# DESIGN OF A HEAT EXCHANGER NETWORK BY GRAPH METHOD

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# ABSTRACT

The problem of synthesizing a network of heat exchangers can be resolved by the temperature – enthalpy diagram or by the table method, but they are not convenient in large heat exchanger network design. In this study, we use graph theory to present and to design a heat exchanger network. The developed method is one way to avoid the difficulties inherent with applying temperature – enthalpy diagram or table method to a large complex network and the presentation of network as a grid in graph theory is very clear and allows for easy modification.

Key words. Design of heat exchanger network

#### **1. INTRODUCTION**

Process streams at high pressure or temperature contain energy that can be usefully recovered. The most common energy recovery technique is to utilize the heat in high temperature process stream to heat a colder stream (saving steam costs, and also cooling water if the hot stream requires cooling). In an industrial process there will be many hot and cold streams and there will be an optimum arrangement of the streams for energy recovery by heat exchange. The problem of heat exchanger network design is to create a minimum cost network of exchangers that will also meet the design specifications on the required outlet temperature of each stream. The problem of synthesizing a network of heat exchangers can be resolved by the temperature – enthalpy diagram or by the table method, but they are not convenient for large heat exchanger network design. However, if the strictly mathematical approach is taken of setting up all possible arrangements and searching for the optimum, the problem, even for a small number of exchangers, would require an inordinate amount of computer time (for a process with four cold and three hot streams,  $2.4 \times 10^8$  arrangements are possible). In this study, we use graph theory to present, to design and to optimize a heat exchanger network. This method is one way to avoid the difficulties inherent with applying temperature – enthalpy diagram and table method to a large complex heat exchanger network. The presentation of a heat exchanger network as a grid in graph theory is very clear and allows for easy its modification.

# 2. BASIC UNDERSTANDING OF GRAPH THEORY FOR HEAT EXCHANGER NETWORK DESIGN

The development and application of the method can be illustrated by considering the problem of integrating the utilization of energy between four process streams: two hot streams which require cooling, and two cold streams that have to be heated. The process data for the streams is set out in Fig. 2. Each stream starts from a source temperature  $T_s$ , and is to be heated or cooled to a target temperature  $T_T$ . The heat capacity of each stream is shown as CP. For stream where the specific heat capacity can be taken as constant, and there is no phase change, CP will be given by:

$$CP = m.C_P \tag{1}$$

where m – mass flow rate, kg/s;  $C_P$  – average specific heat capacity between  $T_S$  and  $T_T$ , kJ/kg.<sup>0</sup>C.

*Number of heat exchanger units.* To understand the minimum number of matches or units in a heat exchanger network, some basic results of graph theory can be used.

A graph in any collection of points in which some pairs of points are connected by lines. Figures 1a and 1b give two examples of graphs. Note that the lines such as BG, CE and CF in Fig.1 are not supposed to cross, that is, the diagram should be drawn in three dimensions. This is true for the other lines in Fig. 1 that appear to cross.

In this context, the points correspond to process and utility streams, and the lines to heat exchange matches between the heat sources and heat sinks.

A path is a sequence of distinct lines that are connected to each other. For example, in Fig. 1a AECGD is a path. A graph forms a single component (sometimes called a separate system) if any two points are joined by a path. Thus, Fig. 1b has two components (or two separate systems), and Fig. 1a has only one.

A loop is a path that begins and ends at the same point, like CGDHC in Fig. 1a. If two loop have a line in common, they can be linked to form a third loop by deleting the common line. In Fig. 1a, for example, BGCEB and CGDHC can be linked to give BGDHCEB. In this case, this last loop is said to be dependent on the other two.

From graph theory, the main result needed in the present context is that the number of independent loops for a graph is given by:

$$N_{\text{UNITS}} = S + L - C \tag{2}$$

where  $N_{UNITS}$  – number of matches or units (lines in graph theory); S – number of streams including utilities (points in graph theory); L – number of independent loops; C – number of component (or number of separate systems).

In general, the final network design should be achieved in the minimum number of units to keep down the capital cost (although this is not the only consideration to keep down the capital cost). To minimize the number of units in equation 2, L should be zero and C should be a maximum. Assuming L to be zero in the final design is a reasonable assumption. However, what should be assumed about C? Consider the network in Fig. 1b that has two components. For there to be two components, the heat duties for stream A and B must exactly balance the duties for stream E and F. Also, the heat duties for stream C and D must exactly balance the duties for streams G and H. Such balances are likely to be unusual and not easy to predict. The safest assumption for C thus appears to be that there will be one component only, that is, C = 1. This

leads to an important special case when the network has a single component and is loop-free. In this case:

$$N_{\rm UNITS} = S - 1 \tag{3}$$

Equation 3 put in words states that the minimum number of units required is one less than the number of streams (including utility streams).

This is a useful result since, if the network is assumed to be loop-free and has a single component, the minimum number of units can be predicted simply by knowing the number of streams. If the problem does not have a pinch, then equation 3 predict the minimum number of units. If the problem has a pinch, then equation 3 is applied on each side of the pinch separately:

 $N_{\text{UNITS}} = (S_{\text{ABOVE PINCH}} - 1) + (S_{\text{BELOW PINCH}} - 1)$ (4)



*Figure 1.* Two alternative graphs



Figure 2. Grid representation for the heat recovery problem

The pinch design method has suggested that a good initialization would be to assume that no individual exchanger should have a temperature difference smaller than  $\Delta T_{min}$ . Having made this assumption, two rules were deducted that if the energy target is to be achieved, the design must not transfer heat across the pinch by: Process – to – process heat transfer; Inappropriate use of utilities.

These rules are necessary for the design to achieve the energy target, given that no individual exchanger should have a temperature difference smaller than  $\Delta T_{min}$ . To comply with these two rules, the process should be divided at the pinch. This is most clearly done by

representing the stream data in the grid diagram. Figure 2 shows a example of the stream data in grid form with the pinch marked. Above the pinch, steam can be used (up to  $Q_{hmin}$ ), and below the pinch cooling water can be used (up to  $Q_{cmin}$ ).

# **3. APPLICATION OF GRAPH THEORY**

# 3.1. Strategy for heat exchanger network design

#### 3.1.1. Start at the pinch

The pinch is the most constrained region of the problem. At the pinch,  $\Delta T_{min}$  exists between all hot and cold streams. As a result, the number of feasible matches in this region is severely restricted. Quite often there are essential matches to be made. If such matches are not made, the result will be either use of temperature differences smaller than  $\Delta T_{min}$  or excessive use of utilities resulting from heat transfer across the pinch. If the design was started away from the pinch at the hot end or cold end of the problem, then initial matches are likely to need follow-up matches that violate the pinch or the  $\Delta T_{min}$  criterion as the pinch is approached. Putting the argument the other way around, if the design is started at the pinch, then initial decisions are made in the most constrained part of the problem. This is much less likely to lead to difficulties later.

#### 3.1.2. The CP inequality for individual matches

Figure 3 shows the temperature profiles for an individual exchanger at the pinch, above the pinch. Moving away from the pinch, temperature differences must increase. Figure 3a shows a match between a hot stream and a cold stream that has a CP smaller than the hot stream. At the pinch, the match starts with a temperature difference equal to  $\Delta T_{min}$ . The relative slopes of the temperature – enthalpy profiles of the two streams mean that the temperature differences become smaller moving away from the pinch, which is infeasible. On the other hand, Figure 3b shows a match involving the same hot stream but with a cold stream that has a larger CP. The relative slopes of the temperature – enthalpy profiles now cause the temperature differences to become larger moving away from the pinch, which is feasible. Thus, starting with  $\Delta T_{min}$  at the pinch, for temperature differences to increase moving away from the pinch:

$$CP_{hot} \le CP_{cold}$$
 (above the pinch) (5)

Figure 4 shows the situation below the pinch at the pinch. If a cold stream is matched with a hot stream with smaller CP, as shown in Fig.4a (i.e. a steeper slope), then the temperature differences become smaller (which is infeasible). If the same cold stream is matched with a hot stream with a larger CP (i.e. a less steeper slope), as shown in Fig. 4b, then temperature differences become larger, which is feasible. Thus, starting with  $\Delta T_{min}$  at the pinch, for temperature differences to increase moving away from the pinch:

$$CP_{hot} \ge CP_{cold}$$
 (6)

Note that the CP inequalities given by equation 5 and 6 only apply at the pinch and when both ends of the match are at pinch conditions.

#### 3.1.3. The "tick-off" heuristic

Once the matches around the pinch have been chosen to satisfy the criteria for minimum energy, the design should be continued in such a manner as to keep capital costs to a minimum.

One important criterion in the capital cost is the number of units. Keeping the number of units to a minimum can be achieved using the *tick-off heuristic*. To tick-off a stream, individual units are made as large as possible, that is, the smaller of the two heat duties on the streams being matched.



Figure 3. Criteria for the pinch matches above the pinch



Figure 4. Criteria for pinch matches below the pinch

Figure 5 shows the matches around the pinch from Fig. 3b with their duties maximized to tick off streams. It should be emphasized that the tick-off heuristic is only a heuristic and can occasionally penalize the design.

The design in Fig. 5a can now be completed by satisfying the heating and cooling duties away from the pinch. Cooling water must not be used above the pinch. Therefore, if there are hot streams above the pinch for which the pinch matches do not satisfy the duties, additional process – to – process heat recovery is required. Fig. 5b shows an additional match to satisfy the residual cooling of the hot streams above the pinch. Again, the duty on the unit is maximized. Finally, above the pinch, the residual heating duty on the cold streams must be satisfied. Since there are not hot streams left above the pinch, hot utility (H) must be used as shown in Fig. 5c.



Figure 5. Sizing the units above the pinch using the tick-off heuristic

#### Design below the pinch

Figure 6a shows the pinch design with the streams ticked off. If there are any cold streams below the pinch for which the pinch matches do not satisfy the duties, then additional process-to-process heat recovery is required, since hot utility must not be used. Figure 6b shows an additional match to satisfy the residual heating of the cold streams below the pinch. Again, the duty on the unit is maximized. Finally, below the pinch, the residual cooling duty on the hot streams must be satisfied. Since there are no cold streams left below the pinch, cold utility (C) must be used (Fig. 6c).



Figure 6. Sizing the units below the pinch using the tick-off heuristic

The final design shown in Fig. 7 amalgamates the hot end design from Fig. 5c and cold end design from Fig. 6c. The duty on hot utility is 7.5 MW ( $Q_{hmin}$ ) and the duty on cold utility is 10.0

MW ( $Q_{cmin}$ ). Note one further point from Fig. 7: the number of units is 7 in total (including the heater and cooler), that agrees with the result predicted by the equation 4.



Figure 7. The completed design for the data from Fig. 2

*Note:* If heat capacities of streams are such that it is not possible to make a match at the pinch without violating the minimum temperature difference condition, then the heat capacity can be altered by splitting a stream. Dividing a stream will reduce the mass flow rates in each leg and hence the heat capacities.

To seek the optimum design for a network

- 1. Start with the design for maximum heat recovery. The number of exchangers needed will be equal to or less than the number for maximum energy recovery.
- 2. Identify loops that cross the pinch. The design for maximum heat recovery will usually contain loops.
- 3. Starting with the loop with the least heat load, break the loops by adding or subtracting heat.
- 4. Check that the specified minimum temperature difference  $\Delta T_{min}$  has not been violated, and revise the design as necessary to restore the  $\Delta T_{min}$ .
- 5. Estimate the capital, operating costs, and the total annual cost.
- 6. Repeat the loop breaking and network revision to find the lowest cost design.
- 7. Consider the safety, operability and maintenance aspects of the proposed design.

#### 3.2. Example for network design for maximum energy recovery

Figure 8 shows the grid for 4 streams problem.

The network design above the pinch

$$CP_{hot} \leq CP_{cold}$$

1. Applying this condition at the pinch, stream 1 can be matched with stream 4, but not with 3.

Matching streams 1 and 4 and transferring the full amount of heat required to bring stream 1 to the pinch temperature gives:

 $\Delta H_{ex} = CP.(T_S - T_{pinch}) = 3.0(180 - 90) = 270 \text{ kW}.$ 

This will also satisfy the heat load required to bring stream 4 to its target temperature:



 $\Delta H_{ex} = 4.5(140 - 80) = 270 \text{ kW}.$ 

Figure 8. Grid for 4 stream problem

2. Stream 2 can be matched with stream 3, whilst satisfying the heat capacity restriction. Transferring the full amount to bring stream 3 to the pinch temperature:

 $\Delta H_{ex} = 1.0(150 - 90) = 60 \text{ kW}$ 

3. The heat required to bring stream 3 to its target temperature, from the pinch temperature, is:

$$\Delta H = 2.0(135 - 80) = 110 \text{ kW}$$

So the heater will have to be included to provide the remaining heat load:

 $\Delta H_{hot} = 110 - 60 = 50 \text{ kW}$ 

This checks with the value given by the problem table method.

The proposed network design above the pinch is shown in Fig.9.

Network design below the pinch

$$CP_{hot} \ge CP_{cold}$$

- 4. Stream 4 is at the pinch temperature,  $T_s = 80^{\circ}C$ .
- 5. A match between streams 1 and 3 adjacent to the pinch will satisfy the heat capacity restriction but not one between streams 2 and 3. So 1 is matched with 3 transferring the full amount to bring stream 1 to its target temperature, transferring:

$$\Delta H_{ex} = 3.0(90 - 60) = 90$$
 kW.



*Figure 9.* Proposed heat exchanger network with  $\Delta T_{min} = 10$  <sup>o</sup>C

6. Stream 3 requires more heat to bring it to the pinch temperature. Amount needed:

 $\Delta H = CP_3 (80 - 20) - \Delta H_{ex} = 3.0(80 - 20) - 90 = 30 \text{ kW}.$ 

This can be provided from the stream 2, as the match will now be away from the pinch. The rise in temperature of stream 3 will be given by:

$$\Delta T = \Delta H/CP$$

So transferring 30 kW will raise the temperature from the source temperature to:

$$20 + 30/2.0 = 35$$
 °C

and this gives a stream temperature difference on the outlet side of the exchanger of:

$$90 - 35 = 55$$
 °C

So the minimum temperature difference condition  $(10 \ ^{0}C)$  will not be violated by this match.

7. Stream 2 will need further cooling to bring it to its target temperature, so a cooler must be included. Cooling required:

 $\Delta H_{cold} = CP_2 (90 - 30) - \Delta H = 1.0(90 - 30) - 30 = 30 \text{ kW}$ 

which is the amount of the cold utility predicted by the problem table method.

The proposed network for maximum energy recovery is shown in Fig. 9.

A loop exists where a close path can be traced through the network. There is a loop in the network shown in Fig. 9, and this loop is also shown in Fig. 9. The presence of a loop indicates that there is scope for reducing the number of exchangers.

#### **4. CONCLUSION**

A good initialization for heat exchanger network design is to assume that no individual exchanger should have a temperature difference smaller than  $\Delta T_{min}$ . Having decided that no exchanger should have a temperature difference smaller than  $\Delta T_{min}$  two rules were deduced: to achieve the energy target there must be no transfer heat across the pinch by: (•) process-to-process heat transfer; (•) inappropriate use of utilities.

These rules are both necessary and sufficient for the design to achieve the energy target given that no individual exchanger should have a temperature difference smaller than  $\Delta T_{min}$ .

The design of heat exchanger networks by the grid method can be summarized in five steps:

- 1. Divide the problem at the pinch into separate problems.
- 2. The design for the separate problems is started at the pinch, moving away.
- 3. Temperature feasibility requires constrains on the CPs to be satisfied for matches between the streams at the pinch.
- 4. The loads on individual units are determined using the tick-off heuristic to minimize the number of units.
- 5. Away from the pinch, there is usually more freedom in the choice of matches. In this case, the designer can discriminate on the basis of operability, plant layout and so on.

Once the initial network structure has been defined, then loops, utility paths and stream splits offer the degrees of freedom for manipulating network cost in multivariable continuous optimization.

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# TÓM TẮT

# THIẾT KẾ MẠNG THIẾT BỊ TRAO ĐỔI NHIỆT BẰNG PHƯƠNG PHÁP LƯỚI

Bài toán tổng hợp mạng thiết bị trao đổi nhiệt có thể được giải quyết bằng phương pháp giản đồ "nhiệt độ - enthalpy" của các dòng hoặc bằng phương pháp bảng. Các phương pháp trên thường không thuận tiện cho mục đích thiết kế và tối ưu hóa mạng thiết bị trao đổi nhiệt. Trong nghiên cứu này chúng tôi sẽ sử dụng lí thuyết lưới để thể hiện và thiết kế mạng thiết bị trao đổi nhiệt. Phương pháp này cho phép tránh được những khó khăn và bất tiện của các phương pháp giản đồ "nhiệt độ - enthalpy" và phương pháp bảng trong thiết kế và tối ưu hóa mạng thiết bị trao đổi nhiệt tạo đổi nhiệt lớn và phức tạp.