

EXAMPLES OF USING MATHEMATICS TO CREATE ARTISTIC PRODUCTS FOR ELEMENTARY STUDENTS

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Artical history

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Abstact

Art has generally been associated with aesthetics, emotion, and senses. On the contrary, mathematics has been associated with logic, precision, and truth. It is necessary to understand that throughout history, the two disciplines have been more united than many might consider. Various characteristics believed to be antagonistic have proved to be more convergent than divergent. Both disciplines use patterns, shapes, lines, fractions, and proportions, etc. These concepts are extremely important to both artists and mathematicians. Furthermore, art is now part of the disciplines in education regarding science, technology, engineering, art, and mathematics (STEAM). Mathematical concepts are explicitly and implicitly used to create visual works. Many masterpieces can be seen to be made up of repeating sequences of simple geometric shapes, lines such as M.C. Escher, Wassily, Kandinsky, Paul Klee, and Dalí. This article offers some products that can involve the knowledge of mathematics to create some simple art products suitable for elementary students' knowledge and skills to help broaden their thinking, increase creativity, and form certain abilities.

Keywords: *Art, education, combine, Mathematics, STEAM.*

MỘT SỐ MINH HOẠ VỀ ỨNG DỤNG TOÁN HỌC VÀO TẠO HÌNH SẢN PHẨM MỸ THUẬT CHO HỌC SINH TIỂU HỌC

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Tóm tắt

Nghệ thuật thường gắn liền với lĩnh vực thẩm mỹ, sáng tạo, cảm xúc và cảm quan. Ngược lại, toán học luôn được xem là lĩnh vực của sự logic, hợp lý, độ chính xác và chân lý. Nhưng trong suốt chiều dài lịch sử của nhân loại, hai ngành đã có nhiều sự thống nhất, liên kết, hỗ trợ, hoà trộn với nhau và có nhiều đặc điểm được cho rằng đối kháng lại được chứng minh là hội tụ. Cả toán học và nghệ thuật đều sử dụng các kiểu mẫu, hình dạng, đường nét, phân số và tỉ lệ... những khái niệm này cực kì quan trọng với cả các nghệ sĩ và các nhà toán học. Trong nghệ thuật, con người đã sử dụng các khái niệm toán học một cách có ý thức hoặc vô thức để tạo ra các tác phẩm thị giác. Chúng ta có thể bắt gặp nhiều tác phẩm mỹ thuật nổi tiếng được tạo thành từ trình tự lặp lại của các hình dạng hình học đơn giản như M.C.Escher, Wassily, Kandinsky, Paul Klee, Dalí... Trong khuôn khổ bài viết, tác giả xin đưa ra một số sản phẩm lấy ý tưởng từ việc kết hợp kiến thức của toán học vào mỹ thuật của một số họa sĩ nêu trên. Từ đó, gợi ý tạo ra những sản phẩm đơn giản phù hợp với kiến thức, kỹ năng của học sinh giúp các em mở rộng tư duy, tăng khả năng sáng tạo và hình thành những năng lực nhất định.

Từ khóa: *Giáo dục, kết hợp, nghệ thuật, STEAM, Toán học.*

1. Introduction

The arts are often considered to have many differences and contrasts with mathematics. Art has generally been associated with aesthetics, emotion, and senses. On the contrary, mathematics has long been associated with logic, precision, and truth (Eisner, 2004). It is necessary to understand that throughout history, the two disciplines have been more united than many might consider. Many characteristics believed to be antagonistic prove to be more convergent than divergent.


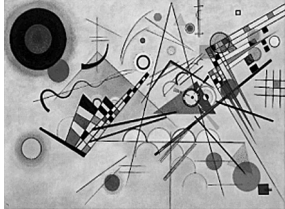
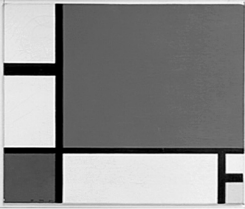
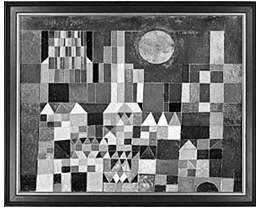
Oostra suggests that this intimate and seductive relationship between the two disciplines has been full of “nuances”. When intermingled, math and art have given way to the creation of infinitely new ways of seeing, feeling, explaining, and thinking about the world, nature, the universe, and reality. The creative process, intuition, and emotions commonly related to art play a fundamental role in mathematical thinking (Oostra, 2004). In contrast, structure, perspective, geometry and arithmetic were used consciously or unconsciously by artists when creating their works, art and mathematics are perfectly articulated through these proofs. Carlos Vasco also focused his interest on the relationship between art and mathematics. In an article, Vasco expressed the relationship in four fundamental sections: art as mathematics, mathematics in art, the art in mathematics, mathematics as art. The central thesis in Vasco’s work is that one must be a mathematician to discover creativity (Oostra, 2004). Seen from many dimensions, as proposed by Bejarano-García, it is possible to approach art through mathematics and vice versa. In its infinite dimensions, both art and mathematics provide commonalities facilitating an internalization of concepts and abstractions (Bejarano-García, 2015). Many examples of this dialogue can be found from different characters throughout history, and it has been possible to observe a trend in which people dedicated to mathematics usually tend to be dedicated to art in one of its multiple expressions. For example, the almighty genius Leonardo Da Vinci is a mathematician, an astronomer, an anatomist, a painter, an architect, a musician. The mathematician Charles Lutwidge was also a writer and photographer. Felix Hausdorff, a founder of topology, dedicated his youth to literature, and Colombian mathematician Otto de Greiff was a music lover.

Oostra (2004) said that: In art, people have

been using mathematical concepts consciously or unconsciously. Both math and art use patterns, shapes, lines, fractions, and proportions. These concepts are extremely important to both artists and mathematicians. We can come across many famous artworks made up of repeating sequences of simple geometric shapes, lines, curves... Or thanks to the concepts of the ancient Greeks such as harmony, symmetry, and symmetry with which mathematics, the study of numbers and related concepts, and art have worked together. Since then, many artists have relied on mathematical concepts as the basis for their work. Without knowledge of geometry, symmetry, and proportions, it is difficult to create aesthetic products. Mariño (2008) refers to the concept of mathematical symmetry as one of the most obvious connection between art and mathematics. In addition, symmetry emphasizes the aesthetic criterion as a basic point of art... It's not difficult to see the famous artworks by M.C. Escher, Wassily, Kandinsky, Paul Klee, Dalí... mathematical elements in their artwork.

MC. Escher (1898-1972) was a Dutch painter, best known for his mathematically-inspired visual illusions, his work often featuring themes of impossible architecture, spatial dimensions, endless repetition, striking an extraordinary visualization of mathematical principles and ideas. Or Russian painter Wassily Kandinsky, famous for his abstract artworks inspired by mathematics such as lines, curves, geometries... Piet Mondrian with works that include geometries and colors. Paul Klee with works composed from basic geometries.

Artworks that are considered aesthetic, suitable for human beauty do not have to rely entirely on creativity, free to fly without standards. Recognized famous works often have ratios that conform to the golden standard (known as the divine ratio, an irrational constant in algebra with an approximate value of 1.618). The sculptor Phidias, who used this ratio to build one of the famous architecture, the Parthenon, or the highly appreciated and loved works of painting, sculpture, and architecture, are attested to what we have in common is that it fits this ratio. The products of the applied arts industry, technology architecture... In addition to the aesthetic beauty, there are requirements for the use function, the structure, the durability of the materials to serve the needs of the customers of human life. They are eloquent examples of the combination of art and science, especially mathematics.

			
<p>Pic 1. M.C. Escher-Convex and Concave, 1955 Source: mcescher.com</p>	<p>Pic 2. Wassily Kandinsky-Composition 8, 1923 Source: wassilykandinsky.net</p>	<p>Pic 3. Piet Mondrian-Red, Blue and Yellow, 1930 Source: pietmondrian.org</p>	<p>Pic 4. Paul Klee-Castle and Sun, 1928 Source: paulklee.net</p>

Therefore, the relationship, impact, and mutual support between them are increasingly appreciated and applied in many industries and fields, especially the field of education. Up to now, it is not difficult to realize that the modern education of the world and Vietnam in particular is moving towards an interdisciplinary and integrated combination, the role of the arts and basic sciences is gradually changing equal positions to train generations to develop comprehensively in thinking and perception.

2. The relationship between art and mathematics in education

“Principles for the Development of a Complete Mind: Study the science of art. Study the art of science. Develop your senses - especially learn how to see. Realize that everything connects to everything else.”- Leonardo da Vinci's Notebook (1452-1519); (Atalay, B., & Wamsley, K., 2008, p. 21). Da Vinci brought science and art together. He felt that artists should be adept at mathematics and geometry so as to accurately demonstrate perspective in their art (depth, relationships between objects).

The use of linkages and interactions between disciplines is currently being focussed. One of the movements advocating this connection is STEAM (short for Science, Technology, Engineering, Arts and Math), which is an initiative to support the integration of the arts into learning traditional science (Ernest, 2016). The main goal in integrating the arts with these disciplines is to promote the transfer of knowledge between different disciplines to enhance student learning (Catterall, J.S.; Chapleau, R.; Iwanaga, 1999); (Deasy, R.J. Critical, 2002).

The intersection between arts and STEM is this: “Without art and science, our world would be a dull place and creativity would see the light of day

less often” (Brown, 2011, p. 24). Most creators do not let their work be limited to one field, they draw inspiration from the connection between fields. This kind of cross-disciplinary thinking resonates with the STEAM model, drawing fundamentals from science and math, and encouraging creativity from the arts (Mishra, Henrickson, & the Deep Play Research Group, 2012).

In the field of education, efforts have been made to establish a dialogue between art and mathematics. Various investigations have been carried out over the years that, in one way or another, have achieved an approach between both types of knowledge. Generally, this approach has been based on finding a method for independently enhancing one of the two disciplines, using one as an instrument of the other. In this way, the relationship cannot be understood as a dialogue between knowledge if one discipline is highlighted over the other. Many different experiments have been carried out to find a method to improve teaching effectiveness between arts and mathematics, using one of two fields as a tool of the other. At the Universidad Nove de Julho in Sao Paulo, Brazil, a research presented the concept of the Fractal (Benoit B. Mandelbrot, 1983) and its potential in the teaching and learning of mathematics (Faria, R.W.S.; Maltempi, M.V. Padrões Fractais, 2012). Research is central to the idea that both mathematics and the arts are related to human activities, with expressions derived from intelligence, desires, ideas, and the need for expression. The findings suggest that the visual aspects related to art and mathematics can begin with the discovery of Fractal patterns.

At the Nove de Julho University in Sao Paulo, Brazil, an investigation was carried out, the main objective of which was to present the concept of a fractal pattern (Benoit B. Mandelbrot, 1983) and its

potential in the teaching and learning of mathematics (Faria, R.W.S.; Maltempi, M.V. Padrões Fractais, 2012). The research was fundamental to the idea that both mathematics and the arts are linked to human activities, with expressions derived from intelligence, desires, ideas, and representational needs. The findings revealed that visual aspects relating art to mathematics can be contemplated from the exploration of fractal patterns.

In Colombia, Zuluaga-Arango and Pérez (2015) applied a series of strategies designed to favor, through the arts, the teaching-learning processes of students from the San Carlos de La Salle Institute. The study sought to show the possibilities for mediating the teaching of mathematics through the plastic and performing arts. In this way, students can approach mathematical knowledge from less instrumental and operational positions, obtaining better results in the development of competencies.

Brooke and Nemirovsky (Ernest, 2016) conducted a study that strengthened the arguments needed to integrate art into math courses. It has been found that by creating math-inspired artwork, students are able to incorporate mathematics into a range of life experiences, exploring real-life situations that encourage mathematical thinking learns.

Many studies have confirmed from various different perspectives the advantages and possibilities that the arts bring in education. For example, Palacios (2006) made many arguments that in art in general and in fine art in particular, it is possible to create new perspectives on a problem. Luis de Tavira (2007) also considered the possibilities of the arts in education and how it can broaden perspectives on diversity. Through art, there are many different creative paths to achieve a goal, in which all factors creating dissimilarity are considered as each different, each taste and that is the driving force of the individual. Each student in the class to give their own views and plans.

At the Primary level, when basic scientific knowledge is gradually equipped to become the foundation for later grades, fine arts in particular and the arts in general provide students with valuable values emotional, aesthetic and creative thinking. The combination of math and art together, from any angle, will bring positive effects on the comprehensive intellectual development of children.

Boruga (2011) worked to reveal the impact origami can have in the educational process concerning behavior and learning. From a behavioral aspect, it was evidenced that students became friendlier and developed more patience and energy to carry out their activities. From a mathematical perspective, it could be demonstrated that logical thinking was enhanced in the students, consolidating in them some mathematical notions, especially geometric ideas. In addition, a better understanding of concepts such as fractions, shapes, and angles was reached by using mathematics in a different context, becoming a meaningful and pleasant experience for students. From an artistic perspective, it was found that students improved their aesthetic sense, creativity, and sensitivity to art, learning an appreciation for the work required to create something and, therefore, to value art.

In essence, man abstracts and expresses his reality through the knowledge found in reason and emotion through art. Diversity in teaching and learning that integrates subjects such as mathematics and the arts is beneficial. Furthermore, this allows instructors to expose students to aesthetic interests and knowledge-seeking experience, making art an effective pedagogical tool, a way out of the rut, providing students' ability to think, feel, and perceive reality and the world without limits or conditions. On the contrary, mathematics becomes the foundation for creativity.

In Vietnam, The 2018 Education Program has been implemented, a number of topics and lessons in the fine art subject 1, 2, 3 are related to mathematical knowledge, for example, the art book **“Ket noi tri thuc va cuoc song”** (Đinh Gia Le and Tran Thi Bien, 2020), there are some titles: “Sang tao tu nhung hinh co ban (Create from basic shapes)”, “Sang tao tu nhung khoi co ban (Create from the basics blocks)” in the 1st grade; “Su ket hop cua cac hinh co ban” (Combination of basic shapes); “Su ket hop thu vi cua khoi (Interesting combination of blocks)” in the 2nd grade; “Ve dep cua khoi (The beauty of the blocks)” in the 3rd grade. Fine art book 1 **“Canh Dieu”** (Pham Van Tuyen and Nguyen Thi Dong, 2020) has the title: “Sang tao voi cac hinh co ban, la cay, Nhung hinh khoi khac nhau (Creative with basic shapes, leaves, different shapes)”. These titles help to support the reciprocity between the two subjects, students are

familiar with basic geometry and blocks from art to shape products, and at the same time help them access knowledge and shapes of shapes, block before studying in higher grades. Thereby, programmers have integrated the knowledge of mathematics into the arts from the first grade, that the basics of mathematics are the premise for creativity in the arts.

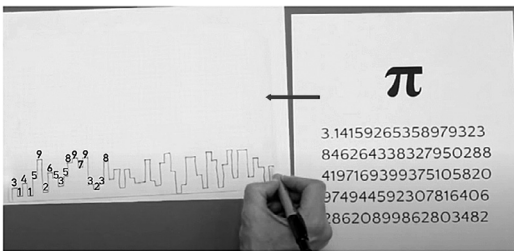



From some knowledge of math and fine arts in Primary School, the author would like to suggest some ideas that combine knowledge of mathematics to create some simple art products, suitable for students' knowledge and skills based on geometric figures, learning tools and some simple mathematical

concepts. When they have hands-on experience with these products, they have just gotten acquainted, reviewed, and experienced the math applied to shaping. This helps them expand their thinking, increase their creativity and for certain abilities.

3. Some examples

Example 1: Art from Pi




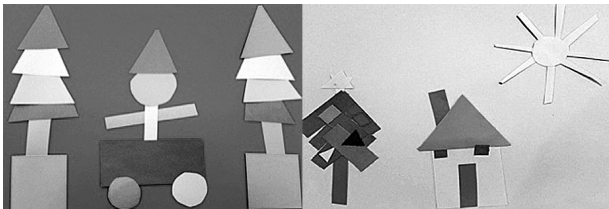
Pi (Symbol: π), also known as Archimedes' constant, is a non-periodic irrational number approximately equal to 3.1415926535897932384626433... represented by the Greek letter π since the mid-18th century. From this irrational number, we can apply it to create art products from its numbers as follows:

Step 1	Preparation: Plotted paper, wax or watercolor, scissors, glue, and Pi number (can be written or printed)	
Step 2	Draw columns with the height of the columns being cells on the drawing paper corresponding to the number in the sequence Pi in the order of the numbers. For example, number 3 will draw a column with 3 cells, 1 will draw a column of 1 cell, 4 will have 4 cells, 9 will have 9 cells as showed in the figure below.	
Step 3	- Color the columns of numbers just drawn and cut out - Draw the sky background on another sheet of paper with the color of your choice	
Step 4	Paste the pictures of the columns of numbers that have just been cut on the background of the paper that has been drawn with the sky background	
Step 5	Draw more symbols on the painted sky background to decorate the drawing more vividly and complete the drawing from π symbol	

Example 2: Creativity from geometries

Inspired by Paul Klee's The Castle and The Sun (paulklee.net), the structure of all objects in nature can

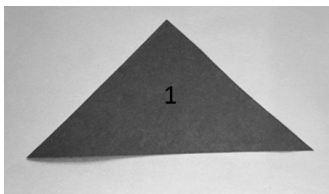
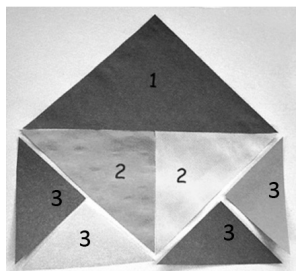
be reduced to basic geometries. From there, with basic geometries such as square, circle, triangle, rectangle, oval... can create countless aesthetic work.

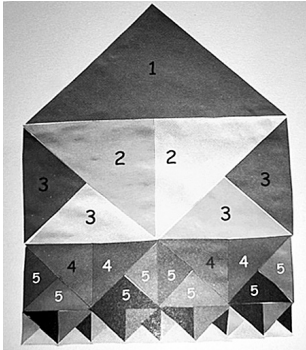
Step 1	Preparation: colored paper, scissors, glue, background paper of any color of your choice	
Step 2	Cut sheets of colored paper into basic shapes: square, circle, triangle, rectangle, ect. with different sizes and colors	
Step 3	Students based on their creativity can assemble basic shapes into many shapes of different things and phenomena in life.	
Step 4	Finally, fix the pictures on the paper, draw more details to complete the product if desired	

Creative use of basic shapes, flexible imagination, these products are not limited to age. From 1st graders to older students can experiment with geometry and art in this way to create individual aesthetic products.

Example 3: Endless triangle

Based on the idea of the applied work "Tesslation" by M.C. Escher (mcescher.com) is inspired by the fantasy of infinite continuity.

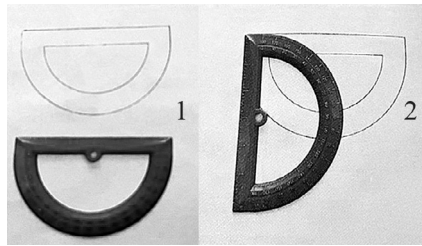
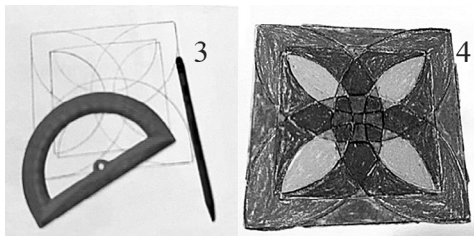
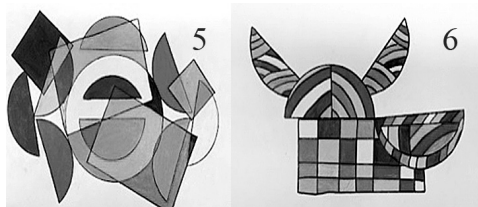
Step 1	Preparation: colored paper of your choice, scissors, glue, ruler, pen	
Step 2	Select the largest sheet of paper, cut it into the largest isosceles triangle with the color of your choice (no.1)	
Step 3	<ul style="list-style-type: none"> - Cut 2 more isosceles right triangles (No. 2) of the same size but different color with side half equal to the hypotenuse of triangle No. 1. Then arrange those 3 triangles to form a square. - Cut 4 triangles with an area equal to half the area of a triangle number 2 and arrange them in the shape as the side (number 3). 	

Step 4	<ul style="list-style-type: none"> - Cut 4 triangles (number 4) with the area half of the triangle number 3 and arrange as showed below. - Cut 8 triangles whose area is half the size of figure 4 (number 5). - Need to cut 8 small triangles half the number 5 and 16 smaller triangles. The picture can stretch to infinity or until you can't cut the following triangles any smaller. - Customize colors according to your preferences and use the color intensity to create visual effects. 	
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Example 4: Creativity from protractor

Inspired by the Takt-Sulayman Variation I in Frank Stella's protractor collection (artnet.com/

artists/frank-stella). This is an example of using a protractor to create shapes, in addition, the artist can use a variety of rulers, different learning tools to make products with arbitrary textures and colors.

Step 1	Prepare: drawing paper, measuring ruler, pencil, drawing color, all kinds of straight ruler, compass...	
Step 2	<ul style="list-style-type: none"> - Use the protractor as shown in the figure below and draw the shape of the protractor as shown in the illustration (Number 1). - Rotate the protractor 90 degrees so that one end of the ruler matches the angle of the first shape and redraw the shape of the ruler as shown (Number 2). 	
Step 3	<ul style="list-style-type: none"> - Repeating the drawing operation in step 2 four times will create a square with intersecting textures inside as shown (Number 3). - Color as you like for the picture you just drew (Number 4). <p>This is a simple example of using the learning tool to create shapes and textures that take advantage of the built-in straight and curved lines of the ruler.</p>	
Step 4	<p>With a variety of combinations of curved rulers, straight rulers, compasses, etc., we can create shapes with many different ideas and color them as we please. Learning tools become tools for unlimited creativity like the example illustrated above (Number 5, 6).</p>	

4. Conclusion

In this way, Best (2011) declares that the confluence between emotion and reason, and art and science, makes human beings believe and express their most intimate thoughts and feelings. In addition, these forms of expression also lead to others to do so, or at least understand and feel their thinking and their affectivity. Dewey (1938) declared that, from this conception where art and science, the subjective and the objective, and dialogue in a search

to generate new possibilities of knowledge, art as a means of expression becomes a new experience of the world, where neither art and science have existed for themselves, but both planes cross generate a series of intersections that open up new possibilities to understand and apprehend the world.

In addition, these forms of expression also make other people or at least understand and feel their thoughts and feelings, and initiatives and inventions are born based on this path. Science becomes the

truth, the basic foundation, and the art of bringing creations and discoveries into reality is based on those foundations. Art in general and the arts in particular has shown to be an approach in a different way from knowledge of mathematical and scientific concepts, not in terms of solving a formula or a problem, but in transforming it from a simple mathematical knowledge to a real understanding by concrete products, creating a connection between general and basic disciplines. With ideas for making art products inspired by the intersection of mathematics and fine arts, showing the connection between these two disciplines and still with the specific goal of developing students' competencies in school both aspects and know how to apply knowledge between these subjects together.

In summary, mathematics has certain roles in art such as music, dance, painting, architecture and sculpture, etc. And these subjects all have a strong connection with mathematics. Especially in connection with visual art, mathematics can provide artists with tools such as basic shapes, lines, angles, balance, proportions, space, etc. The works of art based on the basic elements of mathematics and art open up countless variations, showing the blending of the two disciplines. With some ideas from mathematics to create artistic products, which are illustrative, oriented, and instructive for students, all are based on practicality: easy-to-find, safe tools, suitable to the skills of primary school students, the results are diverse in terms of shaping. Students' creative abilities will be opened up with countless combinations of geometry, lines, colors, etc. The author will continue to have more practical experiments to show that the combination of mathematics and fine arts brings positive impacts to students in developing creative and critical thinking abilities only for primary school students.

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