ACTIVE CONTROL OF EARTHQUAKE-EXCITED STRUCTURES WITH THE USE OF HEDGE-ALGEBRAS-BASED CONTROLLERS

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ABSTRACT

In this paper, we introduce a hedge-algebras-based methodology in vibration control of structural systems to design fuzzy controllers, referred to as hedge-algebras-based controllers (HACs). In this methodology, vague linguistic terms are not expressed by fuzzy sets, but by inherent order relationships between vague terms existing in a term-domain. Semantically quantifying mappings (SQMs), which preserve semantics-based order relationships in term-domains, are defined in a close relationship with the fuzziness measure and the fuzziness intervals of vague terms. Utilizing these SQMs, fuzzy reasoning methods can be transformed into numeric interpolation methods with respect to the points in a multi-dimensional Euclid space defined relying on the if-then rules of the given control knowledge. This provides sound mathematical fundamentals supporting the construction of the control algorithm. The proposed methodology is simple, transparent and effective. As a case study, HACs and optimal HACs have been designed based on this methodology to control high-rise civil structures. They are shown to be more successful in reducing maximum displacement responses of the structure than fuzzy counterparts under three different earthquake scenarios: El Centro, Northridge and Kobe. This demonstrates the effectiveness of the proposed methodology.

Keywords: control theory, approximate reasoning, measure of fuzziness, earthquake engineering, hedge algebra

1. INTRODUCTION

Magnitude earthquakes result in massive movement of the ground and, therefore, cause serious damages to civil structures, in particular, to high-rise buildings. Such situation becomes more hazardous when in each decade, on the average, about 160 to 189 magnitude earthquakes have been recorded on continentals (www.iris.edu). Therefore, the protection of civil structure has been becoming one of the most imperative research tasks since long time ago. Many control strategies and structural control systems have been examined and designed to protect the civil structural systems from the damage caused by earthquake ground motion.

Structural vibration control systems, in general, are classified mainly into active control systems [1, 2, 28] and passive control systems [13, 30, 33]. Passive systems using tuned mass dampers or base-isolation techniques are designed to decrease the response to structural vibration induced by earthquake. They have simple mechanism, require no power to operate and hence are reliable. However, their control capacity and application is limited. Active control systems, including active tendons and active tuned mass dampers, can generate control forces to apply to structural systems through actuators equipped with a designed control algorithm. Given this, they are able to dissipate earthquake energy and reduce structural damage. It has been shown that the active devices are superior to the passive devices in capacity and suitability to high-rise civil structures. However, they do require external power supply and hence their operation may be interrupted during earthquake events, i.e., their reliability is critically decreased. By these reasons, hybrid devices have been developed for designing more effective vibration control systems, called semi-active controllers [6, 7, 12, 14, 17, 32]. They have been shown to be more energy-efficient than active control systems, since they require so little power for operation that they can be able to run on battery power, and become more effective in reducing seismic structural vibrations than passive control systems.

Fuzzy control is an area in which fuzzy logic has been applied successfully since Mamdani's work [16] published in 1974. By applying the theory of linguistic approach and fuzzy inference, one successfully uses 'if–then' rules in the automatic operating control of a steam generator. Since that time, it has been shown that fuzzy logic provides a flexible and effective methodology to solve many practical problems not only in control but also in other application fields, including the problems of protection of civil structures from earthquake. They arise there as a viable design alternative: instead of differential equations to model the structural systems, it uses a control domain knowledge formulated in the form of fuzzy linguistic rules. It does not require an accurate mathematical model as well as precise data describing structural and earthquake-induced vibration characteristics of the complex systems. It can handle non-linear uncertainties and heuristic knowledge effectively considering their ability of convertting the selected linguistic control strategy based on control knowledge to automatic control, whose knowledge base represent the dependencies of the desired control action on the control inputs.

In general, the main advantages of the fuzzy controllers are simplicity and intrinsic robustness, since they are not affected by the selection of the system's models [1]. Subsequently in the last few decades, fuzzy control has attracted considerable attention of researchers in natural-hazard-induced vibration control of structural systems [2, 6-12, 14, 16, 17, 27-29, 32-36].

The key task in the design of fuzzy logic-based controllers is to construct an effective fuzzy reasoning method. In fuzzy control, control knowledge is expressed by the following set of fuzzy rules:

$$\begin{array}{c} \text{If } X_1 \text{ is } A_{11} \text{ and ... and } X_m \text{ is } A_{1m} \quad \text{then } Y \text{ is } B_1 \\ \cdots\cdots\cdots\cdots \\ \text{If } X_1 \text{ is } A_{n1} \text{ and ... and } X_m \text{ is } A_{nm} \quad \text{then } Y \text{ is } B_n \end{array} \tag{1}$$

The rules describe dependencies between linguistic variables X_j, j = 1, ..., m, and Y, where A_{ij}, j = 1, …, m, and B_i , i = 1, …, n, are fuzzy sets whose labels are vague terms of the linguistic variables X_j and Y, respectively. The set of fuzzy rules (1) is called a fuzzy model or a *fuzzy associative memory* (FAM) [31].

In order to determine the numeric output value b_0 of this fuzzy model, for a given input fuzzy sets vector $A_0 = (A_{01}, \ldots, A_{0m})$, the fuzzy rules have to be represented by the respective fuzzy relations $R_i(x_1, \ldots, x_m, y)$, $i = 1, \ldots, n$, utilizing certain fuzzy sets operations and fuzzy implication. Then, b_0 will be produced by exploiting certain composition operation, aggregation operation and defuzzification method. Thus, the constructed reasoning method depends on several factors which make the designer difficult to observe the actual behaviour of the constructed reasoning method and adjust them to enhance the performance of the desired fuzzy controller. Moreover, from our point of view, a fuzzy set regarded as an immediate generation of sets represents the meaning of a vague term in the manner that each value in the reference domain of the linguistic variable is compatible with it to a degree assuming values in the interval [0,1]. That is fuzzy sets associated with each vague terms in the term-domain of a linguistic variable express the meaning of the respective terms individually, but cannot express the relative semantics present between these vague terms. The reason of this fact is that one has not considered term-domains as mathematical structures and, therefore, has to borrow the analytic structure of the set of all fuzzy sets defined on a universe in question. These all lead to some critical disadvantages of fuzzy reasoning mechanisms that may limit the effectiveness of fuzzy controllers, as it will be discussed in this paper.

In our study, we propose to apply the hedge-algebras-based methodology to design fuzzy controllers in fuzzy vibration control of structural systems that utilize the algebraic approach to the semantics of vague terms. In this approach, the meaning of every vague term is not represented by a fuzzy set, but by its inherent semantic-order-based relationships with the remaining ones in the corresponding hedge algebra, which represents much more fuzzy information than each individual fuzzy sets. Based on this approach, fuzzy-rules-based control knowledge is modelled by a numeric hyper-surface established from the fuzzy rules by the quantification of hedge algebras and fuzzy reasoning methods can be developed, utilizing ordinary interpolation methods on this surface. Such fuzzy reasoning methods depend only on two factors, the selected numeric interpolation method and the fuzziness parameters of each linguistic variable. Therefore, they are very simple, transparent and, as it will be shown below, they have many advantages. Especially, it allows not difficultly design optimal controllers based on optimization of their fuzziness parameters. It will be shown that the performance of the controllers designed based on the hedge-algebras-based methodology for the fuzzy vibration control of civil structural systems against earthquakes is better than those designed with traditional fuzzy reasoning methods. The experiments were completed by using the data on ground motion in turn of El Centro, Northridge and Kobe earthquakes. The simulation results for the three earthquakes show that the performance of the hedge-algebra-based controllers, especially the optimal ones, is better than that of the fuzzy controllers.

The paper is organized as follows. In Section 2, the main components of the fuzzy controllers will be described for making some discussion about disadvantages of the fuzzy controllers. An overview of the algebraic qualitative semantics of vague terms is given in Section 3 while quantitative semantics of vague terms is discussed in Section 4. It is characterized by three features, namely fuzziness measure of vague terms, fuzziness intervals of vague terms, and semantically quantifying mappings (SQMs) of terms-domains. Hedge-algebras-based reasoning methods are examined in Section 5. Section 6 is devoted to computer simulations study while conclusions are given in Section 7.

2. FUZZY CONTROLLERS

This section aims to discuss some disadvantages of fuzzy controllers designed by the fuzzy-set-based methodology for a comparison with those designed by the proposed hedge-algebras-based one, called hedge-algebras-based controllers (HACs). At the same time, it aims to ensure that the fuzzy controllers examined in this study are similar to those examined in [6, 9 - 11, 27, 32, 34].

An overall schematic view of fuzzy controllers is shown in Figure 1 [6, 32]. Its main components comprise a fuzzifier, an inference engine and a defuzzifier.

The performance of the designed fuzzy controller is affected by several design tasks related to the above components:

(*C1*) Construction of membership functions for fuzzifier: The fuzzifier is affected by the design of the fuzzy-sets-based semantics of vague terms. The designed membership functions of vague terms may have different forms, say triangular, trapezoidal, Gaussian, etc. The designer has a great level of freedom to construct membership functions for vague terms, provided that they contribute to the enhancement of the performance of fuzzy controller.

(*C2*) Inference engine: The construction of a computational model of the fuzzy model (1) and a reasoning method to determine the output of the controller require determining many factors and operators:

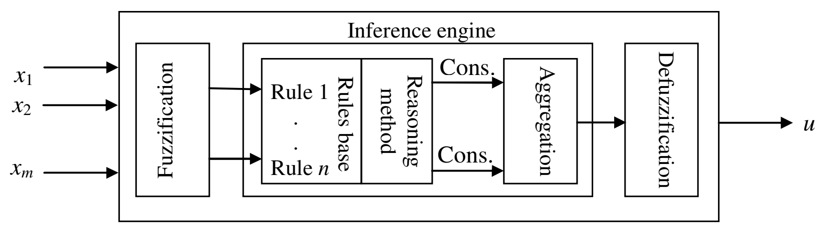

Figure 1. A schematic view of the fuzzy controller

First of all the exploitation of the control knowledge requires interpreting the fuzzy model (1) as one of the two alternatives: (i) conjunctive model and (ii) disjunctive model [15].

(i) In the case of conjunctive model, to compute a desired m-ary fuzzy relation R, which represents dependencies between the variables in (1), each fuzzy rule should be interpreted as a *fuzzy implicator* $\mathrm{I} : [0,1]^2 \rightarrow [0,1]$ by applying an *aggregation operator* to m premise fuzzy sets of the rule and, then, one applies *another aggregation operator* to the obtained implications to produce the relation R. The control action is then calculated by using a *composition operation* of the m-dimensional input vector and the obtained fuzzy relation R. Usually, we encounter here a max-min composition operation. In general, there are many composition operations, using t-conorms and t-norms instead of max and min, respectively.

(ii) The disjunctive model is usually used in fuzzy control. One uses each fuzzy rule to infer its conclusion from the given input data by *a composition inference*. As above, this composition is either in the form of the *max-min composition* or the one in which the max and min are replaced with *t-conorm* and *t-norm*, respectively. The derived consequences are then aggregated by using an *aggregation operator* to calculate the fuzzy control action.

(*C3*) Defuzzifier: This task aims to transform the calculated fuzzy control action into a numeric one. In general, we have a high level of freedom for determining a transformation of the area limited by the membership function of the action into a single numeric value viewed as its representative. Thus, we have *many such transformations*.

Thus, there are so many fuzzy reasoning methods in principle. Therefore, in order to make a comparative simulation study of the two design methodologies relying upon different mathematical bases, the fuzzy controllers in this paper designed by the fuzzy-set-based methodology will follow the following conditions that were applied in several researches (see, e.g. [6, 9-11, 18, 27, 32, 34, 35]):

(*fc1*) Fuzzification: The fuzzy sets of the linguistic terms are assumed to be symmetric triangular fuzzy sets that are equally spread over each range (see Figures 7 – 9). So, once the ranges of the linguistic variable and its number of vague terms are given, these fuzzy sets are completely defined.

(*fc2*) Reasoning method: It is assumed that the set of fuzzy rules in (1) are disjunctive model [15] and the reasoning method is constructed in accordance with (ii) mentioned above.

(*fc3*) Defuzzification is realized as the center of gravity.

Although fuzzy sets have successfully been applied to the fuzzy control, it is worth highlighting some disadvantages of the fuzzy set-based design methodology that may limit the effectiveness of the resulting fuzzy controllers.

(i) The first one lies just in the first design task, the fuzzification procedure. In essence, this is an embedding mapping from a term-set into the set of all fuzzy sets defined on U a reference domain, denoted by $\mathbf{F}(U)$. This means that we had to borrow the mathematical structure of $\mathbf{F}(U)$ to develop various fuzzy reasoning methods. Since term-domains can be considered as at least an order-based structure induced by the inherent meaning of terms, on the mathematical viewpoint, this embedding mapping will only be accepted if it is a homomorphism, i.e. it preserves the order-based structure of terms-domains. However, the fuzzifiers in general do not preserve this structure of term-domains, as it is difficult to define a reasonable order relation on $\mathbf{F}(U)$. As the effectiveness of a fuzzy reasoning method depends strongly on the designed membership functions of vague terms, these embedding mappings which are not homomorphism may limit the performance of designed controllers.

(ii) On the other hand, as discussed above, the performance of fuzzy controllers depends on several independent hard tasks, which have attracted many research efforts so far: a selection of membership functions, fuzzy implicators, t-norms and t-conorms, aggregation operators, composition operations, and defuzzifiers. This may make fuzzy control algorithms to become black boxes whose behaviour is then very difficult to observe by the designer.

To alleviate these difficulties, in the next section we present a development of hedge-algebras-based reasoning methods based on semantic-order-based structure of terms-domains.

3. HEDGE ALGEBRAS: SEMANTIC-ORDER-BASED STRUCTURE MODELLING THE SEMANTICS OF VAGUE TERMS

In the so-called analytic approach, the meaning of vague terms of linguistic variables is represented by fuzzy sets. In a certain aspect, this means that vague terms were understood as being not mathematical objects and, hence, we had to use fuzzy sets to represent their meaning, whose memberships functions are analytical objects of $\mathbf{F}(U)$. The motivation behind the algebraic approach to the semantics of terms comes from the observation that terms-domains of linguistic variables can be considered as partially ordered sets (posets), whose order relations are induced by the inherent meaning of vague terms. For instance, in virtue of vague terms of the linguistic variable VELOCITY in natural language, we have

quick > *medium* > *slow*, *Extremely_slow* < *Very_slow* < *slow*, but that

Little_slow > *Rather_slow* > *slow*, and so on.

So, we have an algebraic approach to the semantics of vague terms. To show its advantages, we provide a brief overview of this approach. Its detailed formal presentation can be found in [20, 22, 24 or 26].

Let X be a linguistic variable, $\boldsymbol{G} = \{g, g'\}$, $g \leq g'$, be the set of its primary terms and $\boldsymbol{H}$ be the set of its hedges. Denote by *Dom*(X) the set of all terms generated from the primary terms by using hedges acting on them in concatenation, i.e. each term in *Dom*(X) can be written in a string $h_n \ldots h_1c$, where $h_i \in \boldsymbol{H}$ and $c \in \boldsymbol{G}$. For convenience in sequel, we assume also that *Dom*(X) contains the specific terms given in $\boldsymbol{C} = \{\boldsymbol{0}, \boldsymbol{W}, \boldsymbol{1}\}$, which are called constants, where $\boldsymbol{0}$ and $\boldsymbol{1}$ is the least and the greatest terms in the structure *Dom*(X) and $\boldsymbol{W}$ is the neutral concept in between the two primary terms. We assume that $\boldsymbol{0} \leq g \leq \boldsymbol{W} \leq g' \leq \boldsymbol{1}$. As discussed above, there exists a semantic order relation $\leq$ on *Dom*(X) and (*Dom*(X), $\leq$) becomes a poset. Thus, the meaning of a term in *Dom*(X) is represented through its order relationships with the remaining terms in *Dom*(X); here we offer a certain view at the semantics of vague terms.

1) Many properties of vague terms discovered and formulated in (*Dom*(X), $\leq$)

In the structure (*Dom*(X), $\leq$) we may discover many essential properties of vague linguistic terms as follows:

(*p1*) *Every term has a semantic tendency expressed through hedges and an "algebraic" sign*: The semantic function of the linguistic hedges is to intensify vague terms, i.e. they either increase or decrease the meaning of vague terms. This implies that the meaning of each term in the structure (*Dom*(X), $\leq$) has a definite semantic tendency, which, while is increased by the ones hedges, is decreased by the others. Based on this idea we can define the following notions, which contribute to describe the semantics of terms:

- The primary terms g and g' have their semantic tendency defined in term of $\leq$. As $g \leq g'$, the semantic tendency of g' is called *positive* and we write $g' = c^+$ and $sign(c^+) = +1$. Similarly, the semantic tendency of g is called *negative* and we write $g = c^-$ and $sign(c^-) = -1$.

- By these tendencies, the set of hedges $\boldsymbol{H}$ can be classified into two sets $\boldsymbol{H}^-$ and $\boldsymbol{H}^+$ defined as follows: $\boldsymbol{H}^- = \{h \in \boldsymbol{H}: hc^- \geq c^- \text{ or } hc^+ \leq c^+\}$, which consists of the hedges that *decrease the semantic tendency* of the both primary terms; while $\boldsymbol{H}^+ = \{h \in \boldsymbol{H}: hc^- \leq c^- \text{ or } hc^+ \geq c^+\}$, i.e. its hedges *increase the semantic tendency* of the primary terms. The elements of $\boldsymbol{H}^-$ are called *negative* hedges and their sign is defined by $sign(h) = -1$. Similarly, every $h \in \boldsymbol{H}^+$ is called *positive* hedge and its sign is defined by $sign(h) = +1$.

For example, for the variable VELOCITY, it can be checked that $\boldsymbol{H}^- = \{R, L\}$ and $\boldsymbol{H}^+ = \{V, E\}$, where R, L, V and E stand for *Rather*, *Little*, *Very* and *Extremely*, respectively. Note that $\boldsymbol{H}^-$ and $\boldsymbol{H}^+$ are also posets. For instance, we have here $R \leq L$ and $V \leq E$.

- For any two hedges h and k, k does either increase or decrease the effect of h. In the former case, we say that *the relative sign* of k with respect to h is *positive* and write $sign(k, h) = +1$. In the second case it is *negative* and we write $sign(k, h) = -1$. This relative sign can be recognized based on order relationships. For instance, if the effect of h acting on x is expressed by $x \le hx$ then $x \le hx \le khx$ implies that k increases the effect of h. Given a set $\boldsymbol{H}$ of hedges, we can always establish a table of the *relative* sign of hedges. For example, it can be seen that the relative sign of the hedges of VELOCITY mentioned above are determined as in Table 1.

Table 1. The relative sign of the hedges in the first column w.r.t. the hedges in the first row

	E	*V*	*R*	*L*
E	+	+	–	+
V	+	+	–	+
R	–	–	+	–
L	–	–	+	–

- The "algebraic" sign of the vague terms: It was shown that each term $x \in Dom(\mathrm{X})$ has a *unique canonical* (string) *representation* $x = h_m \ldots h_1 c$ having the property that for all $i = 1, \ldots, m-1$, $h_{i+1}h_i \ldots h_1 c \ne h_i \ldots h_1 c$. The length of x can then be defined to be the length of the string of the canonical representation of x, denoted by $|x|$. Now, the sign of the term x can be defined as:

$$Sgn(x) = sign(h_m, h_{m-1}) \times \ldots \times sign(h_2, h_1) \times sign(h_1) \times sign(c) \tag{2}$$

It could be shown that

$$(Sgn(hx) = +1) \Rightarrow (hx \ge x) \quad \text{and} \quad (Sgn(hx) = -1) \Rightarrow (hx \le x) \tag{3}$$

For instance, the sign of $x = V\_L\_slow$ of the variable VELOCITY is calculated by $Sgn(V\_L\_slow) = sign(V, L) \times sign(L) \times sign(slow) = (+1)(-1)(-1) = +1$, which implies that $V\_L\_slow \ge L\_slow$.

(*p2*) *Semantic heredity of hedges*: An essential property of hedges is the so-called *semantic heredity*, which states that the terms generated by using hedges from a given term x must inherit the (genetic) core meaning of x. This means that the set $\boldsymbol{H}(x)$ comprises the terms that still contain a core meaning of x. Therefore its hedges cannot change the essential meaning of terms expressed through the semantic order relation (SOR). The semantic heredity of hedges can then be formulated formally in terms of SOR $\le$ as follows:

- For any term x, any hedges h, k, h' and k', where $h \ne k$, if the meaning of hx and kx is expressed by the order relationship $hx \le kx$, then we have

$$hx \le kx \;\Rightarrow\; h'hx \le k'kx.$$

- If the meaning of x and hx is expressed by either $x \le hx$ or $hx \le x$, then we have

$$x \le hx \;\Rightarrow\; x \le h'hx \quad \text{or} \quad hx \le x \;\Rightarrow\; h'hx \le x.$$

It can be seen that these properties viewed as axioms describe the fact that the hedges h' and k' cannot change the semantic relationships of the terms x, hx and kx expressed by the above inequalities in the structure $(Dom(\mathrm{X}), \le)$, when they apply to these terms.

2) Terms-domains of linguistic variables viewed as hedge algebras

Let X be a linguistic variable and $\boldsymbol{X} \subseteq Dom(\mathrm{X})$. From the above discussion, the set $\boldsymbol{X}$ can be viewed as an algebraic structure $AX = (\boldsymbol{X}, \boldsymbol{G}, \boldsymbol{C}, \boldsymbol{H}, \le)$, where the sets $\boldsymbol{G}$, $\boldsymbol{C}$ and $\boldsymbol{H}$ are defined as previously, except that $\boldsymbol{H}$ is assumed for a technical reason that it includes the identity I which is treated as an artificial hedge and defined by $Ix = x$, $\forall x \in \boldsymbol{X}$, and $\le$ is a semantic order relation on $\boldsymbol{X}$. The elements in $\boldsymbol{H}$ are regarded as unary operations of AX. By its semantic effect, I is called "neutral" hedge, since it is neither positive nor negative. Hence, it may be considered as the least element of the both posets $\boldsymbol{H}^-$ and $\boldsymbol{H}^+$. Suppose that $\boldsymbol{X} \setminus \boldsymbol{C} = \boldsymbol{H}(\boldsymbol{G})$, where $\boldsymbol{H}(\boldsymbol{G})$ is the set of all elements generated from the generators in $\boldsymbol{G}$ using operations in $\boldsymbol{H}$, and that $\boldsymbol{0} \leq c^- \leq \boldsymbol{W} \leq c^+ \leq \boldsymbol{1}$. Since $I \in \boldsymbol{H}$, we have $x \in \boldsymbol{H}(x)$.

It is proved that the algebraic structure $AX = (\boldsymbol{X}, \boldsymbol{G}, \boldsymbol{C}, \boldsymbol{H}, \leq)$ can be axiomatized, called *hedge algebra*, which is named by the role of hedges. The hedge algebras have been developed (see e.g. [19-24, 26]) and applied to solve some problems effectively [3, 4, 24, 25]. Here, for reference we recall some facts about hedge algebras. For convenience, for any two subsets U and V of $\boldsymbol{X}$, the notation $U \leq V$ means that $u \leq v$, for $\forall u \in U$ and $\forall v \in V$.

Assume that $\boldsymbol{H}^- = \{h_0, h_{-1}, ..., h_{-q}\}$ and $\boldsymbol{H}^+ = \{h_0, h_1,..., h_p\}$, where $h_0 = I$ and $h_0 < h_{-1} < h_{-2} < ... < h_{-q}$ and $h_0 < h_1 < ... < h_p$. The sets $\boldsymbol{H}(x)$, $x \in \boldsymbol{H}(\boldsymbol{G})$, have the following properties:

- $\boldsymbol{H}(x)$ is partitioned into subsets $\boldsymbol{H}(h_j x)$, $j \in [-q, p]$, where $[-q, p] = \{j \mid -q \leq j \leq p\}$ and, by a convention, $\boldsymbol{H}(h_0 x) = \boldsymbol{H}(Ix) = \{x\}$, i.e. the subsets $\boldsymbol{H}(h_j x)$ are *disjoint* and

$$\boldsymbol{H}(x) = \bigcup_{h \in H} H(hx) \tag{4}$$

- For $Sgn(h_p x) = -1$, $\boldsymbol{H}(h_p x) \leq \ldots \leq \boldsymbol{H}(h_1 x) \leq \{x\} \leq \boldsymbol{H}(h_{-1} x) \leq \ldots \leq \boldsymbol{H}(h_{-q} x)$ (5)
- For $Sgn(h_p x) = +1$, $\boldsymbol{H}(h_{-q} x) \leq \ldots \leq \boldsymbol{H}(h_{-1} x) \leq \{x\} \leq \boldsymbol{H}(h_1 x) \leq \ldots \leq \boldsymbol{H}(h_p x)$ (6)

4. QUANTITATIVE SEMANTICS OF THE VAGUE TERMS

Since in this approach the meaning of terms is not expressed by fuzzy sets, the quantification of hedge algebras has to be overviewed systematically. This quantification is characterized by three concepts: *semantically quantifying mapping* (SQM), *fuzziness measure* and *fuzziness intervals* of vague terms. These concepts have a very close relationship each other and it ensures that the SQMs depend on the fuzziness of terms and can be determined appropriately in fuzzy environments by selecting fuzziness measure values of a few special terms, called fuzziness parameters. As previously, in this section we will give a short overview of necessary knowledge. For more details the reader can refer to [19, 21 or 23-25].

4.1. Semantically quantifying mappings of hedge algebras

Generally, as defuzzifiers in fuzzy control which convert fuzzy sets of terms into numeric values, the quantification of hedge algebra is a mapping from a term-domain into the reference domain of X. Since these mappings in the algebraic approach will be defined in a closed connection with fuzziness measure and fuzziness intervals of terms, which are fundamental characteristics of the semantics of vague terms, they are called *semantically quantifying mappings* (SQMs).

Let us consider a *free* linear hedge algebra AX = ($\boldsymbol{X}$, $\boldsymbol{G}$, $\boldsymbol{C}$, $\boldsymbol{H}$, $\leq$) of a linguistic variable X, where "*free*" means that for every hedge h and every term $x \in \boldsymbol{H}(\boldsymbol{G})$, we always have $hx \neq x$, and $\leq$ is a *linear* order relation on $\boldsymbol{X}$. This implies that all string representations of the vague terms are canonical and every vague term has a unique string representation.

Definition 4.1 An SQM of AX is a mapping $f : \boldsymbol{X} \rightarrow [0,1]$, which satisfies

(i) It is *one-to-one* mapping and $f(\boldsymbol{X})$ is *dense* in [0,1], where [0,1] is the normalization of the reference domain of X;

(ii) It *preserves the order* of $\boldsymbol{X}$. ■

The definition of SQMs is general, but should include their two essential characteristics. The first one is regarded as a consequence the fact that the quantitative meaning of the terms of X should approximate the values of its reference domain. The second is natural: SQMs should preserve the mathematical structure of term-domains.

4.2. Fuzziness model, fuzziness measure and fuzziness interval of vague terms

Since, by the heredity of the hedges, $\boldsymbol{H}(x)$ comprises all the terms that still inherit a core (genetic) meaning of x, it can be taken as a model of the fuzziness of x. It implies that the larger the set $\boldsymbol{H}(x)$ the more fuzziness of the term x. Since for $x = hu$ we have $\boldsymbol{H}(x) \subseteq \boldsymbol{H}(u)$, it follows that the more occurrences of hedges in x, the lower the fuzziness of x. This demonstrates that the use of $\boldsymbol{H}(x)$ as a fuzziness model of x is compatible with our intuition.

Let $f : \boldsymbol{X} \to [0,1]$ be an SQM of AX. Since f preserves the order relation on $\boldsymbol{X}$, for every $x \in \boldsymbol{X}$, the image $f(\boldsymbol{H}(x))$ under f is isomorphic onto $\boldsymbol{H}(x)$ in the category of linearly ordered sets. Thus, since the terms in $\boldsymbol{H}(x)$ are similar with each other and occur consecutively, the size of the set $f(\boldsymbol{H}(x)) \subseteq [0,1]$, i.e. the diameter of $f(\boldsymbol{H}(x))$, can be interpreted as the *fuzziness measure of* x, denoted by $fm(x)$:

$$fm(x) = d(f(\boldsymbol{H}(x))) \in [0,1] \tag{7}$$

This suggests us to introduce a notion of fuzziness interval of the term x, denoted by $\mathfrak{I}(x)$, which is the smallest subinterval of [0,1] including $f(\boldsymbol{H}(x))$. Clearly, $|\mathfrak{I}(x)| = fm(x)$, where $|\mathfrak{I}(x)|$ denotes the length of $\mathfrak{I}(x)$. Since f preserves the semantic order of $\boldsymbol{X}$ and, by (i) of Definition 4.1, $f(\boldsymbol{H}(x))$ is dense in $\mathfrak{I}(x)$, from the semantics of $\boldsymbol{H}(x)$ it follows that $\mathfrak{I}(x)$ comprises the values of [0,1] that are compatible with the meaning of x to a degree indicated by $k = |x|$.

From (4) – (6) and the density of $f(\boldsymbol{X})$ in [0,1], it follows that (see Figure 2)

$$\text{For } Sgn(h_p x) = -1,\ \mathfrak{I}(h_p x) \le \mathfrak{I}(h_{p-1}x) \le \ldots \le \mathfrak{I}(h_1 x) \le \mathfrak{I}(h_{-1}x) \le \ldots \le \mathfrak{I}(h_{-q}x) \tag{8}$$

$$\text{For } Sgn(h_p x) = +1,\ \mathfrak{I}(h_{-q}x) \le \mathfrak{I}(h_{-q+1}x) \le \ldots \le \mathfrak{I}(h_{-1}x) \le \mathfrak{I}(h_1 x) \le \ldots \le \mathfrak{I}(h_p x) \tag{9}$$

$$|\mathfrak{I}(x)| = \sum\{|\mathfrak{I}(h_j x)| \mid j \in [-q\text{^}p]\} \tag{10}$$

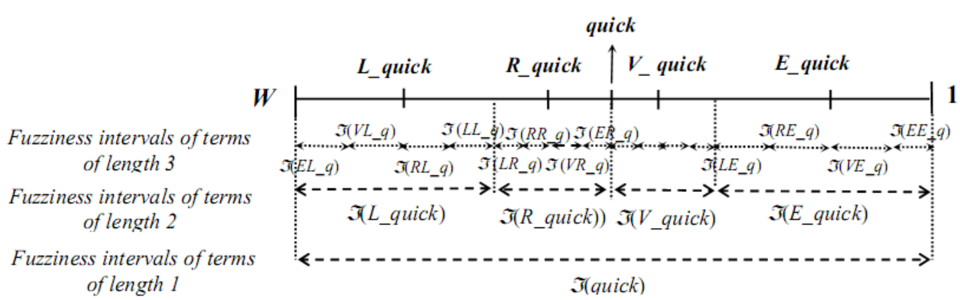

Figure 2. Fuzziness intervals of vague terms of the VELOCITY

Thus, the *fuzziness measure fm* of vague terms satisfies the following properties:

(fm1) $fm(c^-) + fm(c^+) = 1$ and, as a consequence, $fm(\boldsymbol{0}) = fm(\boldsymbol{W}) = fm(\boldsymbol{1}) = 0$.

(fm2) $\sum_{j\in[-q\text{^}p]} fm(h_j x) = fm(x)$, $x \in \boldsymbol{X}$, and $\sum_{x\in X_k} fm(x) = 1$.

It seems natural to assume that the *relative* effect of hedges acting on the terms remains unchanged. This can be expressed by the following expression:

$$\frac{fm(hx)}{fm(x)} = \frac{fm(hy)}{fm(y)} = \mu(h), \text{ for all } x, y \in \boldsymbol{X} \tag{11}$$

The quantity $\mu(h)$ is called the fuzziness measure of h. Then we have

($fm3$) $fm(hx) = \mu(h)fm(x)$, *for* $hx \neq x$, $x \in \boldsymbol{X}$, and, hence, $fm(y) = \mu(h_m) \ldots \mu(h_1)fm(c)$, where $y = h_m \ldots h_1c$ is the canonical representation of y.

($fm4$) $\sum_{-q \le i \le -1} \mu(h_i) = \alpha$ and $\sum_{1 \le i \le p} \mu(h_i) = \beta$, where $\alpha, \beta > 0$ and $\alpha + \beta = 1$.

From these it follows that in order to determine a fuzziness measure of a linguistic variable it merely requires to provide the fuzziness measure values of one primary term and $(p + q - 1)$ hedges, which depend only on the linguistic variable, but not on individual terms. For convenience, we call them *fuzziness parameters* in common. In practice, it is sufficient to assume that p, $q \le 2$. Hence, the number of fuzziness measures of hedges does not exceed 3 and the total number of fuzziness parameters does not exceed 4. On the other hand, since human being uses vague terms in their daily lives, they will have their practical knowledge to define more easily the numeric values of these parameters than to define *individual* fuzzy sets of vague terms. We note that these fuzziness parameters fully determine the quantitative semantics, which comprise the fuzziness measure, fuzziness intervals and semantically quantifying mappings of the linguistic variable in question.

4.3. SQMs induced by a given fuzziness measure of vague terms

It has been seen previously that there is a strict relationship between the notion of SQMs and the notions of fuzziness measure and fuzziness intervals of terms. This relationship is reinforced by the fact that a given fuzziness measure fm will induce an SQM, denoted by υ, so that $fm(x) = d(\upsilon(\boldsymbol{H}(x)))$, the diameter of the image $\upsilon(\boldsymbol{H}(x))$, for $\forall x \in \boldsymbol{X}$. The inequalities in (5), (6), (8) and (9) (refer to Figure 2) suggest that $\upsilon(x)$ should be defined to assume the value lying in-between the fuzziness intervals $\mathfrak{I}(h_{-1}x)$ and $\mathfrak{I}(h_1x)$. Consequently, the mapping υ can be expressed recursively as follows:

(SQM1) $\upsilon(\boldsymbol{W}) = \theta = fm(c^-)$, $\upsilon(c^-) = \theta - \alpha fm(c^-) = \beta fm(c^-)$, $\upsilon(c^+) = \theta + \alpha fm(c^+)$;

(SQM2) $\upsilon(h_jx) = \upsilon(x) + Sgn(h_jx)\left\{\left[\sum_{i=sign(j)}^{j} fm(h_ix)\right] - \omega(h_jx)fm(h_jx)\right\}$

where $\omega(h_jx) = \frac{1}{2}[1 + Sgn(h_jx)Sgn(h_ph_jx)(\beta - \alpha)] \in \{\alpha, \beta\}$, for all $j \in [-q\hat{\ }p]$.

All three quantitative aspects of the terms, the fuzziness measure fm, the fuzziness intervals and the fm-induced SQM υ are completely determined by providing the values of the fuzziness parameters $fm(c^-)$, $fm(c^+)$ and $\mu(h)$, $h \in \boldsymbol{H}$, of X. Using the constraints given in ($fm1$) and ($fm4$), the number of the required fuzziness parameters is $|\boldsymbol{H}| + |\boldsymbol{G}| - 2 = |\boldsymbol{H}|$.

Example 4.1 Consider the linguistic variable VELOCITY, e.g. of motor-bikes, with the hedges examined previously. Suppose that its reference domain is [0, 120] and its fuzziness parameters are provided as follows: $fm(slow) = 0.4$, $\mu(L) = 0.25$, $\mu(R) = 0.20$, $\mu(V) = 0.3$. Hence, we have $fm(quick) = 0.6$ and $\mu(E) = 0.25$ and, hence, $\alpha = 0.45$ and $\beta = 0.55$. Assume that it is required to calculate the quantification values of "*quick*" and "*L_quick*". Then

By (SMQ1), $\upsilon(quick) = 0.4 + (0.25 + 0.20)0.6 = 0.67$.

By (SMQ2), $\upsilon(L\_quick) = 0.67 + (-1)\{[(0.20+0.25)\times 0.6] - 0.55\times(0.25\times 0.6)\} = 0.4825$

since $R = h_{-1}$, $L = h_{-2}$ and we have

$$fm(h_{-1}c^+) + fm(h_{-2}c^+) = [\mu(h_{-1}) + \mu(h_{-2})]\times fm(c^+)$$

and $$\omega(h_{-2}c^+) = ½[1 + sign(Lc^+)sign(E, L)sign(L)sign(c^+)(0.55 - 0.45)] = 0.55$$

Thus, the actual quantification values of *quick* and *L_quick* are 0.67×120 km = 80.4 km and 0.4825×120km = 57.9 km, respectively. ■

It is obvious that when we change the fuzziness parameters, the induced SQM will be changed as well. In order to show how SQMs depend on the structure of hedge algebras, we consider the following example

Example 4.2 Consider again the linguistic variable VELOCITY, but it has only two hedges R and V, i.e. $p = q = 1$, and in the same time we assume that $fm(slow) = 0.4$, $\alpha = 0.45$ and $\beta = 0.55$ that are the same as in Example 4.1. This implies that $\mu(L) = 0.45$ and $\mu(V) = 0.55$. Here, we use the hedge L but not R, since R is usually used in the context of the existence of another negative hedges and, moreover, intuitively its performance is weak. Then the quantification values of "*quick*" and "*L_quick*" will be changed as follows:

By (SMQ1), $\upsilon(quick) = 0.4 + 0.45\times 0.6 = 0.67$, which is the same as above. But (SMQ2), $\upsilon(L\_quick) = 0.67 + (-1)\{[0.45\times 0.6] - 0.55\times(0.45\times 0.6)\} = 0.5485$

since $L = h_{-1}$ we have $fm(h_{-1}c^+) = 0.45\times 0.6$ and

$$\omega(h_{-1}c^+) = ½[1 + sign(Lc^+)sign(V, L)sign(L)sign(c^+)(0.55 - 0.45)] = 0.55$$

Hence the actual quantification value of *quick* is 80.4 km, the same as above, and of *L_quick* is 0.5485×120km = 65.82 km, which is greater than the value 57.9 km above. ■

5. HA-INTERPOLATIVE-REASONING METHODS AND HA-CONTROLLERS

Let us consider a fuzzy model in the form of (1), in which A_{ij}, B_i, $j = 1, .., m$ and $i = 1, \ldots, n$, are, however, not fuzzy sets but vague linguistic terms. Therefore, in the algebraic approach, the set of fuzzy rules in (1) will be called a *linguistic model* of control knowledge.

An essence of the fuzzy controllers is the *fuzzy multiple conditional reasoning* (FMCR) problem [15, 16, 31]. The reasoning method for the given inputs $X_j = A_{0j}$, $j = 1, \ldots, m$, of the linguistic model (1), helps us find an output $Y = B_0$.

In this section, we will present how a fuzzy reasoning method can be constructed to solve a given FMCR problem, utilizing hedge-algebras-based semantics of terms.

5.1 HA-based interpolative reasoning method

We show that based on hedge-algebras-based approach to the semantics of vague terms, we can easily develop HA-based interpolative reasoning methods.

5.1.1. General descriptions of hedge-algebras-based interpolative reasoning method

Although the linguistic model (1) describes a dependency of Y on X_j's, that is it expresses certain domain knowledge of the designer, it does not provide any formal basis for computation.

At first, an exact mathematical model of the domain knowledge represented by (1) has to be constructed. Since a terms-domain of each linguistic variable can be viewed as a subset of a hedge algebra, we may suppose that the linguistic variables X_j and Y appearing in (1) will be associated with certain hedge algebras denoted respectively by $AX_j = (\boldsymbol{X}_j, \boldsymbol{G}_j, \boldsymbol{C}_j, \boldsymbol{H}_j, \leq_j)$ and $AY = (\boldsymbol{Y}, \boldsymbol{G}, \boldsymbol{C}, \boldsymbol{H}, \leq)$ such that $\boldsymbol{G}_j$ and $\boldsymbol{H}_j$ as well as $\boldsymbol{G}$ and $\boldsymbol{C}$ contain all the primary terms and the hedges appearing in (1), $j = 1, 2, \ldots, m$.

Now, if we regard the i^{th}-if-then statement in (1) as a linguistic point $\boldsymbol{A}_i = (A_{i1}, \ldots, A_{im}, B_i)$, then the given linguistic model defines n points in the Cartesian space $\boldsymbol{X}_1\times\ldots\times\boldsymbol{X}_m\times\boldsymbol{Y}$, which describe a *linguistic* surface S_L in this space. The surface S_L can be considered as a mathematical model that simulates approximately the linguistic model given by (1). Since hedge algebras preserve the semantic order relations on the respective term-sets, we have a basis to believe that the surface S_L describes the domain knowledge given by (1) faithfully. Thus, a natural requirement now is to construct a transformation to convert the linguistic surface S_L into a numeric surface S_R in a multiple-dimensional Euclidean space, utilizing SQMs of the hedge algebras in question.

The FMCR problem is now transformed into a classical surface interpolation problem, which will be solved by an interpolation method. A reasoning method described here is called HA-based interpolative reasoning method (HA-IRMd, for short).

5.1.2. Construction of HA-based interpolative reasoning methods

Let be given a linguistic model (1). The methodology for the construction of HA-IRMds comprises the following tasks:

(i) *Determination of hedge algebras associated with linguistic variables*

The expressions of terms of hedge algebras coincide with those in natural languages. Therefore, assume that the linguistic terms used to formulate the fuzzy rules in (1) are terms of certain hedge algebras. Thus, the hedge algebras associated with linguistic variables present in (1) are constructed by the determination of the sets $\boldsymbol{G}_j$, $\boldsymbol{H}_j$, $\boldsymbol{G}$ and $\boldsymbol{C}$, which include respectively the primary terms and the hedges appearing in (1), $j = 1, 2, \ldots, m$. Once $\boldsymbol{G}_j$, $\boldsymbol{H}_j$, $\boldsymbol{G}$ and $\boldsymbol{C}$ are determined, the terms-set $\boldsymbol{X}_j$ is automatically generated. However, as it will be seen, it is necessary to focus attention on only the terms appearing in (1), but not all terms in $\boldsymbol{X}_j$. Notice that since the structure of hedge algebras determines the semantics of their terms (refer also to Example 4.2), it may happen that although some hedges do not appear in (1), they must be included in the respective associated hedge algebra. For example, the absence of the hedge "*rather*" in the context of the presence of "*little*" in a set of fuzzy rules does not mean certainly that the respective hedge algebra does not contain the hedge "*rather*". The presence of the hedge "*rather*" in the algebra is decided by just the semantics of the vague terms, which the application designer wishes to assign to these terms.

In fuzzy control, a FAM-table contains usually vague terms like *positive big* and *negative big* ..., which are compatible with the reference domain $[-1,1]$ while the terms of hedge algebras are compatible with the reference domain $[0,1]$. In the sequel, it is required that the vague terms in a FAM-table must be transformed into linguistic terms in the respective hedge algebras so that the term-transformation should preserve essential order-based semantic properties of terms, including: (i) The semantic order relation between the vague terms and (ii) The symmetric property of the vague terms under consideration, which states that each vague term has its own symmetric term, which is the antinomy or has an opposite meaning of the former one (see

Section 6). For instance, the pair of the terms *positive* and *negative* or of the terms *positive big* and *negative big* is symmetric. The term *zero* in a FAM-table corresponds to the neutral element $\boldsymbol{W}$ in hedge algebras.

(ii) *Determination of SQMs υ_{Xj} and υ_Y and normalized surface S_{norm}*: Since the image domains of an SQM is [0,1], first of all the reference domains of linguistic variables must be normalized. Given a reference domain in the form of an interval $[a, b]$ of a linguistic variable X, the *normalization* of this interval domain is realized by the following linear transformation, which is determined uniquely by the given interval $[a, b]$:

$$g_X : \quad [a,b] \;\rightarrow\; [0,1] \tag{12}$$

The converse mapping g_X^{-1} of g_X is called the *denormalization* mapping of X.

As discussed above, the linguistic model (1) interpreted as n linguistic points simulates a linguistic surface S_L. Let υ_{Xj} and υ_Y be SQMs of the constructed hedges algebras $AX_j = (\boldsymbol{X}_j, \boldsymbol{G}_j, \boldsymbol{C}_j, \boldsymbol{H}_j, \leq_j)$ and $AY = (\boldsymbol{Y}, \boldsymbol{G}, \boldsymbol{C}, \boldsymbol{H}, \leq)$ of the variables X_j and Y, respectively, where $j = 1, 2, \ldots m$. These SQMs transform n points $(A_{i1}, \ldots, A_{im}, B_i)$ in the linguistic space $\boldsymbol{X}_1\times\ldots\times\boldsymbol{X}_m\times\boldsymbol{Y}$ into n points in the Euclidean space $[0,1]^{m+1}$, which simulate a surface in $[0,1]^{m+1}$, called the normalized surface of the linguistic model (1), denoted by S_{norm}. Thus, we can say that the vector $(\upsilon_{X1}, \ldots, \upsilon_{Xm}, \upsilon_Y)$ of the SQMs υ_{Xj}, $j = 1, \ldots, m$, and υ_Y transforms S_L into S_{norm}:

$$(\upsilon_{X1}, \ldots, \upsilon_{Xm}, \upsilon_Y) : \; S_L \;\rightarrow S_{norm}$$

The surface S_{norm} can also be considered as being defined by an m-argument function,

$$v = f_{Snorm}(u_1, ..., u_m),\; v, u_j \in [0, 1],\; j = 1, \ldots, m \tag{13}$$

which satisfies the conditions that $\upsilon_Y(B_i) = f_{Snorm}(\upsilon_{X1}(A_{i1}), ..., \upsilon_{Xm}(A_{im}))$, $i = 1, \ldots, n$. The function f_{Snorm} or S_{norm} can be considered as a normalized numeric model of (1).

Similarly, the vector $(g_{X1}^{-1}, g_{X2}^{-1}, ..., g_{Xm}^{-1}, g_Y^{-1})$ of the denormalization mappings of the respective linguistic variables transforms S_{norm} into a hypersurface S_r in the Euclidean space $[a_{X1}, b_{X1}] \times [a_{X2}, b_{X2}]\times ... \times [a_{Xm}, b_{Xm}] \times [a_Y, b_Y]$, where $[a_{Xj}, b_{Xj}]$ and $[a_Y, b_Y]$ are the reference domains of X_j and Y, respectively, where $j = 1, \ldots, m$. S_R is called a denormalized model of the linguistic model (1).

Next, for convenience, we apply however a selected interpolative reasoning method on the surface S_{norm} instead of S_R.

Since the SQMs preserve the essential semantic properties of linguistic terms, we can state that S_{norm} is similar to S_L, or S_{norm} is a “faithful” computational model of (1). S_L is determined immediately by the given linguistic model (1) or by the fuzzy associative memory (FAM) called in the fuzzy control FAM-table, whose rows are formed by the linguistic terms of the corresponding if-then sentences in (1). Thus, S_{norm} is determined by the quantification of the terms in the FAM-table, which results in a numeric table, called in this study *quantified* FAM-*table* (*q*FAM-table, for short).

In order to construct the mathematical model S_{norm} of (1) it is required to determine the vector of SQMs, $(\upsilon_{X1}, \ldots, \upsilon_{Xm}, \upsilon_Y)$. However, these SQMs will be determined simply by assigning the values to the fuzziness parameters of the respective linguistic variables X_j and Y. In applications, the determination of these parameter values can be provided either by the designer based on his intuitive domain knowledge or by solving an appropriate optimization problem utilizing an evolutionary algorithm. The set of all these parameters consists of the following categories:

- m+1 parameters of the fuzziness measure of primary terms: $\theta_j = fm(c_j^-)$, j = 1, 2, … m, and $\theta = fm(c^-)$.

- $p_j + q_j - 1$ fuzziness parameters of the hedges in $\boldsymbol{H}_j$ of the algebra AX_j, $\mu(h_{j,-qj})$, ..., $\mu(h_{j,-1})$, $\mu(h_{j,1})$, ..., $\mu(h_{j,pj})$, j = 1, 2, … m.

- $p + q - 1$ fuzziness parameters of the hedges in $\boldsymbol{H}$ of the algebra AY, $\mu(h_{-q})$, ..., $\mu(h_{-1})$, $\mu(h_1)$, ..., $\mu(h_p)$.

It is worth emphasizing that, for each j^{th}-dimension, the number of these fuzziness parameters does not depend on the cardinality of the term-set $\boldsymbol{X}_j$, but depends only on the semantics of the linguistic variable X_j. For instance, assume that $p_j = 2$ and $q_j = 2$, the required number of fuzziness parameters for determining the SQM υ_{Xj} is always (1 + 2 + 2 – 1) = 4, for any possible term-set $\boldsymbol{X}_j$. That is it depends on the linguistic variables of interest, but not on a particular set of terms $\boldsymbol{X}_j$.

(iii) *Determination of an interpolation method on* S_{norm}: Suppose in general that input of the linguistic model (1) is a vector $\boldsymbol{A}_0 = (A_{0,1}, \ldots, A_{0,m})$ of m linguistic terms whose meaning is now defined by the structure of their respective hedge algebras $AX_j = (\boldsymbol{X}_j, \boldsymbol{G}_j, \boldsymbol{C}_j, \boldsymbol{H}_j, \leq_j)$ and $AY = (\boldsymbol{Y}, \boldsymbol{G}, \boldsymbol{C}, \boldsymbol{H}, \leq)$, where j = 1, 2, … m. An FMCR problem requires finding an output $\boldsymbol{B}_0$ corresponding to the given input $\boldsymbol{A}_0$.

In the fuzzy control, the input of (1) is a crisp vector, $\boldsymbol{A}_0 = (a_{0,1}, \ldots, a_{0,m})$, $a_{0,j} \in [a_{Xj}, b_{Xj}]$ for j = 1, 2, … m, and the output is required to be a numeric value in $[a_Y, b_Y]$, as well.

Since in the algebraic approach, we will take advantage of the surface S_{norm} and a classical interpolation method on this surface, the vector $\boldsymbol{A}_0$ should be normalized to become $\boldsymbol{A}_{0,norm} = (g_{X1}(a_{0,1}), \ldots, g_{Xm}(a_{0,m})) \in [0,1]^m$, and the calculated input is a numeric value. We can find many interpolation methods and computation tools to solve this problem in the literature. Thus, hedge algebras approach provides another methodology to solve FMCR problems.

Such a constructed HA-IRMd produces a numeric value $b_{0,norm} \in [0,1]$, which is approximately equal to $f_{Snorm}(g_{X1}(a_{0,1}), \ldots, g_{Xm}(a_{0,m}))$, the function described in (13), for a given $\boldsymbol{A}_0$. The actual output value b_0 is calculated from $b_{0,norm}$ as follows:

$$b_0 = g_X^{-1}(b_{0,norm}) \in [a_Y, b_Y] \tag{14}$$

Another way to define HA-IRMd for an application is to transform the surface S_{norm} to a curve C_{norm} in a 2-dimensional Euclidean space and apply a linear interpolation method on C_{norm}. This transformation can be realized by an m-ary aggregation Agg of the quantitative values of the vague terms in each fuzzy rule in (1). Thus, the curve C_{norm} is expressed by the following n calculated points:

$$(Agg(\upsilon_{X1}(A_{i,1}), \ldots, \upsilon_{Xm}(A_{i,m})), \upsilon_U(B_i)), i = 1, ..., n.$$

In this study, the aggregation operator Agg is chosen to be the weighted averaging operation. In this case, the weights are also parameters of the HA-IRMd to be designed or optimized and called also fuzziness parameters of HA-IRMd in common, for convenience.

5.2 Hedge-algebra-based controllers

Based on the HA-IRMds examined above, we introduce a general fuzzy control model based on the theory of hedge algebras, called hedge algebra-based controller (HAC). Figure 3 shows a general schematic view of the HA-control algorithm for HAC. In accordance with the construction of the HA-IRMds described above, there are three components of the HAC modules that are different from the corresponding components of fuzzy control algorithm described in Figure 1. Component (*I*) has the tasks to normalize the reference domains of the linguistic variables and to compute the values of the determined SQMs. Component (*II*) realizes the inference task based on the rules base and the constructed HA-IRMds. The task of Component (*III*) is to calculate the actual numeric value of the control action.

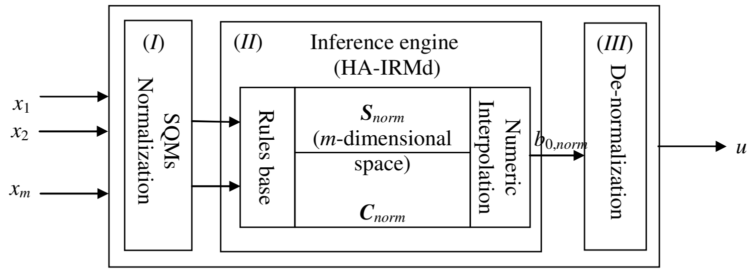

Figure 3. An overview of the HA-control algorithm for HAC

It is obvious that, except its fuzzy rules base, there are only two factors that affect the performance of HACs: (i) The fuzziness parameters of the linguistic variables to calculate SQMs values and (ii) The selected interpolation method on S_{norm}. In the case the designer prefers to use a numeric interpolation method on the curve C_{norm}, an additional factor that the designer must require to pay attention to is the aggregation operation. In comparison with the design of fuzzy controller, there are here only a few factors and it is important that they are much simpler than the factors affecting the effective construction of fuzzy controllers examined in Section 2. Based on the simulation study in Section 6, the designer can adjust these factors to construct a high performance controller.

In addition, as a consequence, the fuzziness parameter optimization problem can easily be solved to enhance its performance. A HAC designed with optimized fuzziness parameters is called optimized HAC or *op*HAC, for short.

The new methodology to construct HA-IRMds and HACs has many significant advantages:

1) The ability to establish a "faithful" mathematical model of (1): Since SQMs are homomorphic in the category of ordered sets, transforming the set of fuzzy rules in (1) into a crisp surface S_{norm} or, equivalently, a function f_{Snorm} in (13), they preserve the essential semantic-order-based structure or essential knowledge information of the linguistic model given by (1). We regard it as an essential factor to enhance the performance of fuzzy reasoning methods.

2) The surface S_{norm} or the function f_{Snorm} is a simple, transparent mathematical model that is easily constructed. At the same time, its construction based on the calculation of SQMs values is very simple. By providing fuzziness parameters of linguistic variables, the SQMs values of vague terms in the linguistic model (1) can be automatically computed.

3) The numerical output of (1) corresponding to the given input vector is calculated utilizing a classical (numeric) interpolation method on the surface S_{norm} or the curve C_{norm}. It is a well-known task and there are many interpolation methods that can be found in the literature. Defuzzification methods are not required here.

4) In the case the designer prefers to realize an interpolation method on the surface S_{norm}, there are only two factors which affect the performance of the designed HACs. Since the factor of the numeric interpolation is well-known, once it is fixed, the fuzziness parameters are the total

parameters that affect the effective construction of HACs, he may concentrate his effort to determine the fuzziness parameters to enhance the performance of the desired controller. This implies also that the fuzziness parameters optimization problem has a significant positive impact on the controller performance.

6. APPLICATIONS

To show the advantages of the proposed methodology, it will be applied to design HACs and opHACs for vibration control of high-rise structural systems with active tuned mass damper (ATMD) against earthquakes. The designed controllers will be simulated with the recorded seismic data of three typical earthquakes, El Centro, Northridge and Kobe to demonstrate their performance and, through this, to explain the advantages of the proposed methodology. In the simulation study, the recorded seismic data of El Centro will be used in the design of the controllers, while the remaining ones will be used for testing their performance.

6.1. Determining the control problem and its discrete control model

For a comparison study between the effectiveness of fuzzy-logic-based controller and HAC, a structural system model similar to those examined in [11] will be considered here. A high-rise building modelled as a structural system with ATMD, which is described in Figure 4, is assumed to have fifteen degrees of freedom all in a horizontal direction. The system is modelled with two active actuators of different types to suppress structural vibrations against earthquakes. Accordingly, one is installed on the first storey and the other on the fifteenth storey, since the maximum inter-storey shear force occurs on the first storey and the maximum displacements and accelerations are expected from the top storey of the structure during an earthquake, assuming equivalent storey stiffness and ultimate capacities. In Figure 4, m_1 is movable mass of the ground storey and m_2, m_3, …, m_{15} are the mass of the remaining storeys, where the mass of all storeys include both the ones of storeys and their walls. The mass m_{16} is of the ATMD installed on the fifteenth storey. The variables x_1, x_2, x_3,…, x_{14} and x_{15} indicate the horizontal displacements and x_{16} indicates the displacement of ATMD. The variable x_0 is the earthquake-induced ground motion disturbance to the considered structural system. All springs and dampers are acting in the horizontal direction. The system and ATMD parameters examined in [11] are given in Table 2.

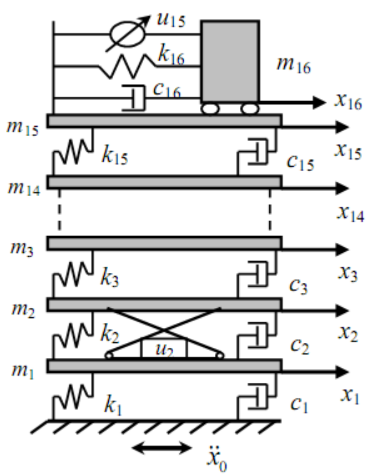

Figure 4. The structure

Table 2. The system parameters with ATMD

Storey i	Mass m_i (10^3 kg)	Damping c_i (10^2 Ns/m)	Stiffness k_i (10^5 N/m)
1	450	261.7	180.5
2-15	345.6	2937	3404
16 (ATMD)	104.918	5970	280

The discrete control model is established based on the dynamic model of fifteen-degrees-of-freedom structural system equipped with ATMD. The equations of motion of the system subjected to the ground acceleration $\ddot{x}_0$ for each earthquake described in Figure 5, with control force vector $\{F\}$, can be described in (15) (see [11]):

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F\}-[M]\{r\}\ddot{x}_0 \tag{15}$$

where $\{x\} = [x_1 \ \ x_2 \ \ x_3 \ \ldots \ \ x_{14} \ \ x_{15} \ \ x_{16}]^T$, $\{F\} = [-u_2 \ u_2 \ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \ u_{15} \ -u_{15}]^T$ and the 16×1 vector $\{r\}$ is the influence vector representing the displacement of each degree of freedom resulting from static application of a unit ground displacement. u_2 and u_{15} are the control forces produced by linear motors; the 16×16 matrices $[M]$, $[C]$ and $[K]$ represent the structural mass, damping and stiffness matrices, respectively.

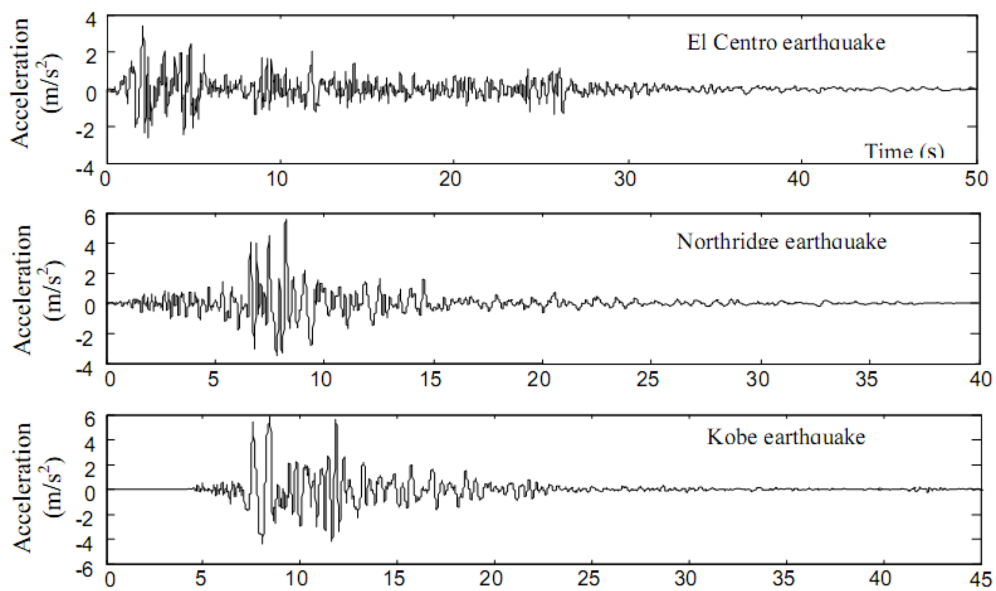

Figure 5. The ground acceleration $\ddot{x}_0$ (m/s^2)

The mass matrix $[M]$ for the high-rise building structure with the assumption of masses lumped at floor levels is a diagonal matrix given in (16), in which the mass of each storey and the ATMD are ordered on its diagonal:

$$[M]=\begin{bmatrix} m_1 & 0 & \ldots & 0 & 0 \\ 0 & m_2 & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & m_{15} & 0 \\ 0 & 0 & \ldots & 0 & m_{16} \end{bmatrix} \tag{16}$$

The structural stiffness matrix [*K*] is formed based on the individual stiffness k_i of each storey is defined by (17):

$$K_{ij}=\begin{cases} k_i+k_{i+1} & i=j\neq 16 \\ k_{16} & i=j=16 \\ -k_i & i-j=1 \\ -k_{i+1} & j-i=1 \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

The structural damping matrix [C] is defined as follows:

$$C_{ij}=\begin{cases} c_i+c_{i+1} & i=j\neq 16 \\ c_{16} & i=j=16 \\ -c_i & i-j=1 \\ -c_{i+1} & j-i=1 \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

Assume that the reference domains of the four state variables of the discrete control model are given by $-a_2 \le x_2 \le a_2$; $-b_2 \le \dot{x}_2 \le b_2$; $-a_{15} \le x_{15} \le a_{15}$ and $-b_{15} \le \dot{x}_{15} \le b_{15}$ and those of the control forces are given by $-c_2 \le u_2 \le c_2$ (N) and $-c_{15} \le u_{15} \le c_{15}$ (N), where a_i, b_i, for $i = 1, \ldots, 15$, indicate respectively the absolute peak displacement and velocity vectors of the uncontrolled state of the structure excited by earthquake ground shaking and c_2 and c_{15} are the maximal values of the control forces of the corresponding storeys.

The goal function *g* of the control is defined as follows:

$$g=\sum_{j=0}^{n}\sum_{i=1}^{15}\left(\frac{x_i^2(j)}{a_i^2}\right) \tag{19}$$

where *n* is the number of control cycles, and the a_i's are specified above.

6.2. Constructing fuzzy controller for the considered structural system

For comparative purposes, based on the discussion in Section 2 and the closed-loop fuzzy control algorithm given in Figure 6, the construction of the fuzzy controllers is realized by the design of the following factors which are the same as examined in [11]:

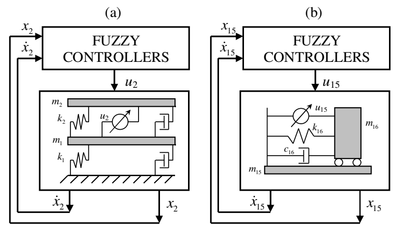

Figure 6. Schematic of fuzzy control algorithm of the structural system, (a) The actuator on the first storey, (b) The actuator on the fifteenth storey (ATMD)

(i) Fuzzifier: The linguistic variables of the variables x_2 and x_{15} are denoted by $\mathcal{X}_2$ and $\mathcal{X}_{15}$, of $\dot{x}_2$ and $\dot{x}_{15}$ by $\mathcal{V}_2$ and $\mathcal{V}_{15}$, respectively, and of u by $\mathcal{U}$. The vague terms of the both $\mathcal{X}_2$ and $\mathcal{X}_{15}$ are NB, NS, Z, PS and PB, of the both $\mathcal{V}_2$ and $\mathcal{V}_{15}$ are N, Z and P and of U are NB, NM, NS, Z, PS, PM and PB. The memberships of these terms are designed as depicted in Figure 7 – 9, which are the same as examined in [11].

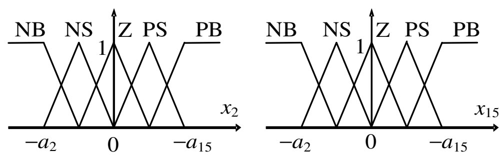

Figure 7. Membership functions for $\mathcal{X}_2$ and $\mathcal{X}_{15}$

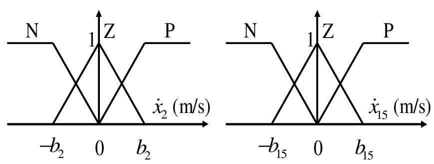

Figure 8. Membership functions for $\mathcal{V}_2$ and $\mathcal{V}_{15}$

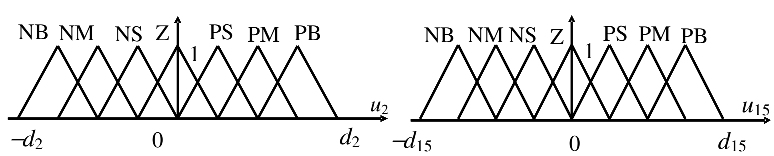

Figure 9. Membership functions for $\mathcal{U}_2$ and $\mathcal{U}_{15}$

Since the recorded seismic data of El Centro earthquake will be used to design controllers, the universes of discourse of four state variables of the discrete control model of the from $-a_2 \le x_2 \le a_2$, $-b_2 \le \dot{x}_2 \le b_2$, $-a_{15} \le x_{15} \le a_{15}$ and $-b_{15} \le \dot{x}_{15} \le b_{15}$ will be determined by, respectively, the absolute peak displacement and velocity vectors of the uncontrolled state of the structure excited by El Centro earthquake ground motion. The control forces u_2 and u_{15} are assumed to subject to the constraints $-3.83\times10^6 \le u_2 \le 3.83\times10^6$ (N) and $-6.9\times10^6 \le u_{15} \le 6.9\times10^6$ (N).

(ii) Inference engine: The construction of the inference engine is also the same as examined in [11]. It comprises two main components, first of which is its fuzzy rules base given in Tables

3 and 4. The remaining one is the fuzzy reasoning method which is selected to be the one of Mamdani.

(iii) Defuzzifier is usually the centre gravity method, which was chosen also in [11].

3) Constructing control algorithm for the desired HAC

The design of HAC in this subsection is based on the HAC scheme depicted in Figure 3.

The following tasks should be implemented:

- *To determine hedge algebras of the considered linguistic variables for representing control knowledge*: The hedge algebras need be determined only for the following variables: x_2, $\dot{x}_2$, x_{15}, $\dot{x}_{15}$, u_2 and u_{15}. The numerical values of the variables corresponding to the remaining storeys are computed by the established discrete control model. As previously, although the linguistic variables under consideration are different, their hedge algebras may be defined with a similar structure as follows: $\boldsymbol{G} = \{small, large\}$, $\boldsymbol{C} = \{\boldsymbol{0}, \boldsymbol{W}, \boldsymbol{1}\}$ and $\boldsymbol{H} = \{h^-, h^+\} = \{L, V\}$, where L and V stand for *Little* and *Very*, respectively, as previously. However, in order to indicate their different quantitative semantics, we denote these hedge algebras by the same notations with different indexes. For instance, the hedge algebra of the variable $\dot{x}_2$ is denoted by AX_{2*} with $\boldsymbol{G} = \{small_{2*}, large_{2*}\}$, $\boldsymbol{C} = \{\boldsymbol{0}_{2*}, \boldsymbol{W}_{2*}, \boldsymbol{1}_{2*}\}$ and $\boldsymbol{H} = \{L_{2*}, V_{2*}\}$. Similarly, in accordance with this convention, the hedge algebra of x_{15} is denoted by AX_{15} with $\boldsymbol{G} = \{small_{15}, large_{15}\}$, $\boldsymbol{C} = \{\boldsymbol{0}_{15}, \boldsymbol{W}_{15}, \boldsymbol{1}_{15}\}$ and $\boldsymbol{H} = \{L_{15}, V_{15}\}$, but for u_{15} it is denoted by AU_{15} with $\boldsymbol{G} = \{small_{u15}, large_{u15}\}$, $\boldsymbol{C} = \{\boldsymbol{0}_{u15}, \boldsymbol{W}_{u15}, \boldsymbol{1}_{u15}\}$ and $\boldsymbol{H} = \{L_{15}, V_{u15}\}$, and so on.

The FAM tables for the fuzzy control of the first and fifteenth storeys examined in [11] are given in Tables 3 and 4, the vague terms in which are only the labels of the designed fuzzy sets defined on symmetric intervals of the form [$-a$, a]. In the algebraic approach, they are however elements of the respective constructed hedge algebras with the qualitative and quantitative semantics with the normalized reference domain [0,1] examined in Sections 3 – 5. Thus, the vague terms in these tables must be transformed into terms of the respective hedge algebras by a term-transformation, which preserves essential order-based semantic properties of vague terms appearing in FAM-tables. Usually, the potential vague terms used in fuzzy control like these FAM tables can be linearly ordered and grouped into pair of terms with opposite meanings, i.e., they are symmetrical with respect to the term ‘*zero*” - the *neutral*. For instance, the pair of terms *positive* and *negative* or of *positive big* and *negative big* is symmetric. The desired term-transformations should preserve their order and symmetry. In this experiment, they are defined by Tables 5 and 6.

Table 3. FAM table for the actuator on the first storey

x_2 \ $\dot{x}_2$	N	Z	P
NB	NB	NM	NS
NS	NM	NS	Z
Z	NS	Z	PS
PS	Z	PS	PM
PB	PS	PM	PB

Table 4. FAM table for the actuator on the fifteenth storey

x_{15} \ $\dot{x}_{15}$	N	Z	P
NB	NB	NM	NS
NS	NM	NS	Z
Z	NS	Z	PS
PS	Z	PS	PM
PB	PS	PM	PB

Table 5. Linguistic transformation for x_2, $\dot{x}_2$, x_{15} and $\dot{x}_{15}$

NB	N	Z	P	PB
small	*Little small*	***W***	*Little large*	*large*

Table 6. Linguistic transformation for u_2 and u_{15}

NVB	NB	N	Z	P	PB	PVB
Very small	*small*	*Little small*	***W***	*Little large*	*large*	*Very large*

- *To construct HA-IRMds for the application under consideration*: For each application, once reference domains of the considered linguistic variables are determined, the normalization transformation for every variable can be automatically produced. The SQMs of the linguistic variables will also easily be determined by providing their fuzziness parameter values, which are either designed by the designer or produced by an evolutionary procedure to solve the fuzziness parameter optimization problem, as discussed previously. Then the required q-FAM tables are constructed.

In this subsection, the HA-IRMds are defined by the linear interpolation with respect to the established hyper-surfaces S_{nor} modelled approximately by the available data given in the q-FAM tables of the first and fifteenth storeys.

6.3. Optimization of fuzziness parameters

In this subsection, we deal with the El Centro, Northridge and Kobe earthquakes, where the seismic data of El Centro earthquake in USA were recorded at the El Centro Terminal Substation Building on May 18th, 1940 with Peak Ground Acceleration (PGA) 0.35g, which can be found at http://www.vibrationdata.com/elcentro.htm, and the seismic data of Northridge earthquake in USA were recorded at the Castaic - Old Ridge Route Station on January 17th, 1994 with PGA 0.57g and the ones of Kobe earthquake in Japan were recorded at the KJMA Station in Kobe on January 16th, 1995 with PGA 0.60g, see http://peer.berkeley.edu/smcat/search.html.

The idea of solving the fuzziness parameter optimization problem here is described as follows: since it is difficult for the designer to determine the appropriate fuzziness parameters for a practical application problem, the data of El Centro earthquake is chosen randomly among three mentioned earthquakes as the training data to determine the near optimal fuzziness parameters for the earthquake protective structural system under consideration. Then, the obtained optimal fuzziness parameters will be used to design the HACs applied to the protective structure in question against other earthquakes in the future. The Northridge and Kobe earthquakes will be used as the testing data for the designed HAC to validate its performance.

Thus, the hedge algebras, the reference domains of the linguistic variables and their normalization transformations and SQMs will be determined utilizing the seismic data of El Centro earthquake. Then, the universes of discourse of four state variables x_2, $\dot{x}_2$, x_{15} and $\dot{x}_{15}$ and of two control force variables u_2 and u_{15} are the same as for the above designed fuzzy controllers.

The goal function is defined by (19), for which the number n of cycles of the whole control process is defined by dividing the total time 50 s of simulation by the time step 0.01 s. So, n = 5000. The fuzziness parameter optimization problem will be solved by utilizing a genetic algorithm (GA) based on the encoding examined in [5] with the following requirements:

- The constraints on fuzziness parameters: $0.3 \le fm(c^-) \le 0.7$ and $0.3 \le \mu(h^-) \le 0.7$.
- Since the main aim of the study is to show the advantages of the proposed methodology, in this simulation only the fuzziness parameters of the algebras AU_2 and AU_{15} are optimized for simplicity.

For the remaining hedge algebras they are assigned with the same values as follows:

$$fm(small) = \mu(Little) = 0.5$$

In despite of this, the simulation experiments and the comparison study below still show the better performance of the designed *op*HAC's than their counterparts in protecting the civil structural system from earthquakes.

Then, the near-optimal fuzziness parameters of AU_2 and AU_{15} shown in Table 7 have been produced by a GA, using the seismic data provided from El Centro earthquake.

Table 7. The optimal parameters of $\mathcal{AU}_2$ and $\mathcal{AU}_{15}$ for the *op*HAC

For the actuator on the 1th-storey, u_2		For the actuator on the 15th-storey, u_{15}	
$fm(c^-)$	$\mu(h^-)$	$fm(c^-)$	$\mu(h^-)$
0.383	0.628	0.620	0.689

6.4. Simulation results and a comparative analysis

- The simulation experiments have been designed in order to show the effectiveness of the hedge-algebra-based methodology applied to this field. The structural system under consideration equipped in turn with the designed fuzzy controller (FC), the designed HAC and the designed *op*HAC has been simulated against the earthquake ground vibrations obtained from the seismic data of the three specific earthquakes - El Centro, Northridge and Kobe. It can be observed from Figures 10, 12 and 14 that all horizontal displacement responses of the fifteen-degree-of-freedom structural system have been taken into account in the simulation experiments.

- All the controllers of three types have been designed based on the recorded seismic data of the El Centro earthquake, including the design of the optimal fuzziness parameters of *op*HACs. This means that the El Centro earthquake data have been used in the training phase and the seismic data of Northridge and Kobe earthquakes have been used in the testing phase to evaluate the effect of the proposed methodology. To offer some comparison, the calculated control forces of the control algorithms of all three controllers should be bounded by the same maximal control forces 1700 kN for the first storey and 3000 kN for the fifteenth storey. The simulation results of all fifteen storeys exhibited in Figures 10, 12 and 14 show that the performance of the designed *op*HACs are always the best and that of the designed fuzzy controllers are always the worst for all the three examined earthquakes, although its optimal parameters are determined by utilizing only the seismic data obtained from the El Centro earthquake. Since in practical applications it is difficult to determine the parameters of the membership functions, in the design of fuzzy controllers, and the fuzziness parameters, in the design of HAC's, these results point out a useful advantage stating that in designing an *op*HAC for a structural system one may determine its optimal fuzziness parameters by an evolutionary technique using the seismic data of a particular earthquake.

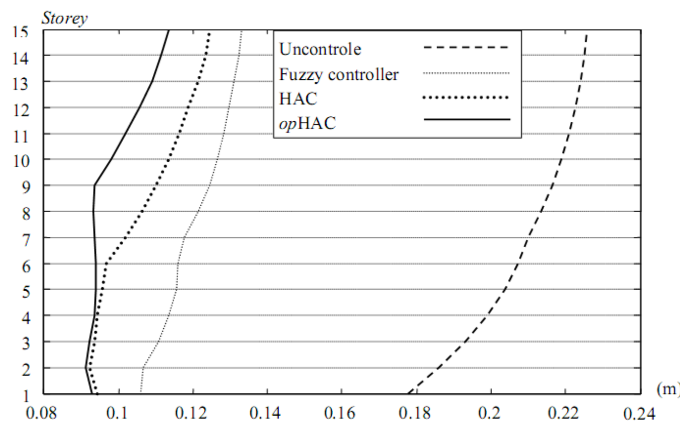

Figure 10. The maximum storey drift of El Centro earthquake

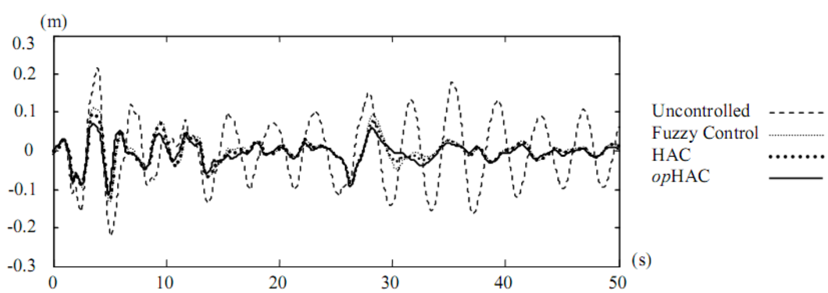

Figure 11. Displacements x_{15} (m) versus time (s) of El Centro earthquake

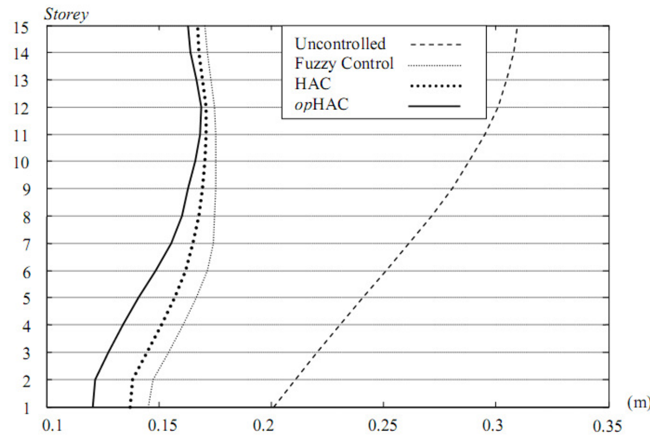

Figure 12. The maximum storey drift – Northridge earthquake

Figures 11, 13 and 15 exhibit comparisons of controlled displacement of the fifteenth storey of the examined structural system calculated by the designed fuzzy controller, HAC, and *op*HAC with the uncontrolled ones for all three earthquakes under consideration.

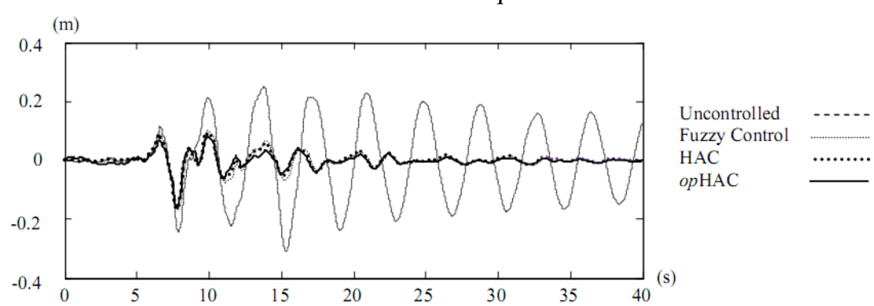

Figure 13. Displacements x_{15} versus time of Northridge. earthquake

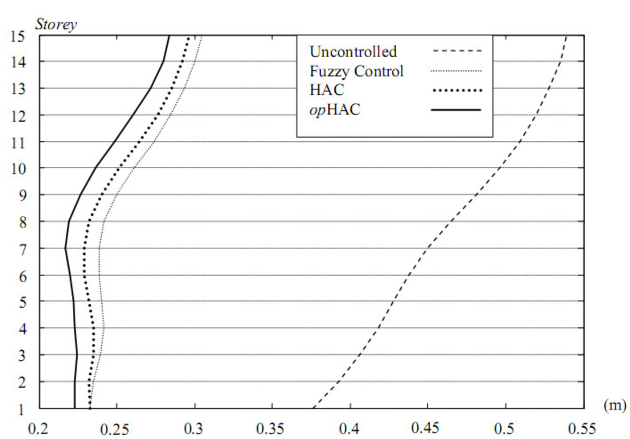

Figure 14. The maximum storey drift – Kobe earthquake

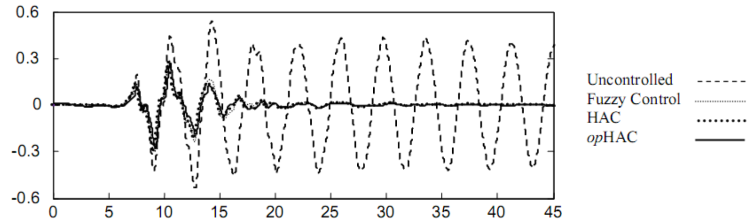

Figure 15. Displacements x_{15} versus time - Kobe earthquake

Table 8. Simulation results

Building Storey	El Centro earthquake				Northridge earthquake				Kobe earthquake			
	Max uncontrolled displacement (m)	Controlled to uncontrolled displacement ratio (reduction ratio)			Max uncontrolled displacement (m)	Controlled to uncontrolled displacement ratio (reduction ratio)			Max uncontrolled displacement (m)	Controlled to uncontrolled displacement ratio (reduction ratio)		
		FC	HAC	opHAC		FC	HAC	opHAC		FC	HAC	opHAC
1	0.178	0.595	0.530	0.523	0.201	0.728	0.681	0.598	0.376	0.618	0.606	0.592
2	0.186	0.573	0.496	0.490	0.211	0.705	0.657	0.576	0.392	0.599	0.591	0.568
3	0.193	0.573	0.485	0.478	0.220	0.705	0.657	0.578	0.406	0.590	0.578	0.552
4	0.199	0.571	0.473	0.469	0.230	0.702	0.657	0.581	0.418	0.578	0.561	0.534
5	0.204	0.566	0.469	0.461	0.241	0.697	0.653	0.585	0.428	0.562	0.541	0.518
6	0.207	0.559	0.466	0.453	0.251	0.687	0.645	0.592	0.438	0.544	0.523	0.503
7	0.210	0.560	0.486	0.445	0.261	0.671	0.633	0.594	0.450	0.531	0.508	0.483
8	0.213	0.569	0.499	0.437	0.271	0.650	0.619	0.591	0.465	0.520	0.499	0.471
9	0.216	0.575	0.509	0.432	0.280	0.629	0.605	0.583	0.481	0.520	0.500	0.471
10	0.219	0.578	0.518	0.447	0.288	0.612	0.592	0.577	0.496	0.527	0.507	0.477
11	0.221	0.580	0.525	0.461	0.295	0.598	0.580	0.570	0.509	0.538	0.519	0.488
12	0.223	0.582	0.533	0.474	0.301	0.584	0.569	0.561	0.519	0.548	0.532	0.502
13	0.224	0.585	0.542	0.486	0.305	0.571	0.557	0.547	0.528	0.556	0.541	0.515
14	0.225	0.587	0.548	0.496	0.308	0.560	0.546	0.533	0.535	0.561	0.546	0.523
15	0.226	0.590	0.551	0.502	0.310	0.555	0.540	0.527	0.539	0.564	0.550	0.526

Table 8 presents a summary of simulation results in view of reducing the displacement response of the examined structural system. For example, for the 1st storey, the response reduction ratio, i.e. the ratio of the controlled to uncontrolled response for maximum displacement is about 59.5%, 53.0% and 52.3% for the designed FC, HAC and *op*HAC, respectively, for El Centro earthquake. That is in view of the reducing the displacement response, the performance of the designed HAC and *op*HAC is better than the performance of the designed FC about 10.92% and 12.1%. The corresponding performance percentages are 15.1% and 17.86%, for Northridge earthquakes, and 1.94% and 4.21%, for Kobe earthquake.

For the 15^{th} storey, the corresponding performance percentages are 6.61% and 14,92%, for El Centro earthquake, 2.7% and 5.06%, for Northridge earthquake, and 2.48% and 6.74%, for Kobe earthquake.

Table 9. CPU computation time of the controllers: a comparative analysis

Earthquake	El Centro	Northridge	Kobe
FC	172.6250	135.3125	165.9357
HAC	17.6250	14.0781	16.4844

In view of the maximum displacement, which is an essential evaluation criterion of the structural control algorithm, it is observed that for El Centro earthquake, the maximum reduction ratio produced by the designed FC is 0.595, by the designed HAC is 0.551 and by the designed *op*HAC is 0.523. The corresponding figures are 0.728, 0.681 and 0.598, for Northridge earthquake, and 0.618, 0.606 and 0.592, for Kobe earthquake. One should emphasize the fact that the maximum displacements of all storeys calculated by the designed controllers for all three examined earthquakes are all decreased in turn from the designed controllers FC to HAC and then to *op*HAC. Notice that although the size of the population is only 80 and the number of generations is only 300, which are still small, the performance of the designed *op*HAC's is already the best in comparison with the remaining controllers.

The comparison above underlines the advantages of the proposed methodology. However, its main benefit is in the simplicity and effectiveness of the control algorithm based on HA-IRMds. This is guaranteed by the advantages discussed in the end of Subsection 4.1 and, hence, it improves the performance of the desired controllers. Table 9 shows that the total CPU computation times of the designed HACs (given in) for the simulation of the structural system against the examined earthquake disturbances are about ten times smaller than those of the designed fuzzy counterpart.

7. CONCLUSIONS

In this paper, the methodology to design fuzzy controllers based on hedge-algebras-based semantics of vague terms has been presented. Its novelty originates from the fact that hedge algebras model directly the qualitative meaning of the vague linguistic terms in term-domains of linguistic variables. The meaning of vague terms is not expressed by fuzzy sets, but by their order relationships with the remaining terms in a terms-domain, called order-based meaning. Thus, we have an algebraic approach to the semantics of vague terms, which provides another mechanism to process fuzzy information.

The proposed methodology exhibits the following main advantages:

(i) Hedge algebras, as a mathematical foundation of the methodology, model the inherent order-based structure of terms-domains of linguistic variables. It may be important to guarantee the effectiveness on the methodology. It has been demonstrated that the fuzziness of vague terms, which is an essential characteristic of fuzzy information, can be modelled in this algebraic approach and based on it fuzziness measure of terms can be defined in an axiomatic way. Moreover, fuzziness intervals of terms, which exhibit a characteristic of the quantitative semantics of terms, can be constructed relying upon properties of fuzziness measure. Semantically quantifying mappings (SQMs), a significant component for developing hedge-algebra-based interpolation reasoning methods (HA-IRMds), can be defined in a close relationship with the fuzziness measure and fuzziness intervals of terms. They are completely determined by providing the fuzziness measure of the primary terms and hedges. These shows that SQMs have been defined in close relation with the semantics of vague terms and hence they can bring useful information conveyed by vague linguistic terms.

(ii) Reasoning methods in this approach, HA-IRMds, are simple and transparent, since they depend only on two factors, the set of fuzziness parameters of SQMs and classical interpolation methods on Euclidean hypersurfaces. We should stress the fact that SQMs are homomorphisms in the category of linearly ordered sets. This ensures that the hypersurface, which is obtained by transforming the given set of fuzzy rules into multiple-dimensional Euclidean space using properly constructed SQMs, can be regarded as a suitable model of available control knowledge.

In fuzzy control, the fuzzy reasoning method, a significant component of the fuzzy engine, depends on many significant factors, one of which concerns the membership functions, as discussed in Section 2. Accordingly, tuning membership functions based on evolutionary techniques has a limited affect on the performance of the desired fuzzy controllers. Instead, in the algebraic approach, tuning of the fuzziness parameters has a significant affect on their performance, since except classical interpolation method to be selected, their performance depends merely on the fuzziness parameters.

The methodology has been applied to the design of HACs and *op*HACs for a problem of active control of earthquake excited high-rise civil structures. The computer simulation completed for the seismic data recorded from El Centro, Northridge and Kobe earthquakes demonstrate the advantages of the proposed methodology and underline the relevance of the algebraic approach.

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TÓM TẮT

ĐIỀU KHIỂN CHỦ ĐỘNG CÁC KẾT CẤU BỊ KÍCH THÍCH DO ĐỘNG ĐẤT BẰNG BỘ ĐIỀU KHIỂN DỰA TRÊN ĐẠI SỐ GIA TỬ

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Trong bài báo này, chúng tôi đưa ra phương pháp luận dựa trên đại số gia tử để thiết kế bộ điều khiển mờ được gọi là bộ điều khiển dựa trên đại số gia tử (HACs) nhằm kiểm soát rung động các hệ kết cấu. Trong phương pháp này, các nhãn ngôn ngữ không thể hiện bằng các tập mờ mà bằng quan hệ thứ tự vốn có giữa các giá trị ngôn ngữ tồn tại trong miền xác định của tập nền. Ánh xạ ngữ nghĩa định lượng (SQMs), bảo toàn mối quan hệ thứ tự dựa trên ngữ nghĩa trong miền xác định, được định nghĩa trong một mối quan hệ chặt chẽ với độ đo mờ và khoảng mờ của các nhãn ngôn ngữ. Qua cách sử dụng SQMs, phương pháp suy luận mờ có thể được chuyển thành các phương pháp nội suy số đối với các điểm trong không gian Euclid đa chiều được xác định dựa trên các luật if-then của các tri thức điều khiển. Điều này cung cấp một cơ sở toán học tốt hỗ trợ cho quá trình xây dựng các thuật toán điều khiển. Phương pháp luận được đề xuất khá đơn giản, minh bạch và hiệu quả. Trong một nghiên cứu cụ thể, các bộ điều khiển HACs và HACs tối ưu đã được thiết kế dựa trên phương pháp luận mới để kiểm soát các kết cấu cao tầng dân sự. Các bộ điều khiển HACs và HACs tối ưu đã thành công hơn các bộ điều khiển mờ trong việc làm giảm các phản ứng dịch chuyển tối đa của kết cấu theo ba kịch bản trận động đất khác nhau: El Centro, Northridge và Kobe. Điều này cho thấy hiệu quả của phương pháp đề xuất.

Từ khóa: lí thuyết điều khiển, suy luận xấp xỉ, độ đo mờ, tính toán kỹ thuật chịu tải động đất, đại số gia tử.