Tạp chí Khoa học và Công nghệ **49** (2) (2011) 9-25

ON THE STATE-OPTIMIZATION APPROACH TO SYSTEM PROBLEMS: CLOSED-LOOP THINKING SOLUTIONS

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Received May 15, 2010

ABSTRACT

State-optimization approach has been proposed to treating various different system problems in optimal projection equations (OPEQ). While the OPEQ for problems of open-loop thinking is found consisting of two modified Lyapunov equations, excepting the rank conditions whereas required in system identification and its related robust problems, the one for closed-loop thinking consists of two modified either Reccatti or Lyapunov equations, excepting conditions for compensating system happened to be in a problem like that of order reduction for controller.

Apart from addditonally constrained-conditions and simplicity in the solution form have been obtainable for each problem, it has been found the system identification problem switching over to computing the solution of OPEQ and the physical nature of medeled states possibly retaining in optimal order reduction problem.

1. INTRODUCTION

System problems may be divided into four major parts which are modeling, setting up the mathematical equations, analysis and design [1]. However, if the discussion is limited to linear systems described in the state space equations, the system problems may be then regarded to belong to either open- or closed-loop thinking ones. There have many research workers been devoted to tackling various different aspects of open- and closed-loop thinking problems from both theoretical and practical angles. Among the myriad references available in literature, two notable methodology contributions related with present paper are from the internally system-theoretic argument and from the treatment in optimal projection equations (OPEQ).

Internal system philosophy based on the contribution of dynamical elements (state variables) to the system input/output relationship has been originated firstly to so-called singular values by Moore in 1981 [2] for an open-loop thinking system and further developed to characteristic values for a closed-loop thinking one by Jonekheere and Silverman [3], and by Mustafa and Glover [4]. The contribution of states to the system input/ouput relationship can be measured on the basics of diagonalizing simultaneous both controllability and observability gramians of the system of any loopwise thinking to the very same diagonalized matrix (internally balanced conditions). This methodology is found promising for system problems of both thinking-wises in the analysis part. However, the major drawback lies on the optimality in designing as no where optimal design gives to troublesome in closed-looping like the one for the controller, especially in a problem of projective control. The component cost ranking principle proposed by Skelton [5] based on determining contributions of dynamical elements to a quadratic errors criterion, from the opinion of the authors, may be regarded as a special method of the earlier philosophy since no rigorous guarantee of optimality is possible although the propose has been guided by an optimality consideration. However, it suggests researches to be carried out on combining an optimality consideration and the internally balanced conditions for the design purpose.

Last more than three decades, an American scientists group (Bernstein, Haddad and Hyland) have devoted a tremendous effort in publishing a series of research papers on different system problems in both loop-wise thinking [6 - 10]. From the first-order necessary conditions for an optimality consideration of each problem, an optimal projection matrix has been realized and used for developing suitable OPEQ. Important significance of treatment in OPEQ philosophy lies on the question of multi-extreme as certain constraint conditions, bounds like internally balanced condition, H_∞ performance bounds, Petersen-Hollt, Guaranteed cost bounds and so on, are able to be accommodated suitably in due OPEQ development course for each problem. This methodology is hence found being applicable to both analysis and design purposes. With a careful analysis, it is found that the minimization has in all the cases been carried out with respect to parameters, which are inherently non-separable from state-variables for an output function. This gives rise to a drawback in regards to some difficulties lying on the complexity of mathematical involvement also on the optimal projection nature, which in most of the cases is an oblique one, leading to the requirement of other conditions for computing the solution of OPEQ. Further, although additionally constraint conditions are able to be facilitated in OPEQ, but not a single provision for retaining the physical nature of desired states in the result. This disvalues significance of the methodology from the analysis point of view.

Concept of state-optimization has been originated by San [11] from the fact that between two systems of sate-variable equations there exists always a non-similarity transformation on each to other state vectors and then the optimality for back-transform is achieved owing the role of pseudo-inverse of that non-similarity. San has shown that for a given system the non-similarity transformation may be freely chosen; hence the retaining physical nature of modeled states is possible in transformed version [17]. If the non-similarity transformation is factorized in terms of a partial isometry, an orthogonal projection matrix can be formed, facilitating the possibility of obtaining a simpler form for OPEQ. Thus, the state-optimization methodology overcomes the drawbacks and enjoys the merits of both early mentioned approaches.

Arrangement of the paper as follows: Two lemmas proposed for preliminary are retaken in II. The first one is related with defining a criterion for the state optimization and the other is with factorizing a non-similarity transformation in terms of a partial isometry. In III, two problems in closed-loop thinking and the respective results [13, 14] are retaken. Both problems have been firstly transferred into open-loop thinking so that the result obtained for the opened-loop thinking can be employed for that in the closed-loop thinking. In the last part of III is used for applications to telecommunication network modeling and the results on robust considerations. Concluding remarks and directions for further researches are mentioned in IV.

2. PRELIMINARY

2.1. Notations

Throughout the paper, following conventions are used

- All systems are taken to be linear, time-invariant, causal and multi-variable.

- Bold capital letters are denoted for matrices, while low-case bolt letters are for vectors.

- $\mathscr{R}$ stands for real, $\mathscr{E}$(.) for either expectation or average value of (.) when t approaches to infinity.

- ρ(.), $(.)^T$, $(.)^+$ stand for rank, transpose, pseudoinverse of (.).

- Stability matrix is the one having all eigenvalues on the left hand side of the S-plane.

- Non-negative (positive) definite matrix is a symmetric one having only non-negative (positive) eigenvalues.

- All the vectors norms are Euclideans or l^2 norms, $\|\mathbf{x}\|^2 = \left(\sum_j |x_j|^2\right)^{1/2}$.

- Controllability and observability gramians of a system are denoted by

$$\mathbf{W}_c = \int_0^t e^{\mathbf{A}t}\mathbf{B}\mathbf{V}\mathbf{B}^T e^{\mathbf{A}^T_t}\,dt\,,\ \mathbf{W}_0 = \int_0^t e^{\mathbf{A}^T_t}\mathbf{C}^T\mathbf{C}e^{\mathbf{A}t}\,dt \tag{2.1}$$

Satisfying dual Lyapunov equations

$$\begin{aligned} \mathbf{A}\mathbf{W}_c + \mathbf{W}_c\mathbf{A}^T + \mathbf{B}\mathbf{V}\mathbf{B}^T &= 0 \\ \mathbf{W}_0\mathbf{A} + \mathbf{A}^T\mathbf{W}_\mathbf{0} + \mathbf{C}^T\mathbf{R}\mathbf{C} &= 0 \end{aligned} \tag{2.2}$$

where $\mathbf{V} = \mathrm{E}(\mathbf{u}\mathbf{u}^T)$, $\mathbf{R}$ is non-negative weighted matrix of order q.

2.2. Introduction to Pseudo-inverse and Transformation in system problems

Concept of generalized inverse seems to have been first mentioned, called as pseudo-inverse by Fredholm in 1903, originating for integral operator. Generalized inverses have been studied extending to differential operators, Green's functions by numerous authors, particularly by Hilbert in 1904, Myller in 1906, Westfall in 1090, Hurwitz in 1912, etc. Generalized inverse has been antedated to matrices on defining first by Moore in 1920 as general reciprocal. The uniqueness of pseudo-inverse of a finite dimensional matrix has been shown by Penrose in 1955, satisfying four equations [12]

$$\mathbf{TXT} = \mathbf{T}\ \text{(i)},\ \mathbf{XTX} = \mathbf{X}\ \text{(ii)},\ \mathbf{(TX)^*} = \mathbf{TX}\ \text{(iii)},\ \mathbf{(XT)^*} = \mathbf{XT}\ \text{(iv)} \tag{2.3}$$

where (.)* denotes for conjugate transpose of (.).

The above four equations are commonly known as Moore-Penrose ones and the unique matrix $\mathbf{X}$ on satisfying these equations is usually referred to as the Moore-Penrose inverse and often denoted by $\mathbf{T}^+$.

Assume that an available system (S) and an invited (or assumed) model (AM) are described in the state-space equations as

$$\text{(S):}\quad \begin{aligned} \mathbf{x}'_n &= \mathbf{A}_n\mathbf{x}_n + \mathbf{B}_n\mathbf{u}_n \\ \mathbf{y}_n &= \mathbf{C}_n\mathbf{x}_n \end{aligned} \tag{2.4}$$

$$\text{(AM):}\quad \begin{aligned} \mathbf{x}'_m &= \mathbf{A}_m \mathbf{x}_m + \mathbf{B}_m \mathbf{u}_m \\ \mathbf{y}_m &= \mathbf{C}_m \mathbf{x}_m \end{aligned} \tag{2.5}$$

where the letters n and m in the subscripts stand for (S) and (AM) also for their order numbers respectively with all of the vectors and matrices are supposed to be appropriately dimensioned.

It was observed that indifferent from orders of the two, there exists always a transformation between two state vectors (referred to as state transformation) and a transformation between two output vectors (named as output transformation). If both (S) and (AM) are subjected to the same input vector, output transformation is seen to be similarity (an invertible matrix) one as dimension of the output vector of (AM) is the same as that of (S), but it is not the case always for state transformation. Even if state transformation is a non-similarity one, the output vectors are match able, however. As non-similarity transformation on state variable vectors is not a bi-directional one, giving rise to the idea of optimization with respect to the state variables.

2.3. Definitions and Lemmas

2.3.1. Definitions

Problem that deals with system be tackled inherently in closed-loop configuration is referred to as closed-loop thinking one [1].

Projection matrix resulted from the first order necessary conditions for an optimality process is termed as an optimal projection. System of equations resulted from the necessary conditions for an optimality expressing in terms of components of optimal projection is called as optimal projection equations (OPEQ) [7, 11].

2.3.2. Lemmas

Lemma 2.1. Let the vector $\mathbf{x}_n$ of n independently specified states of a (S) be given. Assume that an (AM) is chosen having vector $\mathbf{x}_m$ of m independently specified states, m ≤ n. Then there exists a non-similarity transformation $\mathbf{T} \in \mathscr{R}^{m\times n}$, $\rho(\mathbf{T}) = m$, on $\mathbf{x}_n$ for obtaining $\mathbf{x}_m$ such that if the number of (S) output is less than or equal to that of (AM) order, $q \leq m$, then $\mathbf{T}^+\mathbf{x}_m$ leads to the minimum norm amongst the least-squares of output-errors.

Proof. Details can be found in [11]. It is necessary showing that with the condition mentioned in lemma one can easily obtain the weighted least-squares criterion on the output errors

$$J_{\text{Oopt}} = \int_o^\infty (\mathbf{y}_n - \mathbf{y}_m)^T \mathbf{R}(\mathbf{y}_n - \mathbf{y}_m)dt \tag{2.6}$$

from the criterion for state optimization

$$J_{\text{Sopt}} = \int_o^\infty \| \mathbf{x}_n - \mathbf{T}^+\mathbf{x}_m \|_{\mathbf{R}}^2 dt \tag{2.7}$$

with **R** stands for non-negative weighted matrix of the appropriate dimension.

Usually, order n of (S) is not known, order m of (AM) may be highly chosen. In such a case, the validity of the lemma is kept; see the remark II.1 of [11] for the details of argument.

Lemma 2.2. Let the state vector $\mathbf{x}_n$ of (S) be a transformed state vector of (AM) as

$$\mathbf{x}_n = \mathbf{T}^{+}\mathbf{x}_m,\ \mathbf{T} \in R^{mxn},\ \rho(\mathbf{T}) = n<m\,. \tag{2.8}$$

Then $\mathbf{T}$ can be factorized as

$$\mathbf{T} = \mathbf{EG} = \mathbf{HE} \tag{2.9}$$

where, $\mathbf{E} = \mathcal{E}(\mathbf{x}_m\mathbf{x}_n^T) \in \mathcal{R}^{mxn}$ is a partial isometry, $\mathbf{G} = \mathcal{E}(\mathbf{x}_n\mathbf{x}_n^T) \in \mathcal{R}^{nxn}$, $\mathbf{H} = \mathcal{E}(\mathbf{x}_m\mathbf{x}_m^T) \in \mathcal{R}^{mxm}$, both are non-negative definite matrices.

Proof. See [11] for details.

Remark 2.1. It is noted that since $\mathbf{T}$ is constant $\dot{\mathbf{x}}_n = \mathbf{T}^{+}\dot{\mathbf{x}}_m$ is also valid.

It is known that $\sigma_1 = \mathbf{EE}^T, \sigma_2 = \mathbf{E}^T\mathbf{E}$ are optimal in the sense that one state vector is optimized with respect to the other; moreover both are of orthogonal projection matrix.

Although $\mathbf{x}_n$ and $\mathbf{x}_m$ are definitely specified but $\mathbf{T}$ is not unique determined due to mismatch between the dimensions of two state vectors. The question arises regarding the construction of $\mathbf{T}$ so that $\mathbf{x}_n$ is obtainable from the knowledge of $\mathbf{x}_m$.

3. TYPICAL CLOSED-LOOP THINKING PROBLEMS

3.1. Order reduction for state estimation

3.1.1. Statement of the problem

Given an n-th order (S) described by

$$\dot{\mathbf{x}}_n = \mathbf{A}_n\mathbf{x}_n + \mathbf{B}_n\mathbf{u}_n \tag{3.1}$$

$$\mathbf{y}_n = \mathbf{C}_n\mathbf{x}_n \tag{3.2}$$

with order n_e of jointly controllable and observable part of (S) less than n, $n_e \leq n$.

Determine a reduced-order state estimator of order e, $q \leq e$

$$\dot{\mathbf{x}}_e = \mathbf{A}_e\mathbf{x}_e + \mathbf{B}_e\mathbf{u}_e \tag{3.3}$$

$$\mathbf{y}_e = \mathbf{C}_e\mathbf{x}_e \tag{3.4}$$

where $\mathbf{u}_e = \begin{bmatrix}\mathbf{u}_n^T & | & \mathbf{y}_n^T\end{bmatrix}^T \in R^{(p+q)x1}$.

Conditions to be satisfied are

- $(\mathbf{A}_e,\mathbf{B}_e,\mathbf{C}_e)$: Controllable and observable; $(\mathbf{A}_e,\mathbf{B}_e)$: Stabilisable, $(\mathbf{A}_e,\mathbf{C}_e)$: Detectable,
- Maximum value assignable to e,
- L_2 model-reduction criterion on the state-error,
- L_2 model-reduction criterion on the output-error.

3.1.2. Solution of the problem

Theorem 3.1. For a linear, n-th order, time-invariant parameters (S) there exist full row rank matrices $\mathbf{K} \in R^{exn}$, $\mathbf{L} \in R^{q_e xq}$ and a linear combination of (S) outputs **M**, such that the optimal parameters of state estimator of order e are given by

$$\mathbf{A}_e = \mathbf{K}(\mathbf{A}_n - \mathbf{M}\mathbf{C}_n)\mathbf{K}^+,\ \mathbf{B}_e = \mathbf{K}[\mathbf{B}_n \mid \mathbf{M}],\ \mathbf{C}_e = \mathbf{L}\mathbf{C}_n\mathbf{K}^+ \qquad (3.5)$$

The maximum value of e which can be considered for the order of the reduced-order state esti-mator to be controllable and observable is the irreducible order of (S).

Further, there exist a partial isometry $\mathbf{E}_e \in R^{exn}$ and non-negative definite $\mathbf{H}_e \in R^{nxn}$ such that with optimal orthogonal projector $\boldsymbol{\sigma}_e = \mathbf{E}_e^T\mathbf{E}_e \in R^{nxn}$, $r(\boldsymbol{\sigma}_e) = e$ and two non-negative definite matrices $\mathbf{Q}_e^* = \mathbf{H}_e^+\mathbf{E}_e^T\mathbf{Q}_e\mathbf{E}_e$, $\mathbf{P}_e^T = \mathbf{H}_e\mathbf{E}_e^T\mathbf{P}_e\mathbf{E}_e \in R^{nxn}$, the following conditions are to be satisfied

$$\begin{aligned}&\boldsymbol{\sigma}_e\left[\mathbf{H}_e\mathbf{A}_n\mathbf{Q}_e^* + \mathbf{Q}_e^*\mathbf{A}_n^T\mathbf{H}_e + \mathbf{H}_e\mathbf{B}_n\mathbf{V}_1\mathbf{B}_n^T\mathbf{H}_e\right] -\\ &\boldsymbol{\sigma}_e\left[\mathbf{H}_e\mathbf{M}\left(\mathbf{C}_n\mathbf{Q}_e^* - \mathbf{V}_{12}^T\mathbf{B}_n^T\mathbf{H}_e\right) + \left(\mathbf{Q}_e^*\mathbf{C}_n^T - \mathbf{H}_e\mathbf{B}_n\mathbf{V}_{12}\right)\mathbf{M}^T\mathbf{H}_e + \mathbf{H}_e\mathbf{M}\mathbf{V}_2\mathbf{M}^T\mathbf{H}_e\right] = \mathbf{0}\end{aligned} \qquad (3.6)$$

$$\begin{aligned}&\boldsymbol{\sigma}_e\left[\mathbf{H}_e^+\mathbf{A}_n^T\mathbf{P}_e^* + \mathbf{P}_e^T\mathbf{A}_n\mathbf{H}_e^+\right] -\\ &\left[\boldsymbol{\sigma}_e\mathbf{H}_e^+\mathbf{C}_n^T\mathbf{M}^T\mathbf{P}_e^* + \mathbf{P}_e^*\mathbf{M}\mathbf{C}_n\mathbf{H}_e^+\boldsymbol{\sigma}_e + \boldsymbol{\sigma}_e\mathbf{H}_e^+\mathbf{C}_n^T\mathbf{L}^T\mathbf{R}_4\mathbf{L}\mathbf{C}_n\mathbf{H}_e^+\boldsymbol{\sigma}_e\right] = \mathbf{0}\end{aligned} \qquad (3.7)$$

Proof. After some mathematical manipulations, with $\mathbf{L} \in R^{q_e xq}$ (3.1) and (3.2) become

$$\mathbf{x}'_{\sim} = \mathbf{A}_{\sim}\mathbf{x}_{\sim} + \mathbf{B}_{\sim}\mathbf{u}_e \qquad (3.8)$$

$$\mathbf{y}_{\sim} = \mathbf{C}_{\sim}\mathbf{x}_{\sim} \qquad (3.9)$$

Where $\mathbf{x}_{\sim} = \mathbf{x}_n$, $\mathbf{A}_{\sim} = \mathbf{A}_n - \mathbf{M}\mathbf{C}_n$, $\mathbf{M} \in R^{nxq}$, $\rho(\mathbf{M}) = q$, $\mathbf{B}_{\sim} = [\mathbf{B}_n \mid \mathbf{M}] \in R^{nx(p+q)}$, $\mathbf{C}_{\sim} = \mathbf{L}\mathbf{C}_n$.

The closed-loop thinking problem becomes an open-loop one and the result obtained for order reduction of model can be adopted; see [13] for the details.

Remark 3.1. Optimal reduced-order state estimation has been turned to that of unregulated (**S**). If **M** and **L** are assumed to make model described by (3.8) and (3.9) be regulated, the problem is dealing with order reduction for a dynamic compensation. And in such a case, the maximum value that can be assignable to e is n.

Corresponding to the optimal parameters, (3.8) is written for implementing a full order state estimator

$$\mathbf{x}'_e = \mathbf{E}_e\mathbf{H}_e\mathbf{A}_n\mathbf{H}_e^+\mathbf{E}_e^T\mathbf{x}_e + \mathbf{E}_e\mathbf{H}_e\mathbf{M}\left(\mathbf{y}_n - \mathbf{C}_n\mathbf{H}_e^+\mathbf{E}_e^T\mathbf{x}_e\right) + \mathbf{E}_e\mathbf{H}_e\mathbf{B}_n\mathbf{u}_n \qquad (3.10)$$

The error referring to the input side arisen due to non-similarity transformation **M** is given by $\mathbf{E}_e\mathbf{H}_e\mathbf{M}\left(\mathbf{y}_n - \mathbf{C}_n\mathbf{H}_e^+\mathbf{E}_e^T\mathbf{x}_e\right)$.

3.2. Problem of order reduction for controller

3.2.1. Brief summary on the standard Linear Quadratic Gaussian (LQG) problem

If zero mean Gaussian white noises $\mathbf{w}_1(t)$ and $\mathbf{w}_2(t)$ are assumed to be vectors of un-correlated, normalized adding to the input and output of a (S)

$$\mathbf{x}^{\cdot}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\left[\mathbf{u}(t) + \mathbf{w}_1(t)\right] \tag{3.11}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{w}_2(t) \;. \tag{3.12}$$

Then, subject of standard (normalized) LQG is to find out from all linear feedback the one that corresponds to the minimum of a scalar functional cost

$$J = \lim_{t\to\infty} \mathrm{E}\left\{\frac{1}{t}\int_0^t \begin{bmatrix}\mathbf{Cx}\\ \mathbf{u}\end{bmatrix}^{\mathrm{T}} \begin{bmatrix}\mathbf{Cx}\\ \mathbf{u}\end{bmatrix} dt\right\} \tag{3.13}$$

for which the the solution is stated as.

Let ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$) be of minimal. Then, there exists a normalized LQG controller ($\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$, $\hat{\mathbf{C}}$) having parameters determined as

$$\hat{\mathbf{A}} = \mathbf{A} - \mathbf{B}\mathbf{B}^{\mathrm{T}}\mathbf{\Pi} - \mathbf{\Psi}\mathbf{C}^{\mathrm{T}}\mathbf{C},\ \hat{\mathbf{B}} = \mathbf{\Psi}\mathbf{C}^{\mathrm{T}},\ \hat{\mathbf{C}} = -\mathbf{B}^{\mathrm{T}}\mathbf{\Pi} \tag{3.14}$$

where $\mathbf{\Pi}$ and $\mathbf{\Psi}$ are the respective unique positive definite solution of controlling algebraic Riccati equation (CARE) and of filtering algebraic equation (FARE).

$$\mathbf{A}^{\mathrm{T}}\mathbf{\Pi} + \mathbf{\Pi}\mathbf{A} + \mathbf{C}^{\mathrm{T}}\mathbf{C} - \mathbf{\Pi}\mathbf{B}\mathbf{B}^{\mathrm{T}}\mathbf{\Pi} = \mathbf{0} \tag{3.15}$$

$$\mathbf{A}\mathbf{\Psi} + \mathbf{\Psi}\mathbf{A}^{\mathrm{T}} + \mathbf{B}\mathbf{B}^{\mathrm{T}} - \mathbf{\Psi}\mathbf{C}^{\mathrm{T}}\mathbf{C}\mathbf{\Psi} = \mathbf{0} \;. \tag{3.16}$$

Corresponding to the optimum solution, value of the LQG functional cost is

$$J_m = \mathrm{tr}\left[\mathbf{B}^{\mathrm{T}}\mathbf{\Pi}\mathbf{B} + \mathbf{B}^{\mathrm{T}}\mathbf{\Pi}\mathbf{\Psi}\mathbf{\Pi}\mathbf{B}\right] \;. \tag{3.17}$$

Eigenvalues of $\mathbf{\Pi\Psi}$ are known as the LQG-characteristic values of (S) and found similarity invariants. This invariability permits one to obtain an LQG-balanced realization and in such a case, both $\mathbf{\Pi}$ and $\mathbf{\Psi}$ are of the diagonal form. That motivates reduction scheme for closed-loop thinking by truncation off the least significant states. For improving the performance of reduced controller, H∞ bound is adopted [4, 10]. However, still an important question remains to be addressed with respect to the stabilization of (S).

3.2.2. Statement of the controller reduction problem

Given an n-th order (S) having appropriately dimensioned vectors and parameter matrices

$$\mathbf{x}^{\cdot} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{3.18}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{3.19}$$

Determine a controller of order e, q ≤ e ≤ n

$$\mathbf{x}^{\cdot}_{\mathrm{e}} = \mathbf{A}_{\mathrm{e}}\mathbf{x}_{\mathrm{e}} + \mathbf{B}_{\mathrm{e}}\mathbf{u}_{\mathrm{e}},\ \mathbf{u}_{\mathrm{e}} = \mathbf{y} \tag{3.20}$$

$$\mathbf{y}_{\mathrm{e}} = \mathbf{C}_{\mathrm{e}}\mathbf{x}_{\mathrm{e}} \tag{3.21}$$

with p-, q-dimensional vectors $\mathbf{u}_e$, $\mathbf{y}_e$, and appropriately dimensioned matrices $\mathbf{A}_e$, $\mathbf{B}_e$, $\mathbf{C}_e$.

Conditions to be satisfied

- $(\mathbf{A}_e, \mathbf{B}_e, \mathbf{C}_e)$: Controllable and observable; $(\mathbf{A}_e, \mathbf{B}_e)$: Stabilizable, $(\mathbf{C}_e, \mathbf{A}_e)$: Detectable.
- L_2 model-reduction criterion on the state-error for the reduced controller is minimized, with an n-dimensional state vector **x** of the full order LQG controller associated with (S), an exn matrix **T** and a positive definite matrix $\mathbf{R}_3$.
- L_2 model-reduction criterion on the output-error for reduced controller is minimized, with a q-dimensional output vector $\mathbf{y}_n$ of the full order LQG controller associated with (S), an exn matrix **K** of order q, and a positive definite matrix $\mathbf{R}_4$.

3.2.3. Solution to the problem

Theorem 3.2. For a linear, n-th order, time-invariant parameters system there exists a partial isometry $\mathbf{E}_e \in R^{exn}$ and two non-negative definite matrices $\mathbf{Q}_e$, $\mathbf{P}_e$ such that optimal parameters of a jointly controllable and observable controller of order e are given by

$$\mathbf{A}_e = \mathbf{E}_e\mathbf{H}(\mathbf{A} - \mathbf{B}\mathbf{B}^T\mathbf{\Pi} - \mathbf{\Psi}\mathbf{C}^T\mathbf{C})\mathbf{H}^{-1}\mathbf{E}^T,\ \mathbf{B}_e = \mathbf{E}_e\mathbf{H}\mathbf{\Psi}\mathbf{C}^T,\ \mathbf{C}_e = -\mathbf{B}^T\mathbf{\Pi}\mathbf{H}^{-1}\mathbf{E}_e^T \tag{3.22}$$

In which two positive definite matrices $\mathbf{\Pi}$ and $\mathbf{\Psi}$ are the unique solution of CARE and FARE respectively and **H** is related with the states of full order LQG controller.

The following conditions are to be satisfied

$$\boldsymbol{\sigma}_e[\mathbf{H}(\mathbf{B}\mathbf{B}^T\mathbf{\Pi} + \mathbf{\Psi}\mathbf{C}^T\mathbf{C})\mathbf{Q}_e - \frac{1}{2}\mathbf{H}\mathbf{\Psi}\mathbf{C}^T\mathbf{V}\mathbf{C}\mathbf{H}] \geq \mathbf{0} \tag{3.23}$$

$$\boldsymbol{\sigma}_e[\mathbf{H}^{-1}(\mathbf{B}\mathbf{B}^T\mathbf{\Pi} + \mathbf{\Psi}\mathbf{C}^T\mathbf{C})\mathbf{P}_e - \frac{1}{2}\mathbf{H}^{-1}\mathbf{\Pi}\mathbf{B}\mathbf{B}^T\mathbf{\Pi}\mathbf{H}^{-1}] \geq \mathbf{0} \tag{3.24}$$

where $\mathbf{Q}_e = \mathbf{H}^{-1}\mathbf{E}_e^T\mathbf{Q}\mathbf{E}_e$, $\mathbf{P}_e = \mathbf{H}\mathbf{E}_e^T\mathbf{P}\mathbf{E}_e$, $\boldsymbol{\sigma}_e = \mathbf{E}_e^T\mathbf{E}_e$ and **Q**, **P** are controllability, observability gramians of the reduced-order LQG controller.

Proof. Details are available in [14, 22].

3.2.4. Compensating system by reduced controller

Given an n-th order (S) by (3.18) and (3.19) and r-th order (reduced-order) controller obtained by theorem 3.2 denoting by

$$\hat{\mathbf{x}}_r^{\cdot} = \hat{\mathbf{A}}_r\hat{\mathbf{x}}_r + \hat{\mathbf{B}}_r\hat{\mathbf{u}}_r \tag{3.25}$$

$$\hat{\mathbf{y}}_r = \hat{\mathbf{C}}_r\hat{\mathbf{x}}_r \tag{3.26}$$

Conditions $\hat{\mathbf{y}}_r = \mathbf{u}, \mathbf{y} = \hat{\mathbf{u}}_r$ for compensating are performed with respect to normalized LQG by $\mathbf{u} \to \mathbf{u} + \mathbf{w}_1(t),\ \hat{\mathbf{u}}_r \to \hat{\mathbf{u}}_r + \mathbf{w}_2(t)$ on satisfying

$$E\left\{\begin{bmatrix}\mathbf{w}_1(t)\\ \mathbf{w}_2(t)\end{bmatrix}\begin{bmatrix}\mathbf{w}_1^T(t) & \mathbf{w}_1^T(t)\end{bmatrix}\right\} = \begin{bmatrix}\mathbf{I}_p & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_q\end{bmatrix}\delta(t-\tau).$$

That gives rise to a system of six equations reducible to the one of four ones; two are boundary conditions exactly those in (3.23) and (3.24), the other two are modified Lyapunov inequalities

$$\mathbf{B}\mathbf{B}^{\mathrm{T}}\mathbf{\Pi}\mathbf{H}^{-1}\mathbf{E}^{\mathrm{T}}\mathbf{Q}_{12}^{\mathrm{T}} + \mathbf{Q}_{12}\mathbf{E}\mathbf{H}^{-1}\mathbf{\Pi}\mathbf{B}\mathbf{B}^{\mathrm{T}} - \mathbf{B}\mathbf{B}^{\mathrm{T}} \geq \mathbf{0} \tag{3.27}$$

$$\mathbf{C}^{\mathrm{T}}\mathbf{C}\mathbf{\Psi}\mathbf{H}\mathbf{E}^{\mathrm{T}}\mathbf{P}_{12}^{\mathrm{T}} + \mathbf{P}_{12}\mathbf{E}\mathbf{H}\mathbf{\Psi}\mathbf{C}^{\mathrm{T}}\mathbf{C} + \mathbf{C}^{\mathrm{T}}\mathbf{C} \geq \mathbf{0} \tag{3.28}$$

Compensating (S) by a reduced controller has been found not guarantee until proper measures are taken, hence the normalized LQG has been considered, forming an augmented system. From the result of standard normalized LQG problem [17], (reduced-order controller internal stabilizes (S), the augmented system is to be stable) has given rise to a system of six equations deducible to four modified ones; two modified Lyapunov resulted from the reduction and two modified Riccati found being responsible for LQG. However, two of the mentioned four have been found being the conditions (3.23) and (3.24) in theorem 3.2. Other two (Riccati ones) are found decoupling readily due to the role of operational factorization.

Remarks 3.2. Reduced controller can be obtained by state-optimization approach in three steps; LQG is used for obtaining an equivalent open-loop model, reduction is performed on the open equivalent, LQG is used for compensation. This implies that an optimal performance of reduced controller can be obtained by a steep-wise design. This result agrees with that obtained by performing a parameter-optimization simultaneously on LQG and on model reduction, by which an optimal performance of reduced controller can not be achieved by step-wise process.

3.3. A robustness of modeling for electronic interface

3.3.1. Electronic interface modeling

Whenever CPU is still set up on the basics of classical set theory (binary algebraic), then the dynamics of telecommunication nodes is almost kept unchanged; whatever measure have been taken that would consider for interfacing techniques only (a transformation between a set of non- to standard of working conditions required by CPU). State-descriptive model transferring non-standard set $\mathbf{u}_1(t)$ to a standard $\mathbf{y}_1(t)$ of CPU with intercepted noise $\mathbf{w}_1(t)$ is [18, 20]

$$\mathbf{x}_1^{\cdot}(t) = \mathbf{A}_1\mathbf{x}_1(t) + \mathbf{B}_1\mathbf{u}_1(t) + \mathbf{B}_1\mathbf{w}_1(t) \tag{3.29}$$

$$\mathbf{y}_1(t) = \mathbf{C}_1\mathbf{x}_1(t) \tag{3.30}$$

Transferring from standard $\mathbf{u}_2(t)$ of CPU to a non-standard $\mathbf{y}_2(t)$ out side with noise $\mathbf{w}_2(t)$ is

$$\mathbf{x}_2^{\cdot}(t) = \mathbf{A}_2\mathbf{x}_2(t) + \mathbf{B}_2\mathbf{u}_2(t) + \mathbf{B}_2\mathbf{w}_2(t) \tag{3.31}$$

$$\mathbf{y}_2(t) = \mathbf{C}_2\mathbf{x}_2(t) \tag{3.32}$$

On working conditions to be satisfied $\mathbf{u}_1(t) = \mathbf{y}_2(t)$ and $\mathbf{u}_2(t) = \mathbf{y}_1(t)$ then obtains

$$\mathbf{x}_3^{\cdot}(t) = \mathbf{A}_3\mathbf{x}_3(t) + \mathbf{B}_3\mathbf{u}_3(t) \tag{3.33}$$

$$\mathbf{y}_3(t) = \mathbf{C}_3\mathbf{x}_3(t) \tag{3.34}$$

Where

$$\mathbf{x}_3(t) = \left[\mathbf{x}_1^{\mathrm{T}}(t) \quad \mathbf{x}_2^{\mathrm{T}}(t)\right]^{\mathrm{T}},\ \mathbf{u}_3(t) = \left[\mathbf{w}_1^{\mathrm{T}}(t) \quad \mathbf{w}_2^{\mathrm{T}}(t)\right]^{\mathrm{T}},\ \mathbf{y}_3(t) = \left[\mathbf{y}_1^{\mathrm{T}}(t) \quad \mathbf{y}_2^{\mathrm{T}}(t)\right]^{\mathrm{T}},$$

$$\mathbf{A}_3 = \begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1\mathbf{C}_2 \\ \mathbf{B}_2\mathbf{C}_1 & \mathbf{A}_2 \end{bmatrix},\ \mathbf{B}_3 = \begin{bmatrix} \mathbf{B}_1 & 0 \\ 0 & \mathbf{B}_2 \end{bmatrix},\ \mathbf{C}_3 = \begin{bmatrix} \mathbf{C}_1 & 0 \\ 0 & \mathbf{C}_2 \end{bmatrix}.$$

3.3.2. Electronic soft-switch modeling

Since nothing can be done with CPU, the "soft" is in the sense that "soft computing" is applied to the above interfaces to fit electronic switches with various different environments [18, 20]. A specification $\mathbf{u}_4(t) = [\mathbf{u}_{41}(t)\ \ \mathbf{u}_{42}(t)\ ...\ \mathbf{u}_{4q}(t)]^{\mathrm{T}}$ is made in the input to (3.29) for identifying environments and model for interfacing to standard medium $\mathbf{y}_4(t)$ in this case becomes

$$\mathbf{x}_4^{\cdot}(t) = \mathbf{A}_4\mathbf{x}_4(t) + \mathbf{B}_4\mathbf{u}_4(t) + \mathbf{B}_4\mathbf{w}_4(t) \tag{3.35}$$

$$\mathbf{y}_4(t) = \mathbf{C}_4\mathbf{x}_4(t) \tag{3.36}$$

Transferring from the standard medium of CPU back to specified environments is described

$$\mathbf{x}_5^{\cdot}(t) = \mathbf{A}_5\mathbf{x}_5(t) + \mathbf{B}_5\mathbf{u}_5(t) + \mathbf{B}_5\mathbf{w}_5(t) \tag{3.37}$$

$$\mathbf{y}_5(t) = \mathbf{C}_5\mathbf{x}_5(t) \tag{3.38}$$

where $\mathbf{u}_5(t) = [\mathbf{u}_{51}(t) - \mathbf{K}_4\mathbf{x}_4(t)]^{\mathrm{T}}$; $\mathbf{u}_{51}(t)$ consists informations of standard medium, $\mathbf{K}_4\mathbf{x}_4(t)$ carries informations on linear dynamic transfer in according to Riccati equation [21].

On transformation conditions $\mathbf{y}_4(t) = \mathbf{u}_{51}(t)$, $\mathbf{y}_5(t) = \mathbf{u}_4(t)$, one obtains

$$\mathbf{x}_6^{\cdot}(t) = \mathbf{A}_6\mathbf{x}_6(t) + \mathbf{B}_6\mathbf{u}_6(t) \tag{3.39}$$

$$\mathbf{y}_6(t) = \mathbf{C}_6\mathbf{x}_6(t) \tag{3.40}$$

where

$$\mathbf{x}_6(t) = \left[\mathbf{x}_4^{\mathrm{T}}(t) \quad \mathbf{x}_5^{\mathrm{T}}(t)\right]^{\mathrm{T}},\ \mathbf{u}_6(t) = \left[\mathbf{w}_4^{\mathrm{T}}(t) \quad \mathbf{w}_5^{\mathrm{T}}(t)\right]^{\mathrm{T}},\ \mathbf{y}_6(t) = \left[\mathbf{y}_4^{\mathrm{T}}(t) \quad \mathbf{y}_5^{\mathrm{T}}(t)\right]^{\mathrm{T}},$$

$$\mathbf{A}_6 = \begin{bmatrix} \mathbf{A}_4 & \mathbf{B}_4\mathbf{C}_5 \\ \begin{bmatrix} \mathbf{B}_{51}\mathbf{C}_4 \\ -\mathbf{B}_{52}\mathbf{K}_4 \end{bmatrix} & \mathbf{A}_5 \end{bmatrix},\ \mathbf{B}_6 = \begin{bmatrix} \mathbf{B}_4 & 0 \\ 0 & \begin{bmatrix} \mathbf{B}_{51} \\ \mathbf{B}_{52} \end{bmatrix} \end{bmatrix},\ \mathbf{C}_6 = \begin{bmatrix} \mathbf{C}_4 & 0 \\ 0 & \mathbf{C}_5 \end{bmatrix}.$$

"Soft-characterizing" makes rise to presence of $\mathbf{B}_{52}$ in $\mathbf{B}_6$ and $\mathbf{B}_{52}\mathbf{K}_4$ in $\mathbf{A}_6$ and this model is a generalized version of that described by (3.33) and (3.34).

3.3.3. Robust of Electronic interface modeling [20 - 22]

3.3.3.1. Necessary conditions for robust performance *a) Assumption*

- $\mathbf{A}_3$ is a stability matrix,
- $(\mathbf{A}_3, \mathbf{B}_3, \mathbf{C}_3)$: Controllable and observable; $(\mathbf{A}_3, \mathbf{B}_3)$: stabilisable, $(\mathbf{C}_3, \mathbf{A}_3)$: detectable.
- $\mathbf{A}_1\mathbf{A}_1^{\mathrm{T}} = \mathrm{diag}\left(\alpha_{1M}^2, \ldots, \alpha_{1m}^2\right)$, $\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}} = \mathrm{diag}\left(\beta_{1M}^2, \ldots, \beta_{1m}^2\right)$, $\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1 = \mathrm{diag}\left(\gamma_{1M}^2, \ldots, \gamma_{1m}^2\right)$,
- $\mathbf{A}_2\mathbf{A}_2^{\mathrm{T}} = \mathrm{diag}\left(\alpha_{2M}^2, \ldots, \alpha_{2m}^2\right)$, $\mathbf{B}_2\mathbf{B}_2^{\mathrm{T}} = \mathrm{diag}\left(\beta_{2M}^2, \ldots, \beta_{2m}^2\right)$, $\mathbf{C}_2^{\mathrm{T}}\mathbf{C}_2 = \mathrm{diag}\left(\gamma_{2M}^2, \ldots, \gamma_{2m}^2\right)$
- $\mathbf{F}_3(s) = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}s - \mathbf{A}_1 & -\mathbf{B}_1\mathbf{C}_2 \\ -\mathbf{B}_2\mathbf{C}_1 & \mathbf{I}s - \mathbf{A}_2 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{F}_3^{11} & \mathbf{F}_3^{12} \\ \mathbf{F}_3^{21} & \mathbf{F}_3^{22} \end{bmatrix}$
- Conditions for Bi-directional tranferring; i.e., $\left\|\mathbf{F}_3^{12}\right\| = \left\|\mathbf{F}_3^{21}\right\|$.

b) Conditions for each transfering

- For forward transferring

$$\left\|\mathbf{F}_{\mathrm{x}}\right\| \le \left\|\mathbf{C}_1\right\| \left\|\left(\mathbf{I}s - \mathbf{A}_1\right)^{-1}\right\| \left\|\mathbf{B}_1\right\| = \frac{\gamma_{1M}\beta_{1M}}{1+\alpha_{1m}}$$

$$\left\|\mathbf{F}_{3X}^{22}\right\| \le \left\|\mathbf{C}_2\right\| \left\|\mathbf{B}_2\right\| \left\| \left[\left(\mathbf{I}s - \mathbf{A}_2\right) - \mathbf{B}_2\mathbf{F}_{X}\mathbf{C}_2\right]^{-1}\right\| \le \frac{\left(1+\alpha_{1M}\right)\beta_{2M}\gamma_{2M}}{\left(1+\alpha_{1M}\right)\left(1+\alpha_{2m}\right)+\beta_{1m}\beta_{2m}\gamma_{1m}\gamma_{2m}}$$

$$\left\|\mathbf{F}_{3X}^{12}\right\| = \left\|\mathbf{F}_{3X}^{21}\right\| \le \left\|\mathbf{F}_{X}\right\| \left\|\mathbf{F}_{3X}^{22}\right\| \le \frac{\gamma_{1M}\beta_{1M}}{1+\alpha_{1m}} \times \frac{\gamma_{2M}\beta_{2M}\left(1+\alpha_{1M}\right)}{\left(1+\alpha_{1M}\right)\left(1+\alpha_{2M}\right)+\gamma_{1m}\gamma_{2m}\beta_{1m}\beta_{2m}}$$

$$\left\|\mathbf{F}_{3X}^{11}\right\| \le \left\|\mathbf{F}_{X}\right\| \left\{1+\left\|\mathbf{F}_{3X}^{12}\right\|\right\} \le \frac{\gamma_{1M}\beta_{1M}}{1+\alpha_{3m}} \left\{1+\frac{\left(1+\alpha_{1M}\right)}{\left(1+\alpha_{1m}\right)} \times \frac{\gamma_{1M}\beta_{1M}\gamma_{2M}\beta_{2M}}{\left(1+\alpha_{1M}\right)\left(1+\alpha_{2m}\right)+\beta_{1m}\beta_{2m}\gamma_{1m}\gamma_{2m}}\right\}$$

- For back transferring

$$\left\|\mathbf{F}_{\mathrm{N}}\right\| \le \left\|\mathbf{C}_2\right\| \left\|\left(\mathbf{I}s - \mathbf{A}_2\right)^{-1}\right\| \left\|\mathbf{B}_2\right\| = \frac{\beta_{2M}\gamma_{2M}}{1+\alpha_{2m}}$$

$$\left\|\mathbf{F}_{1N}^{11}\right\| \le \left\|\mathbf{C}_1\right\| \left\|\mathbf{B}_1\right\| \left\| \left[\left(\mathbf{I}s - \mathbf{A}_1\right) - \mathbf{B}_1\mathbf{F}_{N}\mathbf{C}_1\right]^{-1}\right\| \le \frac{\left(1+\alpha_{2M}\right)\beta_{1M}\gamma_{1M}}{\left(1+\alpha_{1m}\right)\left(1+\alpha_{2M}\right)+\gamma_{2m}\gamma_{2m}\beta_{1m}\beta_{2m}}$$

$$\left\|\mathbf{F}_{3N}^{21}\right\| = \left\|\mathbf{F}_{3N}^{12}\right\| \le \left\|\mathbf{F}_{N}\right\| \left\|\mathbf{F}_{3N}^{11}\right\| \le \frac{\left(1+\alpha_{2M}\right)}{\left(1+\alpha_{2m}\right)} \times \frac{\gamma_{1M}\gamma_{2M}\beta_{1M}\beta_{2M}}{\left(1+\alpha_{1m}\right)\left(1+\alpha_{2M}\right)+\gamma_{1m}\gamma_{2m}\beta_{1m}\beta_{2m}}$$

$$\left\|\mathbf{F}_{3N}^{22}\right\| \le \left\|\mathbf{C}_4\right\| \left\|\mathbf{F}_{N}\right\| \left\{1+\left\|\mathrm{F}_{3N}^{21}\right\|\right\} \le \frac{\gamma_{2M}\beta_{2M}}{1+\alpha_{2m}} \left\{1+\frac{1+\alpha_{2M}}{1+\alpha_{1m}} \times \frac{\beta_{1M}\beta_{2M}\gamma_{1M}\gamma_{2M}}{\left(1+\alpha_{1m}\right)\left(1+\alpha_{2M}\right)+\beta_{1m}\beta_{2m}\gamma_{1m}\gamma_{2m}}\right\}$$

c) Conditions on applying Nyquist diagrams for each transfering

- For forwarding

$$\frac{\gamma_{1M}\beta_{1M}}{1+\alpha_{1m}}\left\{1+\frac{1+\alpha_{1M}}{1+\alpha_{1m}}\times\frac{\gamma_{1M}\gamma_{2M}\beta_{1M}\beta_{2M}}{(1+\alpha_{1M})(1+\alpha_{2m})+\gamma_{1m}\gamma_{2m}\beta_{1m}\beta_{2m}}\right\}\le 1$$

- For backwarding

$$\frac{\gamma_{2M}\beta_{2M}}{1+\alpha_{2m}}\left\{1+\frac{1+\alpha_{2M}}{1+\alpha_{2m}}\times\frac{\gamma_{1M}\gamma_{2M}\beta_{1M}\beta_{2M}}{(1+\alpha_{1m})(1+\alpha_{2M})+\gamma_{1m}\gamma_{2m}\beta_{1m}\beta_{2m}}\right\}\le 1$$

3.3.3.2. Uncertainty structure

a) Assumption

- Uncertainty parameters

$$(\mathbf{x}_3^{\cdot}(t)+\Delta\mathbf{x}_3^{\cdot}(t))=(\mathbf{A}_3+\Delta\mathbf{A}_3)(\mathbf{x}_3(t)+\Delta\mathbf{x}_3(t))+(\mathbf{B}_3+\Delta\mathbf{B}_3)\mathbf{u}_3(t)$$

$$(\mathbf{y}_3(t)+\Delta\mathbf{y}_3(t))=(\mathbf{C}_3+\Delta\mathbf{C}_3)(\mathbf{x}_3(t)+\Delta\mathbf{x}_3(t))$$

$$\mathbf{F}_3^*(s)=\frac{\mathbf{Y}_3(s)+\Delta\mathbf{Y}_3(s)}{\mathbf{U}_3(s)}=(\mathbf{C}_3+\Delta\mathbf{C}_3)\left[\mathbf{I}s-(\mathbf{A}_3+\Delta\mathbf{A}_3)\right]^{-1}(\mathbf{B}_3+\Delta\mathbf{B}_3)$$

- Consistent conditions and matrix-norm to every blocks of transfer matrix for forward and back transferring are satisfied.

b) Bounded parameters

For each-ward transferring, block components of transfer matrix can be considered as a plant or a controller suitably, and boundary conditions are then obtained.

- For forwarding

$$\frac{\gamma_{3M}\beta_{3M}}{1+\alpha_{3m}}\le 1\,\text{(a) and }\frac{(1+\alpha_{3M})\gamma_{4M}\beta_{4M}}{(1+\alpha_{3M})(1+\alpha_{4m})+\beta_{3m}\beta_{4m}\gamma_{3m}\gamma_{4m}}\le 1\ \text{(b)}$$

- For backwarding

$$\frac{\gamma_{4M}\beta_{4M}}{1+\alpha_{4m}}\le 1\,\text{(a) and }\frac{(1+\alpha_{4M})\gamma_{3M}\beta_{3M}}{(1+\alpha_{3m})(1+\alpha_{4M})+\beta_{3m}\beta_{4m}\gamma_{3m}\gamma_{4m}}\le 1\ \text{(b)}$$

3.3.3.3. Sufficient conditions

a) Assumption

- Parameters are bounded,
- Conditions for Bi-directional tranferring are satisfied also in the case of uncertainty.

b) Conditions obtained

- For forwarding

$$\left(\frac{\Delta\gamma_{1M}}{\gamma_{1M}}+\frac{\Delta\beta_{1M}}{\beta_{1M}}+\frac{\Delta\alpha_{1m}}{1+\alpha_{1m}}\right)\left\{1+\frac{\gamma_{1M}\beta_{1M}}{1+\alpha_{1m}}\times\frac{(1+\alpha_{1M})\gamma_{2M}\beta_{2M}}{(1+\alpha_{1M})(1+\alpha_{2m})+\gamma_{1m}\gamma_{2m}\beta_{1m}\beta_{2m}}\right\}\le 1$$

- For backwarding

$$\left(\frac{\Delta\gamma_{2M}}{\gamma_{2M}}+\frac{\Delta\beta_{2M}}{\beta_{2M}}+\frac{\Delta\alpha_{2m}}{1+\alpha_{2m}}\right)\left\{1+\frac{\gamma_{2M}\beta_{2M}}{1+\alpha_{2m}}\times\frac{(1+\alpha_{2M})\gamma_{1M}\beta_{1M}}{(1+\alpha_{1m})(1+\alpha_{2M})+\gamma_{1m}\gamma_{2m}\beta_{1m}\beta_{2m}}\right\}\le 1$$

3.3.4. Robust of electronic soft-switch modeling [19, 20]

3.3.4.1. Establishing necessary conditions

a) Assumption

- $\mathbf{A}_6$ is a stability matrix,
- $(\mathbf{A}_6,\mathbf{B}_6,\mathbf{C}_6)$: Controllable and observable; $(\mathbf{A}_6,\mathbf{B}_6)$: stabilisable, $(\mathbf{C}_6,\mathbf{A}_6)$: detectable,
- Nominal values of $\mathbf{A}_6$, $\mathbf{B}_6$, $\mathbf{C}_6$ are those coresponding to the case when $\mathbf{w}_6(t) = \mathbf{0}$. With a chosen value of $\mathbf{A}_4$ and $\mathbf{A}_5$, (3.39) and (3.40) become $\mathbf{x}_6^{\cdot}(t)=\mathbf{A}_6\mathbf{x}_6(t)$ and $\mathbf{y}_6(t)=\mathbf{C}_6\mathbf{x}_6(t)$.

b) Paramater estimation process

Minimizing a functional cost weighted with

$$\mathbf{S}_1=\begin{bmatrix}\mathbf{S}_1^{11} & \mathbf{S}_1^{12}\\ \mathbf{S}_1^{21} & \mathbf{S}_1^{22}\end{bmatrix},\ \mathbf{S}_2=\begin{bmatrix}\mathbf{S}_2^{11} & \mathbf{0}\\ \mathbf{0} & \mathbf{S}_2^{22}\end{bmatrix} \text{ and } \mathbf{S}_3=\begin{bmatrix}\mathbf{S}_3^{11} & \mathbf{S}_3^{12}\\ \mathbf{S}_3^{21} & \mathbf{S}_3^{22}\end{bmatrix}$$

$$J(\mathbf{B}_4,\mathbf{B}_{51},\mathbf{B}_{52},\mathbf{C}_4,\mathbf{C}_5,\mathbf{K}_4)=\mathrm{E}\left\{\mathbf{x}_6^{\mathrm{T}}(t)\mathbf{S}_1\mathbf{x}_6(t)+\mathbf{x}_6^{\mathrm{T}}(t)\mathbf{S}_2\mathbf{y}_6(t)+\mathbf{y}_6^{\mathrm{T}}(t)\mathbf{S}_3\mathbf{y}_6(t)(t)\right\}=\mathrm{tr}(\mathbf{Q}_6\mathbf{R}_6)\ (4.41)$$

within the set for $(\mathbf{A}_6,\mathbf{B}_6,\mathbf{C}_6)$ to be controllable and observable, where $\mathbf{Q}_6=\mathrm{E}\left\{\mathbf{x}_6(t)\mathbf{x}_6^{\mathrm{T}}(t)\right\}$, $\mathbf{R}_6=\left\{\mathbf{S}_1+\mathbf{S}_2\mathbf{C}_6+\mathbf{C}_6^{\mathrm{T}}\mathbf{S}_3\mathbf{C}_6\right\}$, the necessary conditions are obtained as [19, 20]

$$\left\{\mathbf{A}_4+\mathbf{B}_4\mathbf{C}_5\mathbf{Q}_6^{21}(\mathbf{Q}_6^{11})^{-1}\right\}\mathbf{Q}_6^{11}+\mathbf{Q}_6^{11}\left\{\mathbf{A}_4+\mathbf{B}_4\mathbf{C}_5\mathbf{Q}_6^{21}(\mathbf{Q}_6^{11})^{-1}\right\}^{\mathrm{T}}+\mathbf{V}_6^{11}=\mathbf{0} \qquad (3.42)$$

$$\left\{\mathbf{A}_4\mathbf{Q}_6^{12}(\mathbf{Q}_6^{22})^{-1}+\mathbf{B}_4\mathbf{C}_5\right\}\mathbf{Q}_6^{22}+\mathbf{Q}_6^{11}\left\{\mathbf{A}_5\mathbf{Q}_6^{21}(\mathbf{Q}_6^{11})^{-1}+\begin{bmatrix}\mathbf{B}_{51}\mathbf{C}_4\\ \text{-}\mathbf{B}_{52}\mathbf{K}_4\end{bmatrix}\right\}^{\mathrm{T}}=\mathbf{0} \qquad (3.43)$$

$$\left\{\mathbf{A}_5+\begin{bmatrix}\mathbf{B}_{51}\mathbf{C}_4\\ \text{-}\mathbf{B}_{52}\mathbf{K}_4\end{bmatrix}\mathbf{Q}_6^{12}(\mathbf{Q}_6^{22})^{-1}\right\}\mathbf{Q}_6^{22}+\mathbf{Q}_6^{22}\left\{\mathbf{A}_5+\begin{bmatrix}\mathbf{B}_{51}\mathbf{C}_4\\ \text{-}\mathbf{B}_{52}\mathbf{K}_4\end{bmatrix}\mathbf{Q}_6^{12}(\mathbf{Q}_6^{22})^{-1}\right\}^{\mathrm{T}}+\mathbf{V}_6^{22}=\mathbf{0} \qquad (3.44)$$

$$\left\{\mathbf{A}_5+(\mathbf{P}_6^{22})^{-1}\mathbf{P}_6^{21}\mathbf{B}_4\mathbf{C}_5\right\}^{\mathrm{T}}\mathbf{P}_6^{22}+\mathbf{P}_6^{22}\left\{\mathbf{A}_5+(\mathbf{P}_6^{22})^{-1}\mathbf{P}_6^{21}\mathbf{B}_4\mathbf{C}_5\right\}+\mathbf{C}_5^{\mathrm{T}}\mathbf{S}_3^{22}\mathbf{C}_5+\mathbf{S}_2^{22}\mathbf{C}_5+\mathbf{S}_1^{22}=\mathbf{0} \qquad (3.45)$$

$$\left\{(\mathbf{P}_6^{22})^{-1}\mathbf{P}_6^{21}\mathbf{A}_4+\begin{bmatrix}\mathbf{B}_{51}\mathbf{C}_4\\ \text{-}\mathbf{B}_{52}\mathbf{K}_4\end{bmatrix}\right\}^{\mathrm{T}}\mathbf{P}_6^{22}+\mathbf{P}_6^{11}\left\{(\mathbf{P}_6^{22})^{-1}\mathbf{P}_6^{21}\mathbf{A}_5+\mathbf{B}_4\mathbf{C}_5\right\}+\mathbf{C}_4^{\mathrm{T}}\mathbf{S}_3^{12}\mathbf{C}_5+\mathbf{S}_1^{12}=\mathbf{0} \qquad (3.46)$$

$$\left\{\mathbf{A}_5+(\mathbf{P}_6^{22})^{-1}\mathbf{P}_6^{21}\mathbf{B}_4\mathbf{C}_5\right\}^{\mathrm{T}}\mathbf{P}_6^{22}+\mathbf{P}_6^{22}\left\{\mathbf{A}_5+(\mathbf{P}_6^{22})^{-1}\mathbf{P}_6^{21}\mathbf{B}_4\mathbf{C}_5\right\}+\mathbf{C}_5^{\mathrm{T}}\mathbf{S}_3^{22}\mathbf{C}_5+\mathbf{S}_2^{22}\mathbf{C}_5+\mathbf{S}_1^{22}=\mathbf{0} \quad (3.47)$$

Using combinations of the necessay conditions, the unknown are determinated.

c) Comments on the uniqueness of parameter set

Different combinations amongst the above necessay conditions may give rise different sets of solutions. The fact is that no sufficient condition has been found; the solution may correspond to a local extreme. Moreover, in the state space description, one solution set may be the image of other set obtaining by a similarity transformation.

However, in the case when $\mathbf{V}_4 = \mathbf{S}_1 = \mathbf{S}_2 = \mathbf{S}_3 = \mathbf{I}$ and diag($\mathbf{Q}_4$) = diag($\mathbf{P}_4$), and unique solution set may be obtained. These are the conditions for bi-directional transformation.

3.3.4.2. Establishing necessary conditions for robustness

a) Assumption

- Uncertain parameters

$$\mathbf{x}_6^{\cdot}(t)+\Delta\mathbf{x}_6^{\cdot}(t)=(\mathbf{A}_6+\mathbf{\Delta A}_6)(\mathbf{x}_6(t)+\mathbf{\Delta x}_6(t))+(\mathbf{B}_6+\mathbf{\Delta B}_6)\mathbf{w}(t)$$

$$\mathbf{y}(t)+\Delta\mathbf{y}(t)=(\mathbf{C}_6+\mathbf{\Delta C}_6)(\mathbf{x}_6(t)+\mathbf{\Delta x}_6(t))$$

$$\tilde{\mathbf{Q}}_6=\mathrm{SupE}\left\{(\mathbf{x}_6+\mathbf{\Delta x}_6)(\mathbf{x}_6+\mathbf{\Delta x}_6)^{\mathrm{T}}\right\},\ \tilde{\mathbf{R}}_6=\mathrm{Sup}\{\mathbf{S}_1+\mathbf{S}_2(\mathbf{C}_6+\Delta\mathbf{C}_6)+(\mathbf{C}_6+\Delta\mathbf{C}_6)^{\mathrm{T}}\mathbf{S}_3(\mathbf{C}_6+\Delta\mathbf{C}_6)\}$$

- Using the same functional cost and constraint conditions,
- Using Petersen-Hollot bound.

b) Sufficient conditions

- Weighted cost function becomes

$$J_{\mathrm{d}}(.,...,.)=\mathrm{tr}(\tilde{\mathbf{Q}}_6\tilde{\mathbf{R}}_6)\geq J(.,...,.)+\mathrm{tr}(\Delta\mathbf{Q}_6\mathbf{R}_6)$$

- $\mathbf{\Delta A}_6$ is a stability matrix of appropriate dimension satisfying the following condition

$$(\mathbf{A}_4+\Delta\mathbf{A}_4)\Delta\mathbf{Q}_4+\mathbf{\Delta Q}_6(\mathbf{A}_6+\mathbf{\Delta A}_6)^{\mathrm{T}}+\mathbf{B}_6\mathbf{B}_6^{\mathrm{T}}+\Omega\left(\mathbf{Q}_6,\mathbf{\Delta A}_6,\Psi(\mathbf{B}_6,\mathbf{\Delta B}_6)\right)-\mathbf{V}_6=\mathbf{0}$$

where $\mathbf{\Omega}(.)=\mathbf{\Delta A}_6\mathbf{Q}_6+\mathbf{Q}_6\mathbf{\Delta A}_6^{\mathrm{T}}+\mathbf{\Psi}_6(.)$ with $\mathbf{\Psi}_6(.)=\mathbf{\Delta B}_6\mathbf{B}_6^{\mathrm{T}}+\mathbf{B}_6\mathbf{\Delta B}_6^{\mathrm{T}}+\mathbf{\Delta B}_6\mathbf{\Delta B}_6^{\mathrm{T}}$

- In the case when an estimation of technical quality, an optimization process is required to carry out with the use of Lagragian function defined as

$$\mathscr{L}(.)=\mathrm{tr}\left\{\boldsymbol{\lambda}_{\mathrm{d}}\mathbf{Q}_6\mathbf{\Delta P}_6+\left[(\mathbf{A}_6+\mathbf{\Delta A}_6)\mathbf{\Delta Q}_6+\mathbf{\Delta Q}_6(\mathbf{A}_6+\mathbf{\Delta A}_6)^{\mathrm{T}}+\mathbf{B}_6\mathbf{B}_6^{\mathrm{T}}\right]\mathbf{\Delta P}_6\right\}$$

where $0 < \boldsymbol{\lambda}_{\mathrm{d}} \leq 1$ and $\mathbf{\Delta P}_6$ is a Lagrangian multipler; they are not 0 in the same time.

c) Comments on the technique used

Robustness of closed loop thinking problems is found complicate one by adopting parameter-optimization technique used by other authors. A great effort would be reduced in tackling the mentioned robustness by adopting the state-optimization approach.

It is found also that robust problems play an important role in estimating technology standard, which is on the direction for further researchs.

4. CONCLUDING REMARKS

Optimal projection equation (OPEQ) has been recognized to play an important contribution to finding the uniqueness amongst multi-extreme in the effect sense of an aditionally constrained condition. However, a complexity happened to be in mathematical involvement of that OPEQ on adopting parameter-optimization process from both aspects; in the establishment and in the solution to the mentioned OPEQ. State-optimization has been found removing that complexity due to the role of factorization in term of a partial isometry and mentioned factorization has an effect of that of an additionally constrained condition to the optimization process.

State-optimization approach can be employed to treating different various problems where an optimization is asked for. In the case of an infinite-dimensional (S) like distributed parameter, non-linear modeled by a series, ect., where partial or functional equations are required, then the concept of generaliazed Green function and its inverse are to be adopted, however. This may gives rise to the concept of a poly-optimization in stead of state-optimization and various researches can be carried out in this direction apart from treating the above mentioned infinite-dimensional (S) also for treating many different optimization problems happened to be in non-finite dimensional space.

Acknowledgment. Authors record their indebtedness to Professor Nguyen Ngoc San, D.Sc.(Eng) of the Posts and Telecommunication Institute of Technology for suggesting about the paper. Also, the authors are thankful to referrers for their useful comments on the paper.

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TÓM TẮT

VỀ PHƯƠNG PHÁP TỐI ƯU THEO TRẠNG THÁI VỚI CÁC BÀI TOÁN HỆ THỐNG: XỬ LÍ THEO TƯ DUY HỆ KÍN

Có thể phân các bài toán thuộc lĩnh vực lí thuyết hệ thống thành 4 nhóm chính: mô phỏng, xác lập phương trình toán học, phân tích hệ và thiết kế hệ thống. Khi giới hạn những bàn luận đối với một hệ thống được mô tả bởi hệ phương trình trong không gian trạng thái thì có thể phân các bài toán thành nhóm phụ thuộc vào kiểu xử lí: cách của tư duy hệ hở và của tư duy hệ kín.

Gần đây nhất, có hai phương pháp tiếp cận đáng chú ý đối với cả hai kiểu xử lí là sử dụng điều kiện cân bằng nội và hệ phương trình quy chiếu tối ưu (OPEQ). Phương pháp đề xuất dùng điều kiện cân bằng nội có ưu điểm nổi trội là sử dụng được tính bất biến về đóng góp của động học vào quá trình tạo ra quan hệ vào ra của hệ, nhưng lại bị hạn chế trên quan điểm tối ưu do không biết được nghiệm tối ưu. Phương pháp xây dựng OPEQ loại bỏ hạn chế về tính tối ưu nhưng lại đối mặt với tính phức tạp về mặt sử dụng toán học trong quá trình phát triển, tìm nghiệm của OPEQ, tuy rằng phương pháp OPEQ được xác định là tìm ra điều kiện ràng buộc thêm vào các điều kiện ban đầu của bài toán tối ưu. Phương pháp tối ưu theo trạng thái được minh chứng đã thụ hưởng các ưu điểm, bỏ lại hạn chế của cả hai phương pháp đã nêu và còn tạo ra hiệu ứng như của một điều kiện ràng buộc mới nhờ việc thừa số hoá phép biến đổi không đồng nhất giữa các vector trạng thái của hai hệ động học theo đẳng cự thành phần (partial isometry).

Bài báo này gồm 4 phần. Phần đầu giới thiệu tổng quát về những nội dung trình bày trong bài báo. Phần thứ hai, tóm tắt những điểm cơ bản liên quan đến tiêu chí tối ưu và thừa số hoá biến đổi không đồng nhất làm sở cứ để giải quyết các bài toán điển hình kiểu xử lí theo cách của tư duy hệ hở đã được trình bày ở bài trước và sử dụng kết quả đã thu được xử lí các bài toán theo cách của tư duy hệ kín. Nội dung áp dụng và các kết quả tương ứng được trình bày trong phần thứ ba của bài báo. Tuy các phép chiếu tối ưu tìm thấy bởi phương pháp tối ưu trạng thái đều vuông, nhưng tính phức tạp về mặt toán học vẫn còn hiện diện khá rõ nét ở quá trình xử lí, xây dựng các hệ phương trình OPEQ đối với các bài toán thuộc kiểu xử lí theo cách tư duy hệ kín và hiện diện ở hầu hết các quá trình xác định nghiệm của các bài toán về tính bền vững.