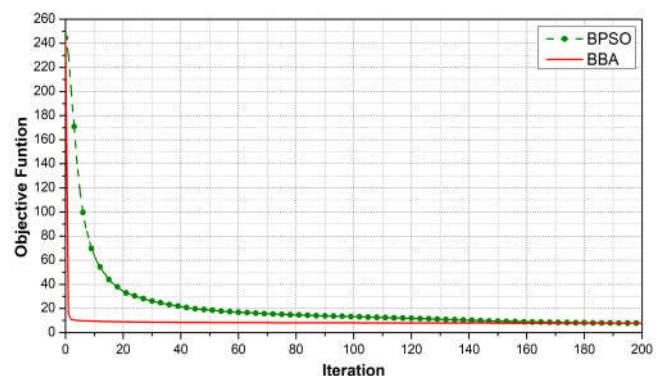


Figure 2. The objective function comparisons of BBA and BPSO

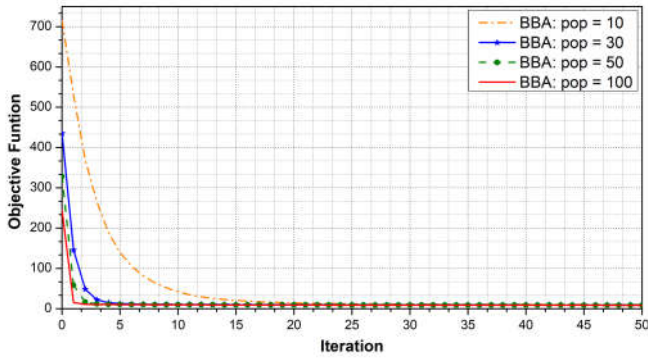


Figure 3. The objective function of BBA-based beamformers with various population sizes

Furthermore, the objective function of the BBA-based beamformer has been implemented with various bat population sizes (pop) and is illustrated in Figure 3. The objective function takes 35 iterations, 8 iterations, 4 iterations, and 2 iterations to roughly converge corresponding to pop = 10, 30, 50, and 100, respectively.

#### 4.2. Optimized patterns with a single null

In Scenario 2, the optimized pattern with a single null has been considered. This null can be arbitrarily set at any angle, which is chosen at the peak of the second sidelobe ( $14.5^\circ$ ) in this test case. The population has been initialized as Chebyshev array weights with  $-30\text{dB}$  SLL, and Figure 4 demonstrates optimized patterns with a single null obtained by BBA and BPSO. The optimized pattern preserves almost all characteristics of the initial Chebyshev pattern such as half-power beamwidth (HPBW =  $6.3^\circ$ ) and SLL ( $-30\text{dB}$ ). The maximum SLL is  $-23.89\text{dB}$ , and the null depth level (NDL) at  $14.5^\circ$  is  $-73.21\text{dB}$ ; additionally, Figure 4 indicates that the pattern with a single null optimized by BBA-based approach is better than that of the BPSO-based one in terms of NDL at the desired direction.

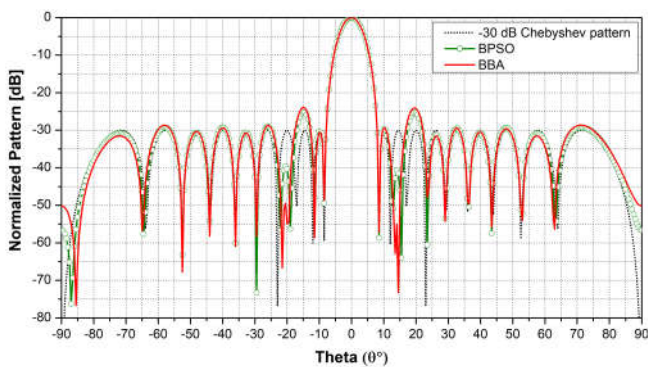


Figure 4. Optimized patterns with a single null at  $14.5^\circ$

#### 4.3. Optimized patterns with multiple nulls

In the third scenario, the proposal will be used for separately imposing multiple nulls at  $-48^\circ$ ,  $20^\circ$ , and  $40^\circ$ , corresponding to the peaks of three sidelobes next to the main lobe of the Chebyshev array pattern. As shown in Figure 5, the patterns with multiple nulls at predefined locations have been obtained. All NDLs are less than

$-46\text{dB}$ , all SLLs are lower than  $-24\text{dB}$ , and HPBW roughly equals that of the Chebyshev pattern. The BBA pattern shows advantages over the BPSO one with respect to NDL.

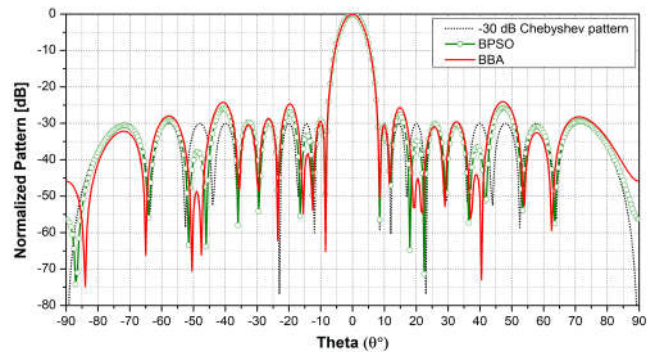


Figure 5. Optimized patterns with three nulls at  $-48^\circ$ ,  $20^\circ$ , and  $40^\circ$

#### 4.4. Optimized patterns with a broad null

In interference suppression applications, if the direction of the interference slightly varies over time or cannot be exactly known, or a null is continuously steered to obtain an appropriate signal-to-noise ratio, a broad null is required. In order to show the ability of broad interference suppression, in the fourth scenario, the pattern with a set broad null at a predefined sector of  $[30^\circ, 40^\circ]$  has been obtained and illustrated in Figure 6. A broad null (minimum NDL  $< -38\text{dB}$ ) on the BBA pattern at that target sector has been successfully placed. The beamwidth is nearly unchanged and the maximum SLL is  $-24.16\text{dB}$ . The results prove that the BBA pattern is better than BPSO one in terms of NDL.

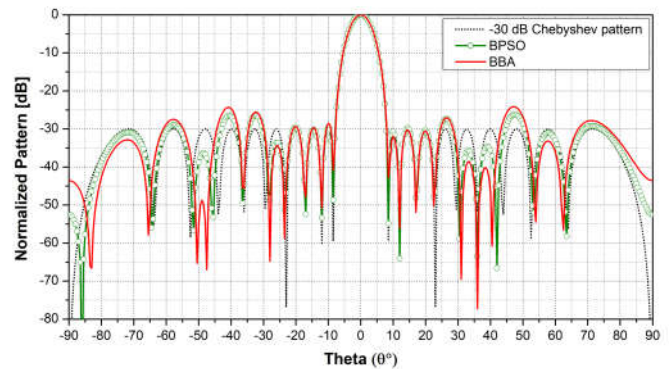


Figure 6. Optimized patterns with a broad null from  $30^\circ$  to  $40^\circ$

#### 4.5. Optimized patterns with the steered main lobe

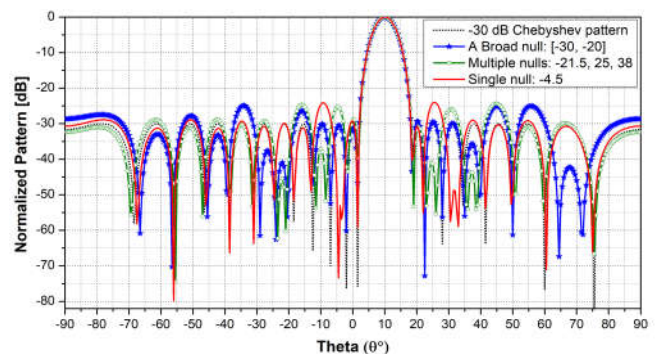


Figure 7. Optimized patterns with the steered main lobe at  $10^\circ$

In the situation of fixed main lobe direction, the efficiency of the proposed beamformer has been demonstrated above. However, this solution is not only constrained by the fixed main lobe direction but also able to apply in main lobe steering. In order to implement pattern nulling while steering the main lobe, the process is the same as mentioned above, but the main lobe is steered to the desired direction. This can be gained by steering the main lobe of the Chebyshev pattern in the objective function to the desired direction before setting nulls toward interferences. In the fifth scenario, three above adaptive pattern nulling cases have been conducted while steering the main lobe to a predefined angle, which is  $10^\circ$  in this test case. The simulation results shown in Figure 7 have been proved that the proposal is capable of performing as well as those in the scenarios presented in Sections 4.2 – 4.4.

## 5. CONCLUSIONS

In this work, a BBA-based beamformer for interferences suppression of ULA antennas has been proposed. The pattern nulling ability of the proposal has been verified by five scenarios consisting of convergence speed, various types of nulls, and the main lobe steering. The results show that the aforementioned nulls can be precisely imposed at arbitrary interference directions using our proposed beamformer while maintaining the HPBW and keeping the low SLL. Furthermore, compared with BPSO-based beamformers, the proposal performs better in terms of execution time and pattern nulling in array pattern synthesis. For future works, the phase change effect and the resolution of phase shifters, real array element pattern, and the directions of interference at the main lobe should be considered.

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