MODIFIED DISTORTED BORN ITERATIVE METHOD WITH MULTI-DISTANCE RECEIVER FOR ENHANCED-QUALITY RECONSTRUCTION IN ULTRASOUND TOMOGRAPHY

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Abstract

Ultrasound tomography would allow ultrasound imaging to reach its full potential, compared to X-ray, MRI, etc. It is based on inverse scattering and has the capability to detect structures whose sizes are smaller than the wavelength of the incident wave, as opposed to conventional one using pulse-echo technique. Some material properties such as sound contrast and attenuation are very useful to detect small objects. The Distorted Born Iterative Method (DBIM) based on first-order Born approximation is an efficient diffraction tomography approach. In this paper, we propose a modified DBIM approach in order to improve the reconstruction quality by using the information of the multi-distance receiver. The normalized error of the proposed approach is significantly reduced.
Keywords: *Ultrasound, tomography, inverse scattering, Distorted Born iterative method (DBIM), multi-distance receiver.*

Tái tạo ảnh siêu âm cắt lớp sử dụng phương pháp lặp vi phân Born sử dụng dữ liệu thu đa khoảng cách

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Tóm tắt

Chụp siêu âm cắt lớp là một phương pháp tạo ảnh tiềm năng so với các phương pháp chụp ảnh X-quang, MRI,... Kỹ thuật này dựa trên sự tán xạ ngược và có khả năng phát hiện các cấu trúc có kích thước nhỏ hơn bước sóng của sóng tới, trong khi đó, kỹ thuật chụp ảnh thông thường sử dụng sự phản hồi của sóng siêu âm không thực hiện được. Một số đặc tính của mô như độ tương phản và độ suy hao âm được sử dụng để phát hiện các đối tượng nhỏ. Phương pháp lặp vi phân Born (DBIM) dựa trên xấp xỉ Born bậc 1 là một phương pháp chụp cắt lớp nhiễu xạ hiệu quả. Trong bài báo này, chúng tôi đề xuất phương pháp DBIM cải tiến nhằm nâng cao chất lượng tái tạo ảnh bằng cách sử dụng dữ liệu thu đa khoảng cách. Kết quả mô phỏng số cho thấy lỗi chuẩn hóa của phương pháp đề xuất giảm đáng kể so với phương pháp DBIM truyền thống.
Từ khóa: *Siêu âm, chụp cắt lớp, tán xạ ngược, phương pháp lặp vi phân Born (DBIM), dữ liệu thu đa khoảng cách.*

1. Introduction

Ultrasound imaging is widely used as a tool for medical diagnosis. The most commonly used ultrasonic imaging method is sonography or B-mode imaging [1]. B-mode imaging uses data in reflection mode to produce anatomical grayscale images. The brightness of each pixel is proportional to the amplitude of the logarithmically compressed envelope of the echoes produced by tissues. Spatial localization is performed using the pulse-echo principle.

However, the propagation of acoustic waves is a much richer phenomenon than simple reflections of acoustic echoes. When an incident acoustic wave encounters an inhomogeneity, some of the energy is scattered in every direction. The acoustic tomography problem consists of estimating the distribution of acoustic parameters (i.e., sound speed, acoustic attenuation, density, etc.) of the scatterer given a set of measurements of the scattered field by inverting the wave equation. Therefore, acoustic tomograms display quantitative information of the object under examination [2].

Inverse scattering-based techniques have high computational complexity which is the biggest barrier to the introduction of commercialized tomography devices. Hence, state-of-the-art inverse scattering techniques focus primarily on reducing the computational complexity and constantly improving the quality of imaging. Most of research works on ultrasound tomography are based on Born approximation. Born Iterative Method (BIM) and Distorted Born Iterative Method (DBIM) are well-known for diffraction tomography [3]. BIM is robust to noise, but its computation is quite high. DBIM is more sensitive to noise, though it offers faster convergence as compared to that of BIM. In addition, the computational complexity of these methods is high due to their use of iterative forward and inverse processes. In [4], authors present two variations of the standard Iterative Born method called the coarse resolution initial value (CRIV) method and the quadric-phase source (QS) method. For low-contrast objects, both algorithms yield accurate reconstructions at lower computational cost. However, for high-contrast objects the QS method fails. Frequency-domain interpolation method in [5] shows that good results in terms of reconstruction quality can be obtained by using bilinear interpolation after increasing sampling density using zero-padding. However, the main limitation of this method is its convergence properties.

In [6], edge detection during the iterative process was introduced in order to speed up the convergence and to enhance the quality of reconstruction, but the complexity and noise sensitivity issues remain. The multi-level fast multi pole algorithm (MLFMA) was applied to the forward solver for further speed up the reconstruction process [7]. However, MLFMA requires high set-up costs that make it difficult to implement in practice. In this paper, we propose a modified DBIM approach to improve the reconstruction quality by using the information of the multi-distance receiver. The normalized error of the proposed approach is significantly reduced.

2. Distorted born iterative method

A measurement configuration is set up for tranducers T-R (i.e. transmitters and receivers), located in a circle around the object in order to obtain the scattered data (see Fig.1). Each tranducer can both transmit and receive. At an instance, only one transmitter and one receiver are active to for a corresponding measured data value. This data was processed using DBIM to reconstruct the sound contrast of scatters. In this way, any tissue can be detected in this medium.

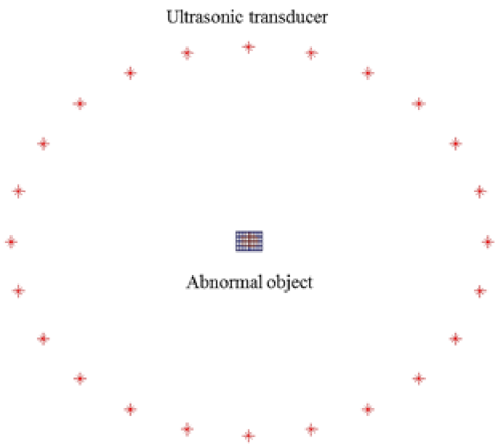


Fig. 1. Geometrical and acoustical configuration

Assuming that there is an infinite space containing homogeneous medium (M_1) such as water whose background wave number is k_0. There is also an object (M_2) with constant density and a wave number k(r) put inside this medium. The wave equation of the system can be shown as:

$$\nabla^2 p(\vec{r}) + k_0^2 p(\vec{r}) = -O(\vec{r})p(\vec{r}), \quad (1)$$

where

$$O(\vec{r}) = k_1^2 - k_0^2 - \rho(r)^{1/2}\nabla^2\rho(r)^{-1/2}, \quad (2)$$

$$k_1(r) = \frac{\omega}{c_1(r)} + i\alpha(r). \quad (3)$$

$O(\vec{r})$ is the ideal object function that needs to be recover (i.e. abnormal object as shown in Fig. 1), $k_1(r)$ is the wave number, $c_1(r)$ is the sound speed, $\alpha(r)$ is the attennuation, $\rho(r)$ is the density, and ω is the angular frequency.

The incident wave is denoted as $p^{inc}(r)$, the scattered wave can then be obtained as follows:

$$p^{sc}(r) = \int_\Omega O(r')p(r')G_0(k_0, r-r')\,dr' \quad (4)$$

where $p(r)=p^{inc}(r)+p^{sc}(r)$ is the total pressure inside the inhomogenerous area Ω and $G_0(k_0, r-r')$ is the Green's function. When the background is homogenerous, G_0 is the 0-th Hankel function of the first kind:

$$G_0(k_0, r-r') = \frac{-i}{4}H_0^{(1)}(k_0|r-r'|) = \frac{-i}{4}\sqrt{\frac{2}{\pi k_0|r-r'|}}\,e^{i(k_0|r-r'|-\pi/4)}. \quad (5)$$

The total pressure can be expressed as

$$p(r) = p^{inc}(r) + \int_\Omega O(r')p(r')G_0(k_0, r - r')\,dr' \quad (6)$$

One of the effective solutions to solve Eq. (6) by discreting is Method of Moment (MoM). The pressure in the grid points (see Fig.1) can be computed in vector form with size $N^2\times 1$:

$$\bar{p} = (\bar{I} - \bar{C}.D(\bar{O}))p^{inc}. \quad (7)$$

The exterior points give scatter vector $N_tN_r\times 1$:

$$\bar{p}^{sc} = \bar{B}.D(\bar{O}).\bar{p}\,, \quad (8)$$

where $\bar{B}$ is the matrix with Green's coefficient $G_0(r,r')$ from each pixel to the receiver, $\bar{C}$ is the matrix with Green's coefficient $G_0(r,r')$ among all pixels, $\bar{I}$ is identity matrix, and D(.) is an operator that transform a vector into a diagonal matrix.

There are two unknown variables are $\bar{p}$

and $\bar{O}$ in equations (7) and (8). In this case, the first Born approximation has been applied and the forward equation (7) and (8) can be rewritten (Lavarello and Oelze, 2009):

$$\Delta p^{sc} = \bar{B}.D(\bar{p}).\Delta\bar{O} = \bar{M}.\Delta\bar{O}, \quad (9)$$

where $\bar{M} = \bar{B}.D(\bar{p})$. For each transmitter and receiver, we will have amatrix $\bar{M}$ and a scalar value Δp^{sc}. Realize that unkown vector $\bar{O}$ has $N \times N$ variables which are equal to the number of pixels in RIO. Object function can be estimated by iterations:

$$\bar{O}^{n} = \bar{O}^{(n-1)} + \Delta\bar{O}^{(n-1)}, \quad (10)$$

where $\bar{O}^{n}$ and $\bar{O}^{(n-1)}$ are object functions at present and previous steps, respectively; $\Delta\bar{O}$ can be estimated by solving Tikhonov regularization problem (Gene et al., 1999):

$$\Delta\bar{O} = \arg\min_{\Delta\bar{O}} \left\| \Delta\bar{p}^{sc}{}_{t} - \bar{M}_t \Delta\bar{O} \right\|_2^2 + \gamma \|\Delta\bar{O}\|_2^2, \quad (11)$$

where $\Delta\bar{p}^{sc}$ is the (N_tN_r×1) vector, contains the difference between predicted and measured scattered ultrasound signals; $\bar{M}_t$ is system matrix ($N_tN_r \times N^2$) formed by N_tN_r different matrixes $\bar{M}_t$; and γ is the regularization parameter. The DBIM procedure is presented in Algorithm 1.

Algorithm 1. The Distorted Born Iterative Method DBIM
Choose initial values: $\bar{O}_{(n)} = \bar{O}_{(0)}$ and $\bar{p}_0 = \bar{p}^{inc}$ (10)
For $n = 1$ *to* N_{DBIM}, **do**
1. Calculate $\bar{B}$ and $\bar{C}$
2. Calculate p, $\bar{p}^{sc}$ corresponding to $\bar{O}_{(n)}$ using (3,
3. Calculate $\Delta\bar{p}^{sc}$ using (7)
4. Calculate $\Delta\bar{O}_{(n)}$ using (9)
5. Calculate $\bar{O}_{(n+1)} = \bar{O}_{(n)} + \Delta\bar{O}_{(n)}$
End For

$$RRE = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{|C_{ij} - \hat{C}_{ij}|}{C_{ij}} \quad (12)$$

3. The proposed method

The complexity of the reconstruction system depends on the number of pixels N×N, the number of iterations N_{iter}, the number of transmitters N_t and receivers N_r. In this paper, we define the number of iterations N_{sum} and the number of pixels N^2 are constants. The number of iterations implemented with the first single-distance receiver (FR) is denoted by N_{r1}, so the number of iterations implemented with the second single-distance receiver (SR) is $N_{r2} = N_{sum} - N_{r1}$. The Relative residual error (RRE) is utilized to evaluate the reconstructed performance. An MDR-DBIM is shown completely in Algorithm 2.

Algorithm 2. Proposed MDR-DBIM
1. Choose initial values: $\bar{O}_{(n)} = \bar{O}_{(0)}$ and $\bar{p}_0 = \bar{p}^{inc}$ as shown in (11)
2. **For** n =1 to N_{fl}, do
3. Calculate p, $\bar{p}^{sc}$, $\bar{B}^r$ corresponding to $\bar{O}_{(n)}$ with FR using (5), (6)
4. Calculate $\Delta\bar{p}^{sc}$ using (7)
5. Update value $\Delta\bar{O}_{(n)}$, satisfying (9)
6. Calculate $\bar{O}_{(n+1)} = \bar{O}_{(n)} + \Delta\bar{O}_{(n)}$
7. **End For**
8. **For** $n = N_{fl}+1$ to N_{iter}, do
9. Calculate p, $\bar{p}^{sc}$, $\bar{B}^r$ corresponding to $\bar{O}_{(n)}$, with SR
10. Calculate $\Delta\bar{p}^{sc}$ using (7)
11. Calculate $\Delta\bar{O}_{(n)}$, satisfying (9).
12. Calculate $\bar{O}_{(n+1)} = \bar{O}_{(n)} + \Delta\bar{O}_{(n)}$
13. Calculate the RRE using (12)
14. **End for**

RRE is investigated by varying Nr1 as shown in Algorithm 3:

Algorithm 3. Investigate the dependence of the reconstructed performance on N_{r1}
1. **For** N_{r1} = 1 to (N_{iter} – 1), do
2. Go to Algorithm 2
3. Restore the computed RRE
4. **End For**
5. Determine the best choice of N_{r1} corresponding to the minimum value of RRE.

4. Simulation and results

4.1. Find the best value of N_{r1}

Simulation parameters: Frequency f = 1 MHz; N_{sum} = 8; N = 22; Scattering area diameter = 10 mm; Sound contrast 30 %; Gaussian noise 10%; Distance from transmitters to the center of the object is 50 mm. Distances from the first and second receivers to the center of the object are 60 mm and 70 mm, respectively.

When using the iteration method for image generation, there are two approaches to exit the loop. First, fix the necessary number of iterations, N_{sum}, and the image generation process will continue until it reaches N_{sum}. Second, set a threshold for the image quality that the system needs to achieve, and then the image generation process will continue until the image quality reaches that threshold. However, in this solution, the method being used may require a small number of iterations to reach the threshold, or it may take a significant number of iterations or never converge, depending on the convergence rate of the method. Therefore, in this work, we choose the fixed number of iterations approach to observe the convergence rate after the first N_{sum} iterations. We choose N_{sum} = 8 because in the traditional DBIM method, convergence occurs very quickly after the first 3 iterations. The next three iterations (from iteration 4 to 6) show continued improvement in convergence speed, although it is significantly slower than the first 3 iterations. Subsequently, from iteration 7 onwards, the normalized error decreases slightly, but not significantly, indicating a saturated state. Continuing to increase the number of iterations does not result in a substantial reduction in the normalized error. Therefore, stopping the iterations in the early stage of the saturation process is necessary (i.e., at iteration 8).

The incident pressure for a Bessel beam of zero order in two-dimensional case is

$$\bar{p}^{inc} = J_0(k_0|r - r_k|), \tag{13}$$

where J_0 is the 0th order Bessel function and $|r\text{-}r_k|$ is the distance between the transmitter and the kth point in the ROI.

Fig. 2 is the ideal object function O(r). The object is placed at the center of the meshing area.

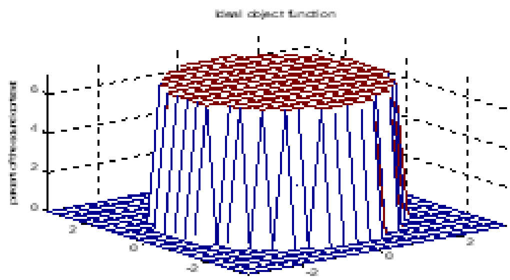


Fig. 2. Ideal object function.

Fig. 3 presents the normalized errors of the MDR-DBIM through iterations corresponding to different N_{r1}s. The value of N_{r1} which offers the smallest error is 4 (i.e. N_{sum}/2).

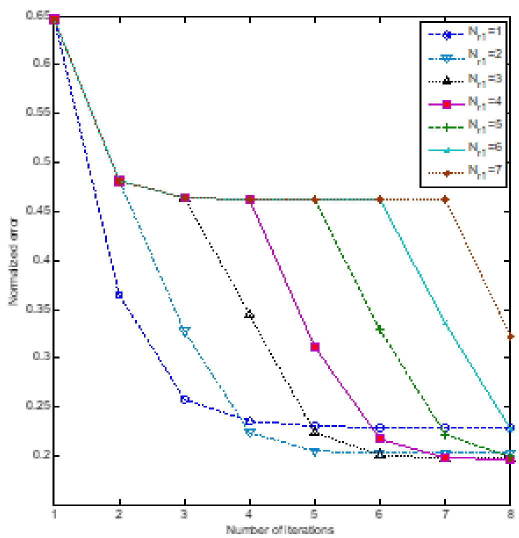


Fig.3. Normalized errors of the MDR-DBIM through iterations corresponding to different N_{r1}s

4.2. Simulation of DBIM and MDR-DBIM

To quantify the efficiency of the proposed method, we acquire the object functions for a series of iterations. Then, the error in the reconstructed image is determined and compared to the original image on each iteration. Suppose that *m* is a V×W original image (i.e. ideal object function) and $\hat{m}$ is the reconstructed image. The error can be defined as:

$$\varepsilon = \frac{1}{V\times W}\sum_{i=1}^{V}\sum_{j=1}^{W}\frac{|m_{ij}-\hat{m}_{ij}|}{|m_{ij}|} \qquad (14)$$

Fig. 4 presents the normalized error performances of three different approaches. The proposed approach offers a better convergence rate and a reduced normalized error. Fig. 5 shows the reconstructed results of the different approaches through iterations. Visually, we can see that the proposed method gives better convergence results and the restored image is quite close to the ideal objective function.

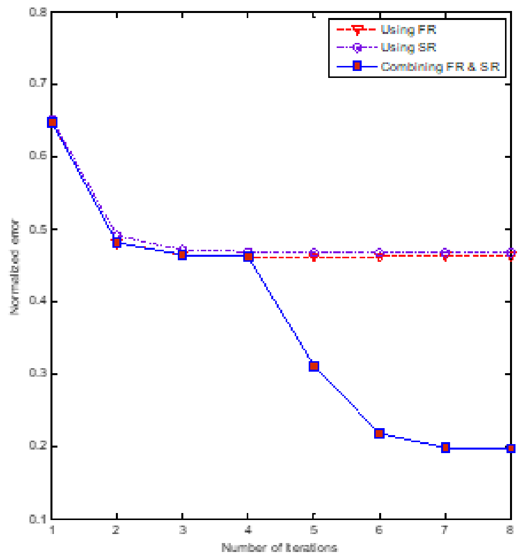
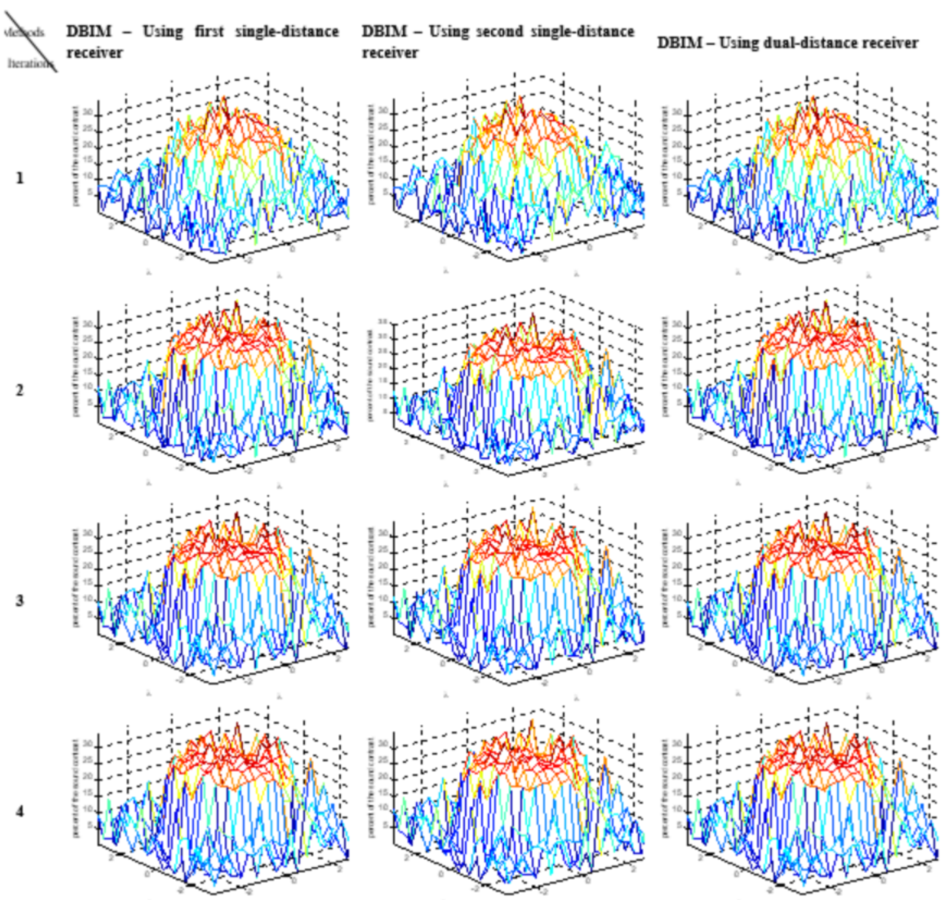


Fig.4. Error comparison of MDR–DBIM and DBIM after N_{iter} iterations



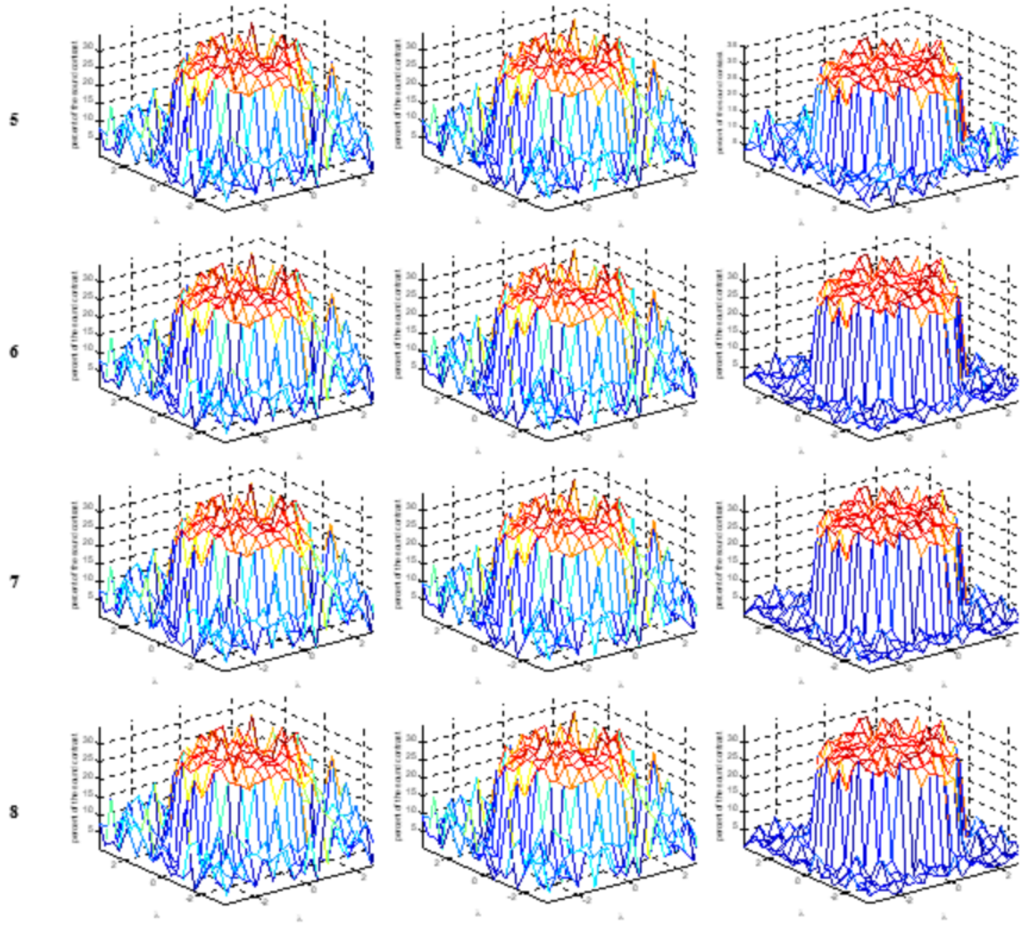


Fig. 5. The reconstructed results of the different approaches through iterations

5. Conclusions

Inverse scattering utilizing DBIM is a popular technique which can be used to resolve structures which are smaller than the wavelength of the incident wave, as opposed to conventional ultrasound imaging using echo method. This paper has successfully applied the information of the multi-distance receiver in order to improve the quality of the image reconstruction. This method also offers a very simple setting compared to the others. Thus, it can avoid many kinds of measurement errors. Simulation scenarios of sound contrast reconstruction were conducted to prove the good performance of this method. The scheme can be further developed by 3D reconstruction and experiment.

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References

[1]. C.F. Schueler, H.Lee, and G. Wade,"Fundamentals of digital ultrasonic processing," *IEEE Transactions on Sonics and Ultrasonics,* vol.31, no.4, pp.195–217, July 1984.

[2]. Lavarello Robert, *"New Developments on Quantitative Imaging Using Ultrasonic Waves",* Doctoral dissertation, University of Illinois at Urbana-Champaign, 2009.

[3]. Devaney AJ, "Inversion formula for inverse scattering within the Born approximation", *Optics Letters* 7:111-112.

[4]. Lujiang Liu, Xiaodong Zhang, and Shira L. Broschat, "Ultrasound Imaging Using Variations of the Iterative Born Technique", *IEEE transactions on ultrasonics, ferroelectrics, and frequency control,* vol. 46, no. 3, May 1999.

[5]. Jonathan Mamou, Michael L.Oelze, "Quantitative Ultrasound in Soft Tissues", Springer Science + Business Media Dordrecht 2013. DOI:10.1007/978-94-007-6952-6_2.

[6]. Haddadin OS, Ebbini ES, "Solution to the inverse scattering problem using a modified distorted Born iterative algorithm", *IEEE Ultrasonics Symposium,* 1995, 1411-1414. DOI: 10.1109/ULTSYM.1995.495819.

[7]. Hesford AJ, Chew WC, "Fast inverse scattering solutions using the distorted Born iterative method and the multilevel fast multipole algorithm", *J Acous Soc America,* 128:679-690.

[8]. Lavarello R, Oelze M, "Density imaging using a multiple-frequency DBIM approach", *IEEE Trans UltrasonFerroelectrFreq Control,* 2010 Nov; 57(11):2471-9. doi: 10.1109/TUFFC.2010.1713.

[9]. Gene H. Golub, Per Christian Hansen, and Dianne P. O'Leary, "Tikhonov Regularization and Total Least Squares", *SIAM Journal on Matrix Analysis and Applications.* Vol. 21 Issue 1, Aug 1999.

[10]. R. J. Lavarello and M. L. Oelze, "Tomographic Reconstruction of Three-Dimensional Volumes Using the Distorted Born Iterative Method", *IEEE Transactions on Medical Imaging*, 28, 2009, pp. 1643-1653.

[11]. Jegou, H., Douze, M., & Schmid, C., "Product quantization for nearest neighbor search. Pattern Analysis and Machine Intelligence", *IEEE Transactions* on, 33(1), 117-128, 2011.