

## MODELING USING 2-D AREAS OF IDEAL CROSS-POINT REGIONS FOR LOSSLESS IMAGES COMPRESSION

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### ABSTRACT

This paper presents a new scheme using 2-D areas of ideal cross-point regions which are a new part in the theory of cross-point regions. When using cross-point regions we need to determine the rule of building cross-point maps, this takes a long time and a big space for storing these maps, this also brings about not high compression ratio when using one dimensional cross-point regions because many coordinates of data points need to be saved for decoding. When these 2-D areas with new features are used, the scheme of 2-DICRIC (2-D Ideal Cross-point Regions for lossless Image Compression) for losslessly encoding and decoding images is used to get higher compression ratio. The base idea of this method is the effect of Gray coding on cross points. Before Gray coding, data sets of cross points are determined, they are called the ideal cross point regions (ICRs). After Gray coding, these regions always contain only 1 bits or 0 bits depending on the number of bit plane after the operation of bit plane decomposition. With the characteristic of images, grey values do not change much in a specific area, especially in medical images which have many regions with the approximate grey levels, the theory with these 2-D areas has important effects on the compression ratio when encoding and decoding processes of lossless image compression for data transmission are proceeded. The scheme can be applied for lossless images compression and cryptography.

**Keywords:** Gray codes, ideal cross-point regions, bit planes decomposition, probability of bits.

### 1. INTRODUCTION

This paper is a development of the papers [1, 2] that presented the definition of cross points and one dimensional cross-point regions, the propositions of bit states of cross points and the consequences about entropy of obtained data in these cross-point regions after Gray code transformation. This paper gives more effect of Gray coding on the calculating process of probability of bits when 2-D areas of ideal cross-point regions are built to use in the new scheme – 2-DICRIC.

Cross points are neighbor points around the points of grey levels  $2^n$  which may or may not exist in data of images. The original data points whose values are less than  $2^n$  have bit states much different from those of the data points greater than or equal to  $2^n$  [3, 4]. The changes of bit

states for Gray coding are studied by the contributions [5 - 9], however the number of bits and the distribution of these bits of Gray codes in the ideal cross-point regions are not mentioned yet. This leads to a new scheme for data compression. The data are arbitrary, but in ideal cross-point regions the change of bit states are systematically determined after Gray coding, so the probability of data bits in ideal cross-point regions is maximal. Finding out cross-point regions for the scheme is very important because it affects the compression ratio via the space for storing the map of cross-point regions, the smaller the size of this map, the higher the compression ratio. Generally, 2-DICRIC uses the entropy coding to losslessly compress images, there are two steps: modeling and coding. In the step of modeling the theory of cross-point regions will provide 2-D areas of ideal cross-point regions before Gray coding [10] with the map of these regions, then other operations are carried out such as Gray coding, bit plane decomposition. After these steps we have the bit planes with the map of ideal cross-point regions, that means 2-D areas of data bits on the different bit planes depending on the values of 2<sup>n</sup> which are the central values of ideal cross-point regions. The probability of data bits in these areas will be optimized for the step of coding which uses some algorithm like arithmetic coding to get a codeword for all of the bit planes of image.

This paper has eight sections. After this introduction, the section 2 discusses the related works – the theory of cross-point regions with ICRs being used for lossless image compression, the disadvantages, and the development from ICRs to 2-D areas of ICRs for the new scheme in this paper. The section 3 mentions the definition of one dimensional ICRs, the proposition about the characteristic of Gray codes of cross points, this section also presents the bit states and the probability of bits in ICRs. Therefore, this section describes the theory used in the step “modeling” of entropy coding [8 - 11]. The coding algorithm in the entropy coding is Jones’ [12] with the supplement of calculating frequencies [13]. Section 4 gives the new concept on effective ideal cross-point regions which is used to build 2-D areas of effective ICRs for the scheme. Section 5 presents the concept of 2-D areas of ICRs, the way to apply these areas to the scheme, and its role in the theory of cross-point regions for image compression. The section 6 introduces the scheme of 2-DICRIC (2-D Ideal Cross-point Regions for lossless Image Compression). This scheme is an entropy coding with the ideal cross-point regions used in the step “modeling”. In the section 7, some results obtained from using the theory above are presented in comparison with the other methods: the results of the previous algorithm of ICRIC [14], Advanced Encryption Standard (WinRar 9.0) [15] and lossless wavelet transform (JPEG 2000) [16]. The section 8 is the conclusion and the scope for future researches.

## 2. RELATED WORKS

The scheme of 2-DICRIC is developed from the algorithm of ICRIC which used normal ideal cross-point regions (ICRs). The related works in the contributions [1] and [2] give us the concept on ICRs from which we improve to get 2-D areas of ICRs applying for the new scheme 2-DICRIC in this paper. The definition, proposition, and consequence for ICRs in [1] and [2] being mentioned in the section 3 below are not exactly the same as the origins in [1] and [2], they are modified for the purpose of developing ICRs to get 2-D ICRs with their effective widths and building the scheme of 2-DICRIC in this paper.

There are three things which need to be presented to make us understand the process of 2-DICRIC, they are the definition of ICRs from what we can determine the set of pixels in the certain range of grey values, the proposition of bit state giving us the existence of 0 and 1 bit in ICRs before and after the Gray coding, and the consequence presenting the ability to decrease

the entropy of new data points on bit planes. One of the advantages of ICRs is that it can give us the ability to encode the regions including edges of objects in images which have obstructed the process of compression for traditional schemes.

However, with many ICRs in a scan line, and many scan line in an image, each ICR needs two pixels for marking, one pixel for the starting and the other for the ending of the ICR, i.e. four coordinates for these pixels, this makes the size of cross-point map for compressing and decompressing bigger, this affects the compression ratio and the time for compressing and decompressing. The effective ICR gives us the ability to aggregate neighbor ICRs to build areas of ICRs with only two pixels needing to be marked in the cross-point map. This makes the size of cross-point map decrease.

The next section gives the concept on normal ICRs which are used to develop to get effective ICRs and then 2-D areas of ICRs for the scheme 2-DICRIC in this paper.

### 3. IDEAL CROSS-POINT REGIONS AND BIT STATES

The definition of ICRs, the proposition of bit states in ICRs, and the consequence about the entropy of data in ICRs here are extracted in the contributions [1] and [2] but not exactly the same.

**Definition 1.** Given the bit depth of data points  $N$ , the ideal cross-point region (ICRs)  $A_0(n, p)$ , where  $n$  is from  $(N - 1)$  down to 1, and  $p$  is from 1 to  $n$ , with  $n \geq p$ , is a set of points grey values of which are among  $(2^n - 2^{n-p})$  and  $(2^n + 2^{n-p} - 1)$ . The data point having value  $2^n$  (if existing) is the central point of that region, and the value  $2^n$  is called the boundary value of region.

With Definition 1, the values of points in ICRs are in the set

$$V_A(n, p) = \{2^n - 2^{n-p}, \dots, 2^n + 2^{n-p} - 1\} \quad (1)$$

The value  $n$  is the exponent of boundary values  $2^n$ , the value  $p$  gives the range of ICRs, that means the number of bit planes are used to encode. The number of the bit plane for data bits compression is  $(n-p)$ . Moreover, the set  $V_A(n, p)$  can be regarded in two groups:  $V_{A_I}(n, p)$  and  $V_{A_G}(n, p)$  with  $V_{A_I}(n, p) = \{2^n - 2^{n-p}, 2^n - 2^{n-p} + 1, \dots, 2^n - 2, 2^n - 1\}$ , and  $V_{A_G}(n, p) = \{2^n, 2^n + 1, \dots, 2^n + 2^{n-p} - 2, 2^n + 2^{n-p} - 1\}$ . These sub regions are very meaningful due to the state of bits inside.

For example, when  $n = 3, p = 1$ ,  $V_A(3, 1) = \{2^3 - 2^{3-1}, \dots, 2^3 + 2^{3-1} - 1\} = \{4, 5, 6, 7, 8, 9, 10, 11\}$ , at that time  $V_{A_I}(3, 1) = \{4, 5, 6, 7\}$ , and  $V_{A_G}(3, 1) = \{8, 9, 10, 11\}$ . In this case, the boundary  $2^n$  exists explicitly. So, data points in the region  $V_{A_I}(3, 1)$ , from 4-7, i.e.  $2^{3-1}, \dots, 2^3 - 1$ , are expanded under the form of a polynomial of radix 2 as the following:

$$0.2^3 + 1.2^{3-1} + x.2^{3-2} + x.2^0. \quad (2)$$

The data points in the region  $V_{A_G}(3, 1)$ , from 8-11, i.e.  $2^3, \dots, 2^3 + 2^{3-1} - 1$  are expanded by the following polynomial:

$$1.2^3 + 0.2^{3-1} + x.2^{3-2} + x.2^0, \quad (3)$$

where  $x$  is 1 or 0.

After Gray coding, (2) and (3) become (4) and (5), respectively:

$$0.2^3 + 1.2^{3-1} + x.2^{3-2} + x.2^0, \quad (4)$$

$$1.2^3 + 1.2^{3-1} + x.2^{3-2} + x.2^0, \quad (5)$$

where  $x$  is 1 or 0.

Equation (5) shows that the region  $V_{Ag}(3, 1)$  on bit plane 2 ( $= 3-1$ ) contains only 1 bits. From (4), we can see a similarity to the region  $V_{Al}(3, 1)$ . By combining (4) and (5), the region  $V_A(3, 1)$  on the bit plane 2 ( $= 3-1$ ) contains only 1 bits.

Let us continue with  $V_A(3, 2)$ . When  $p$  is 2, the data points in the region  $V_{Al}(3, 2)$ , from 6-7, i.e.  $2^3-2^{3-2}$ ,  $2^3-1$  are expanded in the form of a polynomial of radix 2:

$$0.2^3 + 1.2^{3-1} + 1.2^{3-2} + x.2^0 \quad (6)$$

The data points in the region  $V_{Ag}(3, 2)$ , from 8-9, i.e.  $2^3$ ,  $2^3+2^{3-2}-1$  are expanded by the following polynomial:

$$1.2^3 + 0.2^{3-1} + 0.2^{3-2} + x.2^0, \quad (7)$$

where  $x$  is 1 or 0.

After Gray coding, (6) and (7) become

$$0.2^3 + 1.2^{3-1} + 0.2^{3-2} + x.2^0, \quad (8)$$

$$1.2^3 + 1.2^{3-1} + 0.2^{3-2} + x.2^0. \quad (9)$$

Equation (8) shows that the region  $V_{Al}(3, 2)$  on the bit plane 1 ( $= 3-2$ ) contains only 0 bits. From (9), we can see a similarity to the region  $V_{Ag}(3, 2)$ . By combining (8) and (9), the region  $V_A(3, 2)$  on the bit plane 1 ( $= 3-2$ ) contains only 0 bits. The proposition below generalizes these bit states.

| $n/2^p$ | $7/2^7$            | $6/2^6$          | $5/2^5$          | $4/2^4$          | $3/2^3$         | $2/2^2$        | $1/2^1$        |
|---------|--------------------|------------------|------------------|------------------|-----------------|----------------|----------------|
| 1       | $n-p=6$<br>64-191  | $n-p=5$<br>32-95 | $n-p=4$<br>16-47 | $n-p=3$<br>8-23  | $n-p=2$<br>4-11 | $n-p=1$<br>2-5 | $n-p=0$<br>1-2 |
| 2       | $n-p=5$<br>96-159  | $n-p=4$<br>48-79 | $n-p=3$<br>24-39 | $n-p=2$<br>12-19 | $n-p=1$<br>6-9  | $n-p=0$<br>3-4 |                |
| 3       | $n-p=4$<br>112-143 | $n-p=3$<br>56-71 | $n-p=2$<br>28-35 | $n-p=1$<br>14-17 | $n-p=0$<br>7-8  |                |                |
| 4       | $n-p=3$<br>120-135 | $n-p=2$<br>60-67 | $n-p=1$<br>30-33 | $n-p=0$<br>15-16 |                 |                |                |
| 5       | $n-p=2$<br>124-131 | $n-p=1$<br>62-65 | $n-p=0$<br>31-32 |                  |                 |                |                |
| 6       | $n-p=1$<br>126-129 | $n-p=0$<br>63-64 |                  |                  |                 |                |                |
| 7       | $n-p=0$<br>127-128 |                  |                  |                  |                 |                |                |

Figure 1. Ranges of 8-bit values in ICRs.



Figure 1 presents the ranges of 8-bit values of data points in ICRs from the bit planes  $(n-p)$  6 down to 0 for all central values from  $2^7$  to  $2^1$ . Actually, with the scheme of 2-DICRIC these data values for ICRs could not be used directly because of the dispersion of the starting points of ICRs, they themselves need to be grouped in order to form ICRs with effective widths, and then these regions become cores for the scheme of 2-DICRIC.

**Proposition 1.** Given the exponent  $n$  of the boundary value  $2^n$  in the region  $A_o(n, p)$ , with  $n$  from  $(N-1)$  down to 1,  $p$  from 1 to  $n$ , and  $n \geq p$ ,  $N$  is the bit depth of data. Gray codes in the region  $A_o(n, p)$  always contain 1 bits (when  $p = 1$ ) or 0 bits (if otherwise) on the bit plane  $(n-p)$ .

*Proof of Proposition 1.*

From Definition 1, with a value of  $n$  from  $(N-1)$  to 1 and  $p = 1$ , the values of data points in  $A_o(n, 1)$  are from  $(2^n - 2^{n-1})$  to  $(2^n + 2^{n-1} - 1)$ , so they can be written under the form of polynomials of radix 2 (10) (for data points in  $V_{A_1}$ ) and/or (11) (for data points in  $V_{A_2}$ ) as the followings:

$$0.2^{N-1} + \dots + 0.2^n + 1.2^{n-1} + x.2^{n-2} + \dots + x.2^0, \quad (10)$$

$$0.2^{N-1} + \dots + 0.2^{n+1} + 1.2^n + 0.2^{n-1} + x.2^{n-2} + \dots + x.2^0, \quad (11)$$

where  $x$  are 1 or 0.

After Gray coding, (10) and (11) become (12) and (13) respectively:

$$0.2^{N-1} + \dots + 0.2^n + 1.2^{n-1} + x.2^{n-2} + \dots + x.2^0, \quad (12)$$

$$0.2^{N-1} + \dots + 0.2^{n+1} + 1.2^n + 1.2^{n-1} + x.2^{n-2} + \dots + x.2^0. \quad (13)$$

By combining (12) and (13), we can see the region  $A_o(n, 1)$  on the bit plane  $(n-1)$  always contains 1 bits.

For example, when  $n = 3$ , the boundary value is  $2^3$ ,  $A_o(3, 1) = \{4, 5, 6, 7, 8, 9, 10, 11\}$ . After Gray coding, the Gray codes of those values are 6, 7, 5, 4, 12, 13, 15, 14 respectively, all of them have 1 bits on the bit plane 2 ( $= 3 - 1$ ). This affects the process of compressing because the probability of 1 bit in those ICRs is always 1, and the probability of 0 bit is always 0.

When  $p = 2, 3, \dots$ , or  $n$ , the values of data in  $A_o(n, p)$  are from  $(2^n - 2^{n-p})$  to  $(2^n + 2^{n-p} - 1)$ , so they can be expanded by (14) and/or (15):

$$0.2^{N-1} + \dots + 0.2^n + 1.2^{n-1} + 1.2^{n-2} + \dots + 1.2^{n-p} + x.2^{n-p-1} + \dots + x.2^1 + x.2^0, \quad (14)$$

$$0.2^{N-1} + \dots + 0.2^{n+1} + 1.2^n + 0.2^{n-1} + 0.2^{n-2} + \dots + 0.2^{n-p} + x.2^{n-p-1} + \dots + x.2^1 + x.2^0, \quad (15)$$

where  $x$  is 1 or 0.

After Gray coding, (14) and (15) become

$$0.2^{N-1} + \dots + 0.2^n + 1.2^{n-1} + 0.2^{n-2} + \dots + 0.2^{n-p} + x.2^{n-p-1} + \dots + x.2^1 + x.2^0, \quad (16)$$

$$0.2^{N-1} + \dots + 0.2^{n+1} + 1.2^n + 1.2^{n-1} + 0.2^{n-2} + \dots + 0.2^{n-p} + x.2^{n-p-1} + \dots + x.2^1 + x.2^0. \quad (17)$$

Equation (16) indicates that the points satisfies  $V_{At}$  on the bit plane  $(n-p)$  are always 0 bits. By combining (16) and (17), the region  $A_o(n, p)$  on the bit plane  $(n-p)$  always contains 0 bits too.

For example, when  $n = 3, p = 2$ , we have  $A_o(3, 2) = \{6, 7, 8, 9\}$ , Gray codes of these decimal values (5, 4, 12, 13) have 0 bits on the bit plane 1 ( $= 3 - 2$ ).

This statement is favorable for the process of optimizing the probability of data bits in the schemes of lossless image compression, especially in the step of modeling of entropy coding.

**Consequence 1.** After Gray coding, the entropy of data obtained in the ideal cross-point regions  $A_o$  on a certain bit plane is minimum.

*Proof of Consequence 1.*

We can see in the ICRs  $A_o(n, p)$  on the bit plane  $(n-p)$  before Gray coding, the probability of 1 bit and 0 bit are random, therefore the entropy of the bit string is  $H = -P(1) \cdot \log_2 P(1) - P(0) \cdot \log_2 P(0) > 0$ .

After Gray coding, these ICRs contain all the same bits (1 bits or 0 bits), so the probability of 1 bit in the regions is 1, and the probability of 0 bit is 0 there, and inversely depending on the number of bit plane being estimated. This is why the entropy of bit string is always  $H = -P(1) \cdot \log_2 P(1) - P(0) \cdot \log_2 P(0) = 0$ , i.e. the average information of these regions is 0.

#### 4. EFFECTIVE IDEAL CROSS-POINT REGIONS

Let  $w$  be the width of an typical ICR whose coordinates of starting point and ending point are  $(X_s, Y_s)$  and  $(X_e, Y_e)$  correspondingly, of course,  $Y_s = Y_e$ . We have  $w = X_e - X_s + 1$ , this value  $w$  expresses the maximal number of cross points in the ICR we can use generally, i.e. the number of cross points satisfying (1) of the whole ICR. In practice, depending on a concrete scheme we can use this ICR with the different concepts of width, i.e. the number of cross points in ICR used in the scheme may be different from another scheme. The concept of effective width of ICR is defined as the following.

**Definition 2.** Let  $w$  be the width of an ICR being estimated. The effective width  $w_e$  of that ICR is a value in  $[1, w]$ , i.e.  $1 \leq w_e \leq w$ , and determined by the concrete conditions of the scheme. The ICR with effective width  $w_e$  like that is called the effective ICR.

This definition gives a capability of using the width of an ICR depending on the idea of algorithm we want. Concretely, when we use entire cross-point regions in [11] with cores being ideal cross-point regions inside, we have the effective width of ICRs from 1 to  $2^{n-p+1}$  points around the central value  $2^n$ ,  $n$  is the exponent of the central value, and  $p$  is the factor to gain the range of encoding like in Definition 1.

In the scheme ICRIC [14], we use the ICRs with the effective width  $w_e$  being the whole width  $w$  of ICRs on scan lines, each ICR is individual with others while being encoded. This is an advantage over other schemes when we overcome obstacles on compressing edges of object on images, especially medical and multiple detailed images. But, there is a difficulty for this scheme when it has a big map of ICRs for encoding and decoding, this brings about low compression ratio.

In this paper, we propose the effective width of an ICR in the relation with the ICRs above and below it, i.e. ICRs in different scan lines. The effective width  $w_e$  is determined flexibly but in the range  $1 \leq w_e \leq w$ . This helps us build up big 2-D areas of effective ICRs on the different scan lines, and each 2-D area with many effective ICRs like those is encoded at the same time. This can give higher compression ratio, especially for medical and multiple detailed images. The next section presents the method to gain 2-D areas of effective ICR in the relation with other ICRs.

## 5. 2-D AREAS OF IDEAL CROSS-POINT REGIONS

**Definition 3.** The area of ideal cross-point regions  $AR(n)$  is a rectangle consisting of effective ideal cross-point regions with the same width  $w_e$  so that the number of cross points in the area is maximal.

By this definition, there are many effective ICRs in the area  $AR(n)$ , they may not be the whole ICRs, that means these effective ideal cross-point regions may not consist of all cross points of the ICRs being estimated. From Definition 1 with Fig. 1 and Proposition 1 in Section 3, we can see the ICRs  $A_o(n, 1)$  have much significance because they include all the remaining ICRs  $A_o(n, p)$ . So, the area  $AR(n)$  is built by effective ICRs of these ICRs  $A_o(n, 1)$ . Concretely, suppose that after finding out ICRs  $A_o(n, 1)$ , we see a coarse area of effective ICRs  $\{(X_1, Y_1), (X_2, Y_2)\}$ , where  $(X_1, Y_1)$ ,  $(X_2, Y_2)$  are the coordinates of left-top point and right-bottom point of this rectangle.

The number of cross points  $w_{ei}$  in the effective ICR  $i$  in the above coarse area of ICRs is  $w_{ei} \leq (X_2 - X_1 + 1)$ . Then, the number of cross points  $Q_{CP}$  in that coarse area is

$$Q_{CP} = \sum_{i=0}^{Y_2 - Y_1 + 1} w_{ei}, \quad (18)$$

where  $Q_{CP} \leq (X_2 - X_1 + 1) \cdot (Y_2 - Y_1 + 1)$ .

In practice, we want to look for the area  $AR(n)$  as large as possible from the effective ICRs within it, i.e. from the left-top point at the coordinate  $(X_1, Y_1)$  we have many coarse areas  $AR(n)$  because of the ability to append effective ICRs to get the larger  $AR(n)$ , and we must choose one rectangle for the largest  $AR(n)$ . Call  $Q$  the number of cross points in the area we want to select from the coarse areas above, we need to compute  $Q$  as follows

$$Q = \max \{ Q_{CP1}, Q_{CP2}, \dots, Q_{CPi}, \dots \}. \quad (19)$$

Finally, the area of cross-point regions determined by the center value  $2^n$   $AR(n)$  is

$$AR(n) = \{(X_1, Y_1), (X_2, Y_2)\}, \quad (20)$$

with the rule (19) for the eventual coordinates  $(X_1, Y_1)$ ,  $(X_2, Y_2)$ .

In the area  $AR(n)$  every data points satisfy the characteristics of ICRs, i.e. their grey values lay in the range (1). This plays an important role in our scheme of lossless image compression, 2-DICRIC. Instead of using whole individual ideal cross-point regions on each scan line to optimize the probability of data bits, we use the areas of effective ICRs  $AR(n)$  to do this on all bit planes ( $n-p$ ) when  $p = 1, 2, 3, \dots, n$ .

For the scheme of the paper, the 2-D areas of ideal cross-point regions  $A_o(n, 1)$  need to be found out, and we call them areas of effective ICRs which can be used for optimizing probability of data bits in them at one time instead of using data bits in cross-point regions themselves on each scan line at different times. The algorithm to look for these areas is also presented in the next section. This can reduce the size of cross-point map and increase the compression ratio.

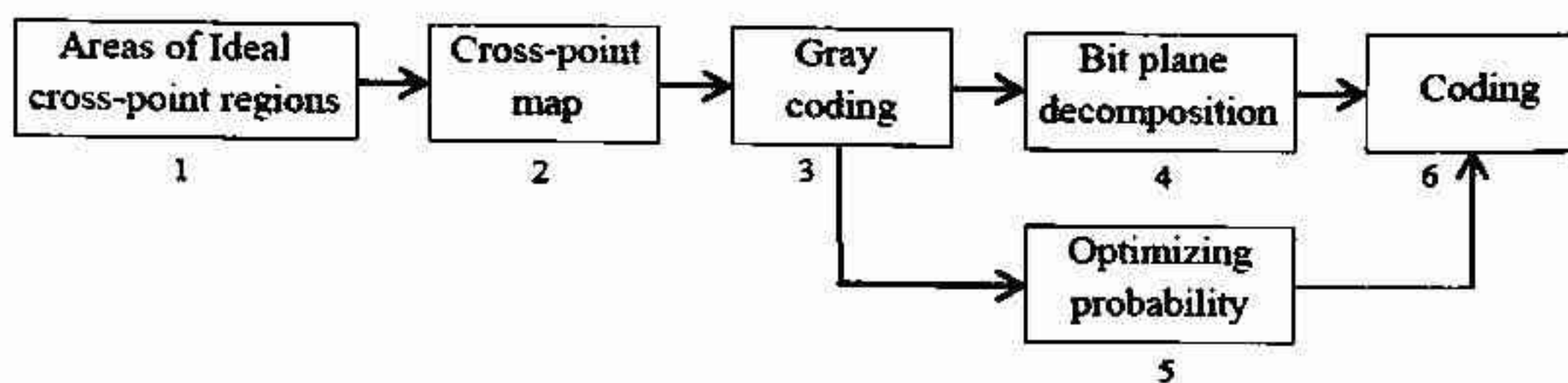
In practice, if we use cross-point regions for losslessly compressing data part of the file of image we can get high compression ratio without counting the header of that image file and the map of cross-point regions. When we count them, the compression ratio for the whole file becomes worse because of the big size of the map of cross-point regions. That is the reason why 2-D areas of ideal cross-point regions are built to get higher compression ratio for the whole file of image, and this ratio is really important when estimating a scheme for data compression.

## 6. SCHEME OF 2-DICRIC FOR LOSSLESS IMAGE COMPRESSION

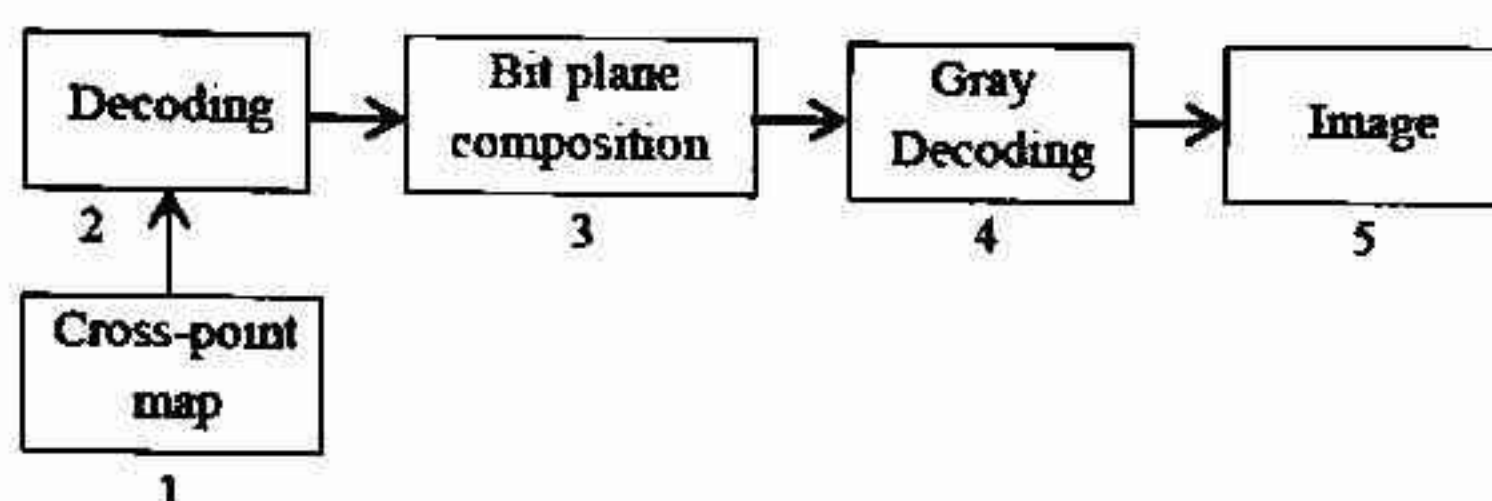
Figure 2 presents the scheme of 2-DICRIC for image compression (a) and decompression (b) generally. With the process of compression (a), each step is numbered according to the sequence of the scheme, we have 6 steps from 1 to 6. The first step, Step 1 (Areas of Ideal cross-point regions) looks for 2-D areas of ideal cross-point regions where we can optimize the probability of data bits. Grey values of data points in these regions satisfy (1) for ICRs. Step 2 (Cross-point map) will establish the map of the 2-D areas of ideal cross-point regions. Notice that with each central value  $2^n$  we have many different ideal cross-point regions  $A_o(n, p)$  on bit planes ( $n-p$ ), and so many areas of these regions, in those areas we encode data bits. This step evaluates and ignores small 2-D areas of ideal cross-point regions containing 1, 2 or 3 data points in each ICR depending on the number of bit plane ( $n-p$ ), these areas don't affect much the compression ratio of processes of coding and decoding. The third step, Step 3 - Gray coding, makes Gray code transformation [3]. Step 4 - Bit plane decomposition decomposes image data into separate bit planes that are numbered from 0 to  $N-1$ , the number of bit planes depends on the significance of bits, where  $N$  is the bit length of pixels of image. The next step, Step 5 - Optimizing probability, calculates the probability of data bits outside ICRs and optimizes probabilities of data bits in the areas of ICRs by Proposition 1. This optimization is based on the cross-point map in Step 2, and implemented in Step 6 to compute frequencies of data bits. The last step, Step 6 - Coding, uses some algorithm in the process of entropy coding, like arithmetic coding. With the experimental results achieved in the next section, we use Jones' algorithm [12]. The process of encoding data bits should be carried out from the most to the least significant bit planes because of the random of data on the less significant bit planes.

Therefore, this scheme is a process of entropy coding with two stages: modeling (from Step 1 to Step 5), and coding (Step 6).





(a)



(b)

Figure 2. The scheme of 2-DICRIC with two processes - (a) Compression - (b) Decompression.

With the process of decompression (b), there are 5 steps numbered from 1 to 5, and in the opposite with the process of compression. In the process, the cross-point map is very important for the scheme, we use it to locate the 2-D areas of ICRs where we optimized the probability of data bits in the process of compression.

To obtain the coarse areas of ideal cross-point regions which are “rectangles” from many scan lines of cross-point regions, we need to deploy Section 4 via the procedure to get the cross-point map of the 2-D areas.

Horizontally, we omit cross-point lines which are too short. The results may be illustrated by Fig. 3 as the following.

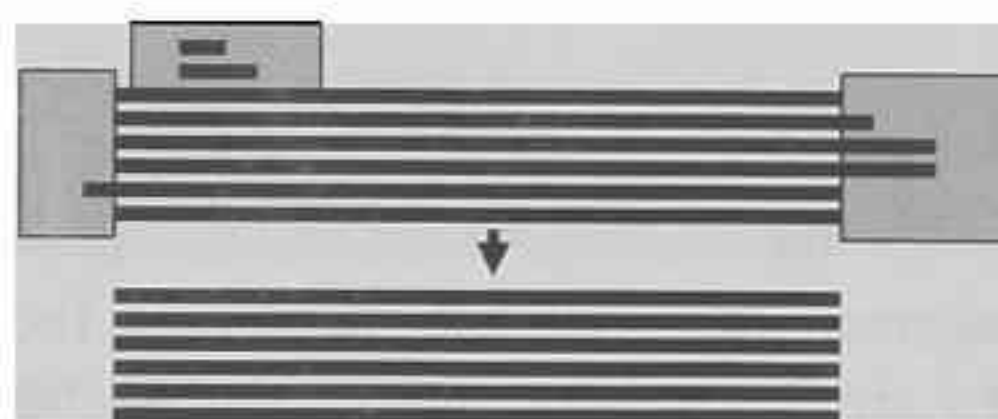


Figure 3. Results of omitting short cross-point scan lines.

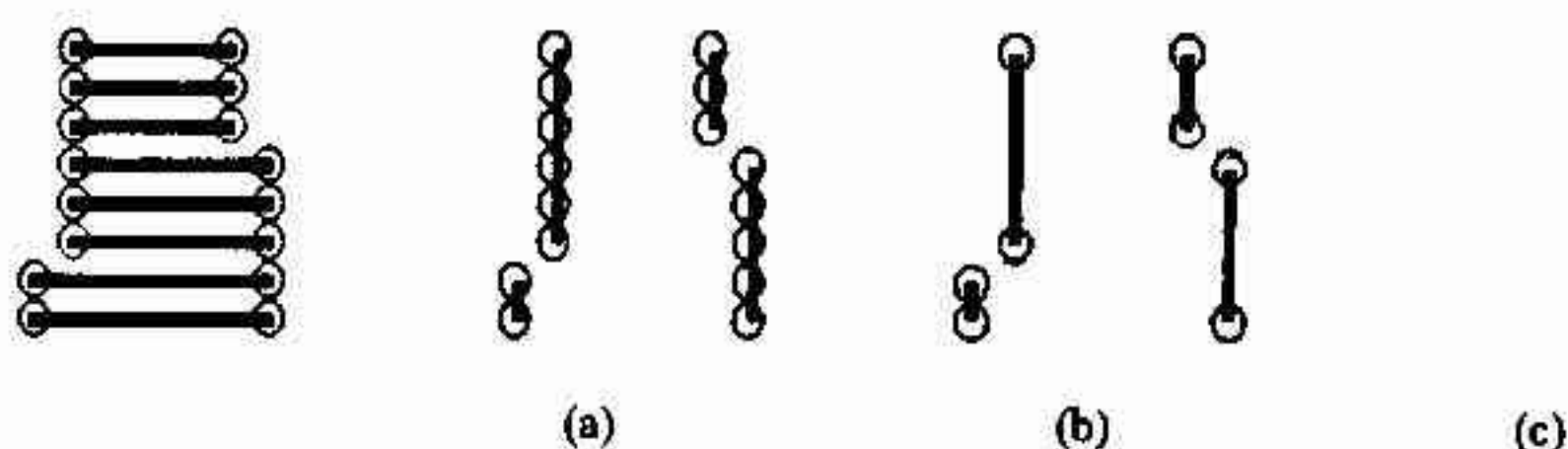


Figure 4. 2-D areas of ideal cross-point regions with their sides.

Actually, it is difficult to get areas of ICRs as rectangles, we must ignore many parts of ICRs, or small ICRs, or ICRs alone from others in the compression processing. We can accept the shape of 2-D areas flexibly by getting positions of data points for the map. For example, we

have an area of ICR like in Fig. 4a. At that time, cross-point regions create straight lines in Fig. 4b, these lines are considered as sides of smaller rectangles in Fig. 4c, in order we save beginning points and ending points to compressed files.

## 7. EXPERIMENTAL RESULTS

Table 1 presents the results when the images in Fig. 5 are compressed by the scheme of 2-DICRIC. The compression ratio used here is the ratio between the image file including header of original image and the file of image compressed. With these results we can see that the scheme of 2-DICRIC is good at images in which there are many same grey levels, especially medical images containing backgrounds with not much different grey levels.

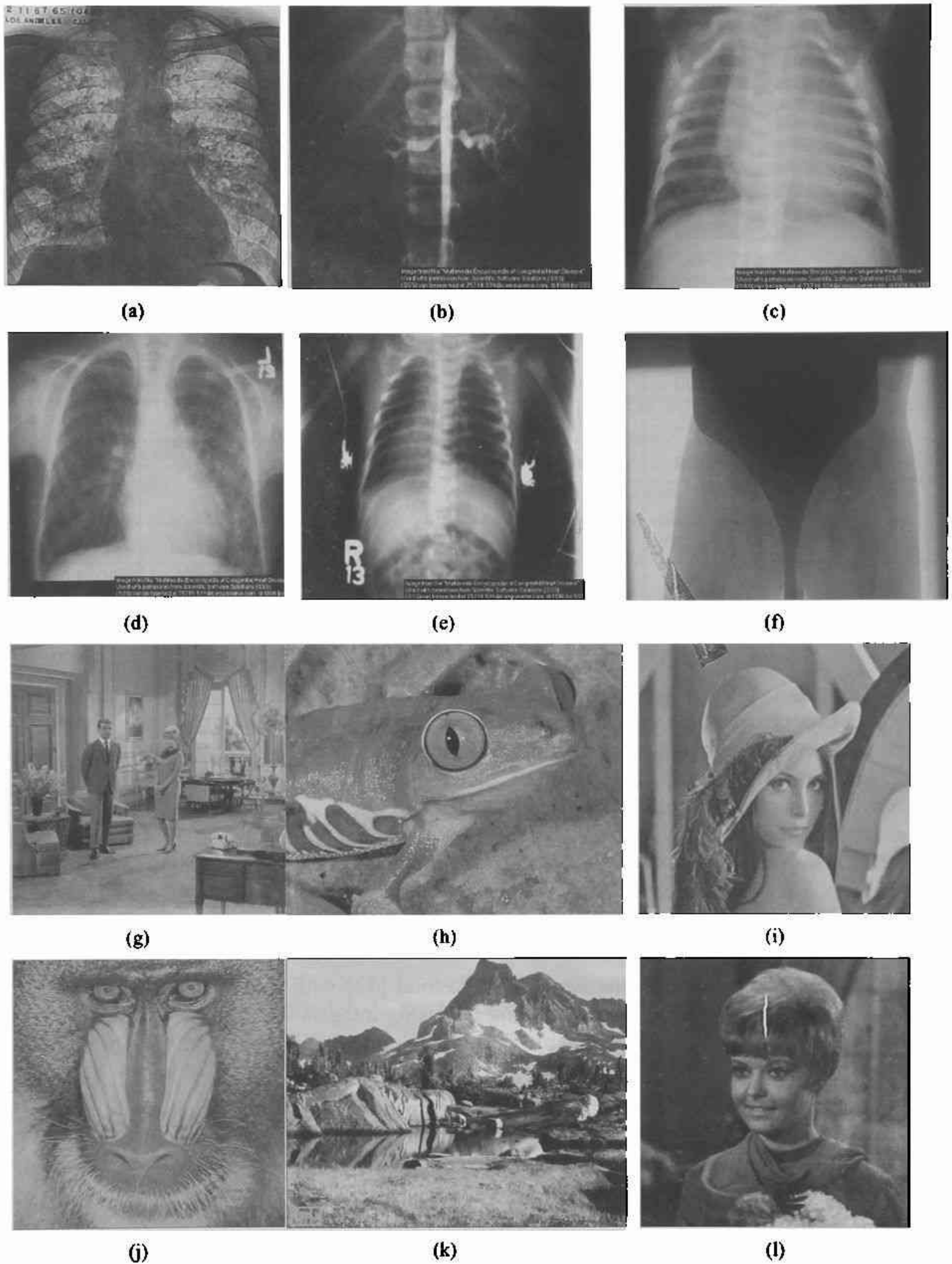
Table 1. Experimental results of 2-DICRIC compared with other methods (for whole image file).

| Images   | Size      | 2-DICRIC  | ICRIC [14] | AES<br>(WinRAR 3.40)<br>[15] | JPEG 2000<br>[16] |
|----------|-----------|-----------|------------|------------------------------|-------------------|
| Chest    | 256 x 256 | 1.82529:1 | 1.70347:1  | 1.573:1                      | 1.243:1           |
| Chest1   | 512 x 480 | 1.98693:1 | 1.90962:1  | 1.673:1                      | 1.877:1           |
| Chest2   | 512 x 480 | 1.90227:1 | 1.81781:1  | 1.457:1                      | 1.874:1           |
| Chest3   | 512 x 480 | 2.36946:1 | 2.25995:1  | 2.201:1                      | 2.268:1           |
| Chest4   | 512 x 480 | 2.01013:1 | 1.93215:1  | 1.728:1                      | 1.922:1           |
| Joint    | 512 x 400 | 2.18503:1 | 2.07995:1  | 1.932:1                      | 2.225:1*          |
| Couple   | 512 x 512 | 1.66307:1 | 1.61817:1  | 1.427:1                      | 1.513:1           |
| Frog     | 621 x 498 | 1.57793:1 | 1.55513:1  | 2.121:1*                     | 1.217:1           |
| Lena     | 512 x 512 | 1.66024:1 | 1.61457:1  | 1.567:1                      | 1.752:1*          |
| Mandrill | 512 x 512 | 1.25378:1 | 1.23687:1  | 1.206:1                      | 1.291:1*          |
| Mountain | 640 x 480 | 1.49581:1 | 1.49086:1  | 1.520:1*                     | 1.177:1           |
| Zelda    | 256 x 256 | 1.88730:1 | 1.72553:1  | 1.511:1                      | 1.603:1           |

\* These results are better than those of 2-DICRIC.

The algorithm of the scheme of 2-DICRIC here uses the fourth-order estimate of the source entropy [10], so there are 3 neighbor bits of the bit being encoded, they are chosen beforehand. We can take a different order estimate depending on the key we use in the scheme of encoding and cryptography. Notice that the value  $p$  gives the number of bit planes ( $n-p$ ) on which we encode the bit string,  $p$  is from 1 to  $n$ . Table 1 gives the results with a typical value of  $p$ ,  $p = 2$ . From the results in Table 1 we can see the new results of the scheme of 2-DICRIC are better than those of the old ICRIC when we do not use 2-D areas of ideal cross-point regions. On the other hand, if using this scheme ICRIC to losslessly compress medical images (Chest, Chest1, Chest2, Chest3, Chest4, Joint) we almost obtain higher compression ratios than other algorithms like AES (Advanced Encryption Standard) [15] used in the software WinRAR, and JPEG 2000 using lossless wavelet transform [16]. With photographic images (Mandrill, Couple, Lena, Zelda, Frog, Mountain), the scheme of 2-DICRIC may give better or worse results than another methods. This depends on the analyses of each method.





**Figure 5.** Images for experimental results  
 (a) Chest, (b) Chest 1, (c) Chest 2, (d) Chest 3, (e) Chest 4, (f) Joint,  
 (g) Couple, (h) Frog, (i) Lena, (j) Mandrill, (k) Mountain, (l) Zelda.

Table 2. Experimental results – Compression and Decompression time of 2-DICRIC and ICRIC.

| Images   | Size      | 2-DICRIC               |                          | ICRIC [14]             |                          |
|----------|-----------|------------------------|--------------------------|------------------------|--------------------------|
|          |           | Compression time (sec) | Decompression time (sec) | Compression time (sec) | Decompression time (sec) |
| Chest    | 256 x 256 | 0.17                   | 0.16                     | 0.16                   | 0.13                     |
| Chest1   | 512 x 480 | 0.56                   | 0.48                     | 0.59                   | 0.50                     |
| Chest2   | 512 x 480 | 0.58                   | 0.50                     | 0.59                   | 0.49                     |
| Chest3   | 512 x 480 | 0.56                   | 0.52                     | 0.60                   | 0.50                     |
| Chest4   | 512 x 480 | 0.57                   | 0.49                     | 0.60                   | 0.48                     |
| Joint    | 512 x 400 | 0.48                   | 0.41                     | 0.49                   | 0.40                     |
| Couple   | 512 x 512 | 0.63                   | 0.55                     | 0.66                   | 0.53                     |
| Frog     | 621 x 498 | 0.74                   | 0.65                     | 0.77                   | 0.63                     |
| Lena     | 512 x 512 | 0.62                   | 0.52                     | 0.65                   | 0.53                     |
| Mandrill | 512 x 512 | 0.66                   | 0.54                     | 0.69                   | 0.56                     |
| Mountain | 640 x 480 | 0.75                   | 0.65                     | 0.77                   | 0.63                     |
| Zelda    | 256 x 256 | 0.16                   | 0.13                     | 0.16                   | 0.13                     |

Table 2 gives the results of two schemes - 2-DICRIC and ICRIC, with two parameters: the compression time and the decompression time. The program for 2-DICRIC uses the computer with the configuration as the followings: CPU Intel(R) Core (TM) i7-3770 3.4 GHz, Memory 8.00 GB (3.47 GB usable), HDD 750 GB, the operating system of Windows 7 Ultimate 32 bit. From these results we can see when 2-D areas of ideal cross-point regions are used, the compression ratio is better than this when only cross-point regions being used. This is because we use many more cross-point regions for modeling in ICRIC. This brings about the cross-point map is bigger, so the compression ratio for whole files with ICRIC is worse than this with 2-DICRIC. On the other hand, the compression time of 2-DICRIC is almost shorter than this of ICRIC.

The encoding process here uses Jones' method [12] with some supplementaries in [10]. This algorithm is like the arithmetic coding but using integers for processing. At the moment, the program for deploying the scheme 2-DICRIC can try 8-bit grey images of bitmap file format.

The images restored must be identical to the original images. This problem is strictly carried out by comparing the original image with the image decompressed for every pixel because of the characteristic of lossless image compression or cryptography.

## 8. CONCLUSION

The new scheme of 2-DICRIC and some results are presented. This scheme with 2-D areas of ideal cross-point regions is now the new part of the theory of cross-point regions which is mentioned in [11], and [14]. It has illustrated the use of the theory of cross-point regions with



optimizing probabilities of data bits only in the 2-D areas of ideal cross point regions with Definition 3, presenting the rule to determine these 2-D areas. In this paper, we have used ideal cross-point regions on bit planes ( $n - p$ ), this is the development from [1]-[2], and [11], [14]. We may choose the number of bit planes ( $n - p$ ) before encoding, that may make the compression ratio decrease due to bigger and bigger cross-point maps, but this is better and better for cryptography. The data received from the scheme is reversible, so besides for image compression this scheme can be strongly used in cryptographic systems with keys depending on  $n$ -order estimating, number of bit planes, number of ideal cross-point regions; this problem will be mentioned in other papers. Generally, the scheme of 2-DICRIC is the process of entropy coding, it includes two parts: modeling (Step 1-5) and coding (Step 6). The theory of cross-point regions can be used in the first part in order to reduce interpixel redundancy; the second part uses some algorithm, like arithmetic coding or Jones' method to reduce coding redundancy.

Based on the theory of cross-point regions, the scheme of 2-DICRIC upgraded with using 2-D areas of ICRs for optimizing probabilities of cross points proves a meaningful improvement, the compression ratio obtained has been compared with the other authors' methods and the old scheme – ICRC, it gives the better results, but has been being developed, so it has not become the standard for everybody to use yet. From these concrete bases, the problem of improving the compression ratio of image compression and transmission can be further developed in future in order to be used popularly.

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## TÓM TẮT

### MÔ HÌNH SỬ DỤNG VÙNG CROSS-POINT LÍ TƯỜNG 2 CHIỀU CHO NÉN ẢNH KHÔNG TỒN HAO

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Bài báo này trình bày sơ đồ mới với việc sử dụng vùng cross-point lí tường hai chiều, là một phần mới trong lí thuyết vùng cross-point. Trong thực tế khi sử dụng các vùng cross-point chúng ta cần một thuật toán để xác định các bản đồ của vùng cross-point, điều này mất khá nhiều thời gian và không gian để lưu trữ bản đồ này, và kéo theo tỉ số nén không cao khi sử dụng vùng cross-point một chiều vì tọa độ các điểm dữ liệu cần phải được lưu trữ để giải mã rất nhiều. Khi vùng hai chiều được sử dụng, sơ đồ 2-DICRIC (vùng Cross-point lí tường hai chiều cho nén ảnh không tồn hao) để mã hóa và giải mã ảnh không tồn hao được sử dụng để có tỉ số nén cao hơn. Ý tưởng cơ bản của phương pháp này là ảnh hưởng của mã Gray lên các cross points. Trước khi biến đổi Gray được thực hiện, tập dữ liệu của các điểm cross points cần được xác định, chúng tạo ra các vùng cross-point lí tường (ICRs). Sau biến đổi Gray, các vùng này luôn chỉ chứa các bit 1 hoặc bit 0 tùy thuộc vào số hiệu mặt phẳng bit sau khi phép phân tích mặt phẳng bit được thực hiện. Với đặc trưng của dữ liệu ảnh, mức xám không thay đổi nhiều trong một vùng xác định, đặc biệt là trong các ảnh y tế có nhiều vùng trong đó mức xám xấp xỉ nhau, lí thuyết với vùng cross-point hai chiều có ảnh hưởng quan trọng tới tỉ số nén khi quá trình mã hóa và giải mã nén ảnh không tồn hao khi truyền dữ liệu được tiến hành. Sơ đồ này có thể được sử dụng cho nén không tồn hao dữ liệu ảnh hay mật mã dữ liệu.

*Từ khóa:* mã Gray, vùng cross-point lí tường, phân tích mặt phẳng bit, xác suất của các bit.