

EVALUATE ERROR CORRECTION PERFORMANCE OF BINARY REPEAT ACCUMULATE CODE AND QUASI CYCLIC LOW-DENSITY PARITY-CHECK CODE IN 5G NEW-RADIO

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ABSTRACT

Low-density parity-check (LDPC) codes have been accepted as the standard codes for the 5th-generation New Radio (5G NR) standard. The performances of a linear block code LDPC based on the sparse matrix are extremely near to the Shannon limit. Ever since LDPC was invented, continuous encryption algorithms have been introduced, in which two more dominant ones, Quasi-cyclic (QC) LDPC and Repeat Accumulate (RA) LDPC, are referred to as the simple decoding algorithms. In this paper, the construction of the two algorithms will be analyzed, and the performances of these algorithms will be compared on the 5G standard. From there, it can be assessed when transmitting information on 5G standard, QC code will achieve better performances, but it also gets higher complexity compared to RA code.

Keywords: Low-density parity check code (LDPC), Repeat Accumulate (RA) code, Quasi-cyclic (QC) code, 5G standard, BER-SNR performance.

1. INTRODUCTION

A low-density parity-check (LDPC) code is a decoding algorithm introduced in the late 20th century by Gallager [1]. After being demonstrated by Mackay and Neal for superior ability to check and correct signal errors showing the ability to approach the Shannon limit, extensive research was conducted. In last decades, these codes have been used in various communication systems, including digital video broadcasting (DVB) [2], the wireless local area network (WLAN, IEEE 802.11n) [3], wireless radio access network (WRAN, IEEE 802.22) [4], and the advanced television system committee (ATSC) [5]. More specifically, LDPC codes have been selected as the coding scheme for the 5G enhanced mobile broadband (eMBB) data channel [6].

LDPC code is a block encoding and decoding algorithm, which expands from parity bit to parity matrix. This construction increases the complexity of the algorithm, on the other hand, the quality of the transmission of digital information is better. After being introduced by Gallager in 1963, the decoding method is from the H matrix using reducing operator Gauss Jordan to convert it into a generator matrix G then using a matrix multiplication algorithm. This solution shows the high complexity of coding, so two low complexity algorithms were born, RA code and QC code.

Iterative code - RA code, which combines aspects of both Turbo and LDPC codes, was first introduced by Abbasfar *et al.* in 2007 [7]. RA codes are high-speed capacity-achieving codes with bounded complexity per message bit for both encoding and decoding. Therefore,

these codes have been used for encoding on many communication standards such as DVB-S2 [2], WiMax [8], and so on. More and more research is being conducted to expand and develop an RA code algorithm or an RA based on the IRA code to increase the quality of the algorithm [9]. Besides being an algorithm with a simple structure, fast decoding speed, the algorithm also shows that it has a relatively good performance.

Quasi-cyclic LDPC (QC-LDPC) codes are generally specified by an array of circulants [10, 11]. They possess various advantages such as easy hardware implementations of encoding and decoding, fast decoding convergence, and lower error-floor using simple shift registers and logic circuits. QC code is a sub-class of LDPC code, which has a simple decoding algorithm with high performances [10, 12]. This algorithm has been performed at different standards. QC-LDPC codes have extremely been investigated and found in many applications such as different digital communication and storage systems [13].

Based on the 3rd generation partnership project 3GPP specification 38.212. Rel 15 [6, 14], the choice of a matrix for 5G will also be mentioned in this article. One of the goals in today's transmission of information is reliability or, in other words, the quality of channel encoding. The selection of the optimal algorithms in both encoding and decoding is essential in addition to improving the quality of the transmitter and receiver hardware. There are several studies underway to apply different encoding algorithms on different transmission standards, or even two algorithms on the same transmission standard as [8, 10, 15-18]. However, there is no project to evaluate the ability and code quality affecting the decoding process on the receiver. To select the most efficient algorithm, quality analysis based on bit-error-rate (BER) and signal-to-noise ratio (SNR) will clarify aspects of the codeword length and bit rate of each LDPC code.

In this article, we will describe how to build two algorithms RA code and QC LDPC code over galois fields GF(2) (binary). We will also show how to build a parity matrix according to 5G NR standards based on the 3GPP specification. The highlight in this paper is comparison quality between two encoding algorithms, the RA and QC code, to evaluate which algorithm should be selected to use for the 5G standard. Messages are decoded on two algorithms using binary phase shift keying (BPSK) modulation, transmitted over the additive white gaussian noise (AWGN) channel, and the belief propagation (BP) decoding algorithm.

The rest of this paper is organized as follows. RA-LDPC encoding algorithms, graph model for construction RA code are outlined in section 2. Section 3 presents the definitions, basic concepts, and some analysis capabilities to reduce the complexity of QC-LDPC codes. The way to build base graph (BG) BG1 and BG2 for a parity matrix will be discussed in Section 4. The quality (BER versus SNR) of both LDPC codes will be analyzed with different code lengths and rates in section 5. Finally, section 6 concludes this paper.

2. REPEAT ACCUMULATE ENCODER

Repeat-accumulate codes (RA codes) are a low complexity class of error-correcting code. The RA code was invented by Abbasfar *et al.* [7]. In the RA code, the construction of a parity matrix is divided into two parts, including a square matrix and a matrix defined based on the interleaver.

The RA code has a H matrix consisting of two parts:

$$H = (H_1, H_2) \tag{2.1}$$

Where the matrix H_2 is a lower triangular matrix, which is shown in Figure 1, its size $m \times m$. This is a fixed matrix form for different RA codes and whose size depends on the codeword length that we need to design. When H_1 is a sparse matrix (by definition of LDPC), then the computation is just the addition (or XOR). Matrix H_1 is an interleaver-based matrix.

When we have interleaver $\Pi = (\pi_1, \pi_2, \dots, \pi_n)$ with $\pi_i \neq \pi_j, \forall i \neq j$ and $n = Kq$. Each π_i specifies that the (i/a) -th row of H_1 has one in the (π_i/q) -th column, where x denotes the smallest integer greater than or equal to x [15].

$$H_2 = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & 1 & 0 \\ 0 & 0 & 0 & \dots & 0 & 1 & 1 \end{bmatrix}$$

Figure 1. Matrix H_2 for RA code

The indices a and q are the weights of all row and column in the matrix H_1 , respectively. The relationship between the weights of the matrix H_1 and code rate is represented by the formula:

$$N = K \left(1 + \frac{q}{a}\right) \quad (2.2)$$

$$R = \frac{a}{a+q} \quad (2.3)$$

RA code or (q, a) is regular if the weights of all the rows of H_1 are the same, a , and the weights of all the columns of H_1 are the same, q .

In the RA code, the H matrix has a fixed, besides, the construction of the Tanner graph is also relatively different from other codes. Figure 2 shows the Tanner diagram of the RA code, which shows the simple coding capabilities of the RA code, reduces the computation complexity of the encoder, thereby building the encoder at a high speed. From this figure, we see the relationship between LDPC code and Turbo code [16]. The reason is that the RA code is a Turbo-like code, but when applied to LDPC, it is expected to see its ability to quickly encode. It also shows how to build an encoder circuit for RA code [17].

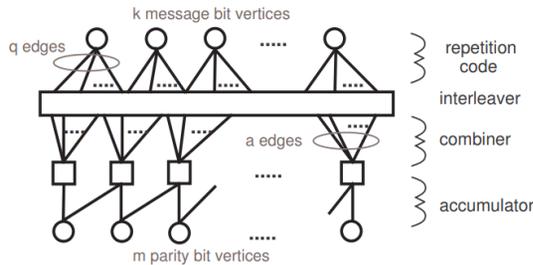


Figure 2. RA code Tanner graph.

The construction of the RA code consists of the following steps:

- Take K source bit
- Each bit is repeat q times
- Construction interleaver
- Combiner
- Mapping repeated from step 2 to combiner

The following coding flow will show specific each step and encoder circuit will show in Figure 3:

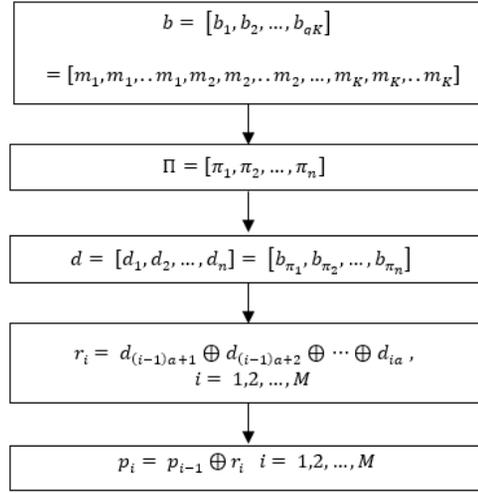


Figure 3. Coding flow for RA code.

As coding flow show that for repeating bits q times, in the hardware design aspect we only need to use buffer gates to build. The quality of the RA code is largely determined by the indices a and q as well as by the interleaver [18]. It is important in the RA code to construct a H_1 matrix based on an interleaver. For a large code, a randomly ordered interleaver will show high performance. The way to build a suitable interleaver includes the π -rotation, combinatorial interleaver methods mentioned in [7]. The combiner and mapping are built on modulo 2 addition or hardware XOR gates. Through the above analysis, we see the ability of RA to simplify the encoder while achieving high performance.

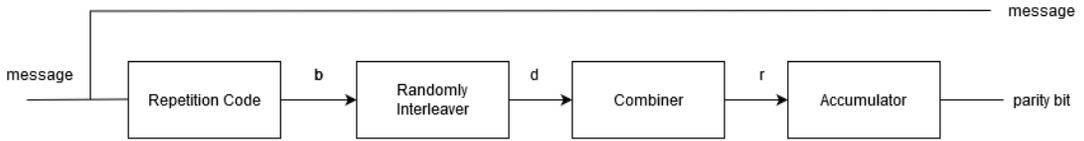


Figure 4. RA encoder block diagram.

3. QUASI-CYCLIC LDPC CODE

Several attempts have been made to build the QC code over $GF(2)$ [12, 19]. A code is called quasi-cyclic when with a cyclic movement of a codeword with a displacement of 1, a codeword is obtained. The simplest quasi-cycle is the coded row, which has weigh L , described by the parity check matrix [10, 20].

$$H = [H_1, H_2, \dots, H_L] \quad (3.1)$$

where H_1, H_2, \dots, H_L are binary $v \times v$ circulant matrices

QC code has been studied by Townsend and Weldon [19], Chen [15] and after its invention, there are many other research articles such as [10, 11, 19] gradually improving QC code. According to the information and coding theory, one of the things to keep in mind when designing channel coding is Hamming distance. In simple terms, Hamming distance or maximum distance is the difference between codewords. When the decoder executes, the larger the Hamming distance, the less likely it is to convert the correct codeword to another

codeword that also satisfies the error check condition in LDPC. The maximum distance of the QC code mentioned in [19] shows that the QC code is a good code with Hamming distance much improved compared to other algorithms.

In matrix H presented in Equation 3.1, in sub-matrices always exists a reversible matrix, called H_L . Thus, we can infer the generator matrix G based on the formula 3.2 [12, 13, 20].

$$G = \begin{bmatrix} & (A_l^{-1}A_1)^T \\ I_{v(l-1)} & (A_l^{-1}A_2)^T \\ & \vdots \\ & (A_l^{-1}A_{l-1})^T \end{bmatrix} \quad (3.2)$$

Besides, due to the unique characteristics of the code, circulant matrices can be represented by the polynomial as follows: $a(x) = a_0 + a_1x + \dots + a_{v-1}x^{v-1}$ with indicators a_0, a_1, \dots, a_{v-1} are the first row in the H_i matrix. Thus, instead of dealing with matrices, we can replace signal processing on algebraic equations. Furthermore, we can find the inverse matrix corresponding to finding an inverse equation is mentioned in the study of Baldi *et al.* [21].

Because QC code has its characteristics in construction, with a codeword length or a specified code rate, there will be a separate matrix standard for the QC code. In which, 3GPP is one of the associations that regulates communication structures, such as the QC code matrix for WiMax, DVBS2, 5G standards [3, 8, 14, 22, 23]. Thus, instead of the traditional method of building the parity matrix H and using a complex Gauss-Jordan elimination to create a generator matrix G , with a definite structure QC code makes the LDPC encoding faster and simpler.

Nowadays, due to traditional standards and large information transmission needs, it is difficult to create the matrix H . Therefore, instead of giving the matrix H , we build a base graph matrix (BG matrix) containing the shift parameters, this is also known as the QC code block. Each translation index in BG will correspond to a unit matrix of size Z_C with the number of shifts. The next section will show how to build BG in the 5G standard.

4. 5G-NR LDPC BASE GRAPH CONSTRUCTION

Information message for the 5G network is large, thus the decoding is complicated. Instead of giving a very large H matrix, it is replaced by a base matrix, and from this base matrix with Z_C , size of each unit matrix, we will deduce matrix H . There are two types of base matrices BG1 and BG2, which are adopted for 5G LDPC codes.

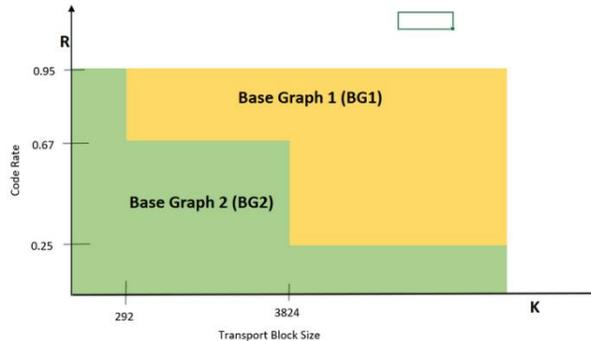


Figure 5. Base graph selection based on block size and code rate [23]

These two matrices have a similar structure. BG1 with a matrix size of 46×68 entries is designed for large transport block, information lengths up to 8448, and code rates from $1/3$ to $8/9$, while BG2 with a matrix size of 42×52 entries is targeted for smaller block lengths and code rates from $1/5$ to $2/3$.

The construction on BG1 and BG2 is mentioned in [6, 14, 22] with general following steps:

- Step 1: Base on the size of message K and code rate R , we can determine the type of base graph.
- Step 2: We find the number of information circulant columns K_b with given K and R .
- Step 3: We determine suitable shift coefficient Z_C by searching the following table:

Table 1. Choose Z_C base on index i_{LS} table [6].

Set index (i_{LS})	Set of lifting sizes (Z)
0	{2, 4, 8, 16, 32, 64, 128, 256}
1	{3, 6, 12, 24, 48, 96, 192, 384}
2	{5, 10, 20, 40, 80, 160, 320}
3	{7, 14, 28, 56, 112, 224}
4	{9, 18, 36, 72, 144, 288}
5	{11, 22, 44, 88, 176, 352}
6	{13, 26, 52, 104, 208}
7	{15, 30, 60, 120, 240}

- Step 4: Searching standard table and the result will be BG for 5G-NR LDPC.

In this paper, data will be used on BG2 with $K = 2304$, $K = 1944$ and code rate $= \frac{1}{2}$ and $\frac{2}{3}$, respectively for both QC and RA codes.

5. SIMULATION RESULTS AND DISCUSSION

This section will build the RA code and QC code based on the method mentioned in sections 2 and 3. To examine the quality of both RA and QC codes over GF(2), we encode the same message with both RA and QC codes with binary phase shift keying (BPSK) modulation and transmit over the AWGN channel, and use the Min Sum LDPC decoder in 10 iterations at receiver, conducted by MATLAB software. For QC-LDPC, BG2 was generated based on the 3GPP standard mentioned in section 4, and for RA LDPC, the interleaver was randomly selected because, for the 5G standard, the size of data is large.

Figure 6 shows the performances of both algorithms at the same code length $N = 2304$ and rate $R = \frac{1}{2}$. In general, we can see that when transmitting information over 5G standard, if the information is encoded by QC code, the performance will be much higher than that of the RA code, approximately $\Delta \approx 3.10^{-5} \div 10^{-5} \approx 3$ times at SNR = 3 dB, and deviation distance Δ between QC code and RA code is almost maintained during the change of SNR. If we consider the quality difference between RA and QC code, it can be seen that the larger the SNR, the higher the QC quality compared to RA code.

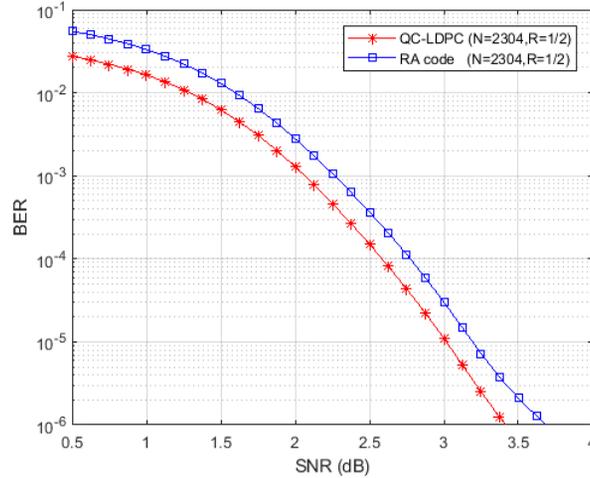


Figure 6. BER vs SNR of both codes with $N = 2304, R=1/2$.

To have more specific comments, we proceed to analyze both codes on different code lengths and code rates. We will change the code rate and length parameters to get a better overview of the performance in two encoders.

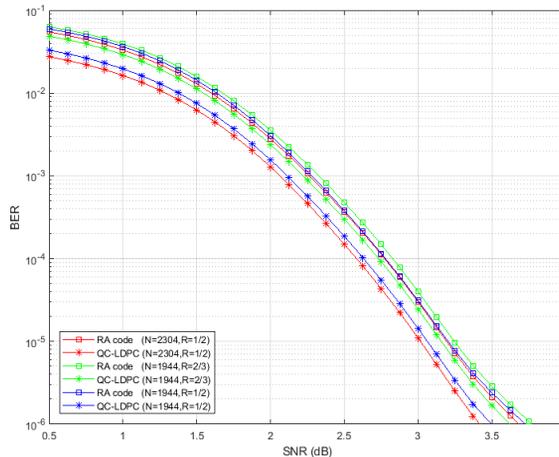


Figure 7. BER performance comparison of RA and QC in different code length and rate.

Figure 7 shows the simulation results when we set the code rates $R = \frac{1}{2}, \frac{2}{3}$ and code length 2304, 1944 for each code, respectively. We can see that at the same code rate $R = \frac{1}{2}$ when the code length increases from 1944 to 2304 which leads to better BER performance. This is seen in both codes. Thus, BER performance will be inversely proportional to the length of the codeword. When we observe with the same code length $N = 1944$, the quality of both codes is declined as the code rate increases.

For a better view between BER curve of each code, we zoom the results found with a BER range from 10^{-6} to 10^{-4} and a SNR range from 2.5 dB to 4 dB. Thus, we get the result as shown in the Figure 8. In general, we see that all three cases of QC code give better results than the RA code.

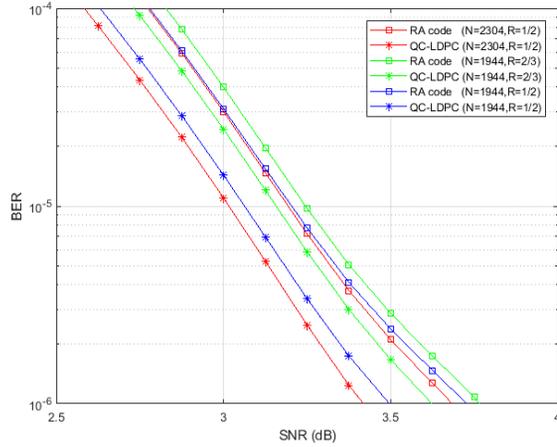


Figure 8. Comparison performance of both codes in different length and rate with BER from 10^{-6} to 10^{-4} .

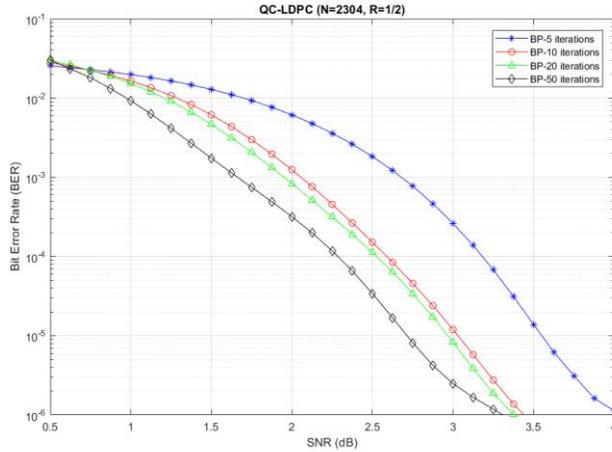


Figure 9. BER performance of QC LDPC codes with $N = 2304$, $R=1/2$ in different iteration number.

To further demonstrate the performance difference among various iteration number, we consider simulating QC-LDPC code with $N = 2304$ and $R = \frac{1}{2}$. Figure 9 shows that as the iteration number increases, BER performance gets better. It is shown that the iteration number of 50 has about 0.25 dB improvement compared to the number of 20 while the number of 10 has about 0.5 dB improvement compared to the number of 5. This improvement is observed at the BER of about 10^{-5} .

As analyzed above, we found that the quality of the QC code with $N = 1944$ and $R = \frac{2}{3}$ is the lowest in the case set of QC code; and the RA code with $N = 2304$ and $R = \frac{1}{2}$ is the best in the set of RA code. However, we can see that the quality of those two cases is approximately the same, showing the superiority of the QC code compared to the RA code. One thing to notice when we observe two BER curves are QC code with $N = 1944$; $R = \frac{2}{3}$ and RA code with $N = 1944$; $R = \frac{1}{2}$, with the same information, the same length, although the speed of the QC code is greater than that of the RA code, the quality is still greater than the RA

code. Thus, with the same message, one of the ways to increase the code rate while keeping the performance or even better is to change the encoding method from an RA encoder to a QC encoder.

6. CONCLUSION

In this paper, the construction of the RA code and QC code on the algebraic method over GF(2) is presented. We implement comparisons of the quality of both codes in the case of changes in code rate and code length. When considering the same speed and length, it is clear that the BER performance of the QC code will give better quality than the RA code approximately 3 times. Whereas if we look at the overview on the different rates and different lengths of the QC and RA codes, we find that with the same code length N , we can increase the code rate but keep the BER performance when changing the encoder from RA code into QC code. The code rate plays a very important role in the digital information age. Although the complexity of the QC code is higher than the RA code due to the implementation of matrix multiplication, the performance of the QC code is much better than the RA code, so the engineer needs to consider choosing the QC code or RA code. Choosing the appropriate algorithm for encryption will contribute to increased efficiency in information transmission in 5G, a standard that Vietnam wants to achieve in Industry 4.0.

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TÓM TẮT

ĐÁNH GIÁ CHẤT LƯỢNG SỬA LỖI KHI SỬ DỤNG MÃ REPEAT ACCUMULATE VÀ MÃ QUASI CYCLIC LOW-DENSITY PARITY CHECK TRONG MẠNG 5G

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Mã LDPC (Low-density parity check) là mã sửa sai mật độ thấp đã được chọn dùng cho mạng 5G vì khả năng tốc độ xử lý cao, hiệu suất về diện tích và năng lượng cao. Mã LDPC đã được chứng minh cho kết quả sửa lỗi tiến tới giới hạn Shannon kể từ khi LDPC được phát minh ra, có nhiều thuật toán mã hóa cũng được công bố, trong đó có mã Quasi Cyclic LDPC (mã QC) và mã Repeat Accumulate LDPC (mã RA). Hai mã này được sử dụng nhiều trong việc mã hóa và giải mã bởi độ phức tạp của chúng thấp. Trong bài báo này, nhóm tác giả tiến hành xây dựng hai mã trên, khảo sát và phân tích hiệu suất của cả hai mã với yêu cầu ứng dụng chiều dài khối lớn trong mạng 5G.

Từ khóa: Mã LDPC, mã RA, mã QC, chuẩn 5G, hiệu suất BER so với SNR.