HIGGS SECTOR IN THE 3-3-1 MODEL WITH AXION DARK MATTER

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Abstract: *The Higgs sector in the 3-3-1 model with axion dark matter is presented. The diagonal- ization of 4 × 4 square mass matrix for the CP-odd sector is exactly fulfilled. Our results show that the axion is mainly contained from the CP-odd part of the singlet φ, while the CP-even component of the later is the inflaton of the model. The positivity of the masses leads to constraints for some couplings of the Higgs sector. PACS numbers: 11.30.Fs, 12.15.Ff, 12.60.-i*

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1. INTRODUCTION

Being as Dark Matter (DM) candidate, nowadays, axion is very attracted subject in Particle Physics. The axion is a hypothetical CP-odd scalar, protected by a shift symmetry and derivatively coupled to Standard Model (SM) fields. It is predicted by the Peccei-Quinn solution to the strong CP problem [1, 2].

Among the beyond SM, the models based on $SU(3)_C \times SU(3)_L \times U(1)_X$ (3-3-1) gauge group [3] have some intriguing properties. It is emphasized that Peccei-Quinn symmetry is automatically satisfied in the 3-3-1 models [4]. That is why the 3-3-1 models is attractive for the axion puzzles.

In the framework of the 3-3-1 models, the axion has been studied in the papers [5–8]. In [5, 7], the axion is massless field and its mass is generated by quantum gravity effects. In addition, in diagonalization of square mass matrix for CP-odd scalars, the mixing matrix is not unitary leading to extra states such as PS_1 and PS_2. In this paper, these eros are corrected.

2. CONTENT

2.1. Brief review of the model

As usual, fermion content satisfying all the requirements is

$$\begin{gathered}\psi_{aL} = (\nu_a, e_a, \mathrm{N}_a)_L^T \sim (1,3,-1/3), \quad e_{aR} \sim (1,1,-1),\\ Q_{3L} = (u_3, d_3, U)_L^T \sim (3,3,1/3), \quad Q_{\alpha L} = (d_\alpha, -u_\alpha, D_\alpha)_L^T \sim (3,3^*,0),\\ u_{aR}, U_R \sim (3,1,1/3), \quad d_{aR}, D_{\alpha R} \sim (3,1,-1/3),\end{gathered} \tag{1}$$

where α = 2, 3 and a = 1, 2, 3 are family indices. The quantum numbers as given in parentheses are respectively based on $SU(3)_C$, $SU(3)_L$, $U(1)_X$ symmetries. The U and D are exotic quarks, while N_R are right-handed neutrinos. The above model is named by the 3-3-1 model with right-handed neutrinos.

The model with right-handed neutrinos requires three triplets:

$$\chi \sim (1,3,-1/3)\,, \eta \sim (1,3,-1/3)\,, \rho \sim\sim (1,3,2/3), \tag{2}$$

with expansions as follows

$$\begin{gathered}\chi = \begin{pmatrix} \frac{1}{\sqrt{2}}(R_\chi^1 + iI_\chi^1) \\ \chi^- \\ \frac{1}{\sqrt{2}}(v_\chi + R_\chi^3 + iI_\chi^3) \end{pmatrix},\\ \eta = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_\eta + R_\eta^1 + iI_\eta^1) \\ \eta^- \\ \frac{1}{\sqrt{2}}(R_\eta^3 + iI_\eta^3) \end{pmatrix}, \rho = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(v_\rho + R_\rho + iI_\rho) \\ \rho_3^+ \end{pmatrix}.\end{gathered} \tag{3}$$

In addition, ones introduce a singlet

$$\phi = \frac{1}{\sqrt{2}}\left(v_\phi + R_\phi + iI_\phi\right) \sim (1,1,0).$$

The full potential invariant under 3−3−1 gauge and $Z_{11} \otimes Z_2$ discrete symmetries is determined as [5]

$$\begin{aligned}V = {} & \mu_\phi^2 \phi^* \phi + \mu_\chi^2 \chi^\dagger \chi + \mu_\rho^2 \rho^\dagger \rho + \mu_\eta^2 \eta^\dagger \eta + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\eta^\dagger \eta)^2 \\ & + \lambda_3 (\rho^\dagger \rho)^2 + \lambda_4 (\chi^\dagger \chi)(\eta^\dagger \eta) + \lambda_5 (\chi^\dagger \chi)(\rho^\dagger \rho) + \lambda_6 (\eta^\dagger \eta)(\rho^\dagger \rho) \\ & + \lambda_7 (\chi^\dagger \eta)(\eta^\dagger \chi) + \lambda_8 (\chi^\dagger \rho)(\rho^\dagger \chi) + \lambda_9 (\eta^\dagger)(\rho^\dagger \rho).\end{aligned} \tag{4}$$

Substitution of (3) into (4) leads to the following constraints at the tree level as follows

$$\mu_\rho^2 + \lambda_3 \upsilon_\rho^2 + \frac{\lambda_5}{2}\upsilon_\chi^2 + \frac{\lambda_6}{2}\upsilon_\eta^2 + \frac{\lambda_6}{2}\upsilon_\phi^2 + \frac{L}{\upsilon_\rho^2} = 0,$$
$$\mu_\eta^2 + \lambda_2 \upsilon_\eta^2 + \frac{\lambda_4}{2}\upsilon_\chi^2 + \frac{\lambda_6}{2}\upsilon_\rho^2 + \frac{\lambda_{12}}{2}\upsilon_\phi^2 + \frac{L}{\upsilon_\eta^2} = 0,$$
$$\mu_\chi^2 + \lambda_1 \upsilon_\chi^2 + \frac{\lambda_4}{2}\upsilon_\eta^2 + \frac{\lambda_5}{2}\upsilon_\rho^2 + \frac{\lambda_{11}}{2}\upsilon_\phi^2 + \frac{L}{\upsilon_\chi^2} = 0, \tag{5}$$
$$\mu_\phi^2 + \lambda_{10} \upsilon_\phi^2 + \frac{\lambda_{11}}{2}\upsilon_\chi^2 + \frac{\lambda_{12}}{2}\upsilon_\rho^2 + \frac{\lambda_{13}}{2}\upsilon_\eta^2 + \frac{L}{\upsilon_\phi^2} = 0,$$

where $L \equiv \lambda_\Phi\, v_\Phi v_\chi v_\eta v_\square$.

2.2. Charged scalar sector

In this sector we have two square mass matrices. One of them is as follows: In the base (η_1^-, δ_1^-) ones get square mass matrix as

$$M_c = \begin{pmatrix} \frac{\lambda_9 \upsilon_\rho^2}{2} - \frac{L}{2\upsilon_\eta^2} & \frac{\lambda_9 \upsilon_\rho \upsilon_\eta}{2} - \frac{L}{2\upsilon_\rho \upsilon_\eta} \\ \frac{\lambda_9 \upsilon_\rho \upsilon_\eta}{2} - \frac{L}{2\upsilon_\rho \upsilon_\eta} & \frac{\lambda_9 \upsilon_\eta^2}{2} - \frac{L}{2\upsilon_\rho^2} \end{pmatrix} = -\frac{\left(L - \lambda_9 \upsilon_\rho^2 \upsilon_\eta^2\right)}{2} \begin{pmatrix} \frac{1}{\upsilon_\eta^2} & \frac{1}{\upsilon_\eta \upsilon_\rho} \\ \frac{1}{\upsilon_\eta \upsilon_\rho} & \frac{1}{\upsilon_\rho^2} \end{pmatrix}. \tag{6}$$

This matrix has one massless G_1^- and one massive H_1^- with mass equal to

$$m_{H_1^-}^2 = -\frac{\left(L - \lambda_9 \upsilon_\rho^2 \upsilon_\eta^2\right)}{2} \cdot \frac{\left(\upsilon_\rho^2 + \upsilon_\eta^2\right)}{\upsilon_\rho^2 \upsilon_\eta^2}. \tag{7}$$

From (7) it follows

$$\lambda_9 > \lambda_\phi \frac{\upsilon_\phi \upsilon_\chi}{\upsilon_\eta \upsilon_\eta}. \tag{8}$$

The physical fields are given as

$$\begin{pmatrix} G_1^- \\ H_1^- \end{pmatrix} = \begin{pmatrix} \cos\theta_\alpha & \sin\theta_\alpha \\ \sin\theta_\alpha & \cos\theta_\alpha \end{pmatrix} \begin{pmatrix} \rho_1^- \\ \eta^- \end{pmatrix}, \tag{9}$$

where

$$\tan\theta_\alpha = \frac{\upsilon_\eta}{\upsilon_\rho} \tag{10}$$

In the limit $v_\rho \gg v_\eta$ we have

$$G_1^- = \rho_1^- \simeq G_{w^-}. \tag{11}$$

In the base (χ^- , ρ_3^-) ones get square mass matrix as

$$M_{c2} = \begin{pmatrix} \frac{\lambda_8 v_\rho^2}{2} - \frac{L}{2v_\chi^2} & \frac{\lambda_8 v_\rho v_\chi}{2} - \frac{L}{2v_\rho v_\eta} \\ \frac{\lambda_8 v_\rho v_\chi}{2} - \frac{L}{2v_\rho v_\chi} & \frac{\lambda_8 v_\chi^2}{2} - \frac{L}{2v_\rho^2} \end{pmatrix} = -\frac{\left(L - \lambda_8 v_\rho^2 v_\chi^2\right)}{2} \begin{pmatrix} \frac{1}{v_\chi^2} & \frac{1}{v_\chi v_\rho} \\ \frac{1}{v_\chi v_\rho} & \frac{1}{v_\rho^2} \end{pmatrix}. \tag{12}$$

This matrix has one massless G_2^- = and one massive H_2^- with mass equal to

$$m_{H_2^-}^2 = -\frac{\left(L - \lambda_8 v_\rho^2 v_\chi^2\right)}{2} . \frac{\left(v_\rho^2 + v_\chi^2\right)}{v_\rho^2 v_\chi^2} \tag{13}$$

From (13) it follows

$$\lambda_8 > \lambda_\phi \frac{v_\phi v_\eta}{v_\chi v_\rho}. \tag{14}$$

The physical fields are given as

$$\begin{pmatrix} G_2^- \\ H_2^- \end{pmatrix} = \begin{pmatrix} \cos\theta_\beta & \sin\theta_\beta \\ \sin\theta_\beta & \cos\theta_\beta \end{pmatrix} \begin{pmatrix} \chi^- \\ \rho_3^- \end{pmatrix}, \tag{15}$$

where

$$\tan\theta_\beta = \frac{v_\rho}{v_\chi}. \tag{16}$$

In the limit $v_\rho \gg v_\eta$ we have

$$G_2^- = X_1^- \simeq G_y^-. \tag{17}$$

2.3. CP-ODD sector

For CP-odd scalars, in the base (I_1^x, I_1^3) ones get square mass matrix as

$$M_A\left(I_\chi^1, I_\eta^3\right) = \begin{pmatrix} \frac{\lambda_7}{4} v_\eta^2 - \frac{L}{2v_\chi^2} & -\frac{\lambda_7}{4} v_\chi v_\eta + \frac{L}{2v_\chi v_\eta} \\ -\frac{\lambda_7}{4} v_\chi v_\eta + \frac{L}{2v_\chi v_\eta} & \frac{\lambda_7}{4} v_\chi^2 - \frac{L}{2v_\eta^2} \end{pmatrix}. \tag{18}$$

Diagonalization of matrix in (18) yields one massless scalar G_1 and one massive field A_1 with mass as follows

$$m_{A_1}^2 = -\frac{\left(L - \lambda_7 v_\eta^2 v_\chi^2\right)}{2} . \frac{\left(v_\eta^2 + v_\chi^2\right)}{v_\eta^2 v_\chi^2}. \tag{19}$$

From (19) it follows

$$\lambda_7 > \lambda_\phi \frac{\upsilon_\phi \upsilon_\rho}{\upsilon_\chi \upsilon_\eta}. \tag{20}$$

The physical fields are

$$\begin{pmatrix} G_1 \\ A_1 \end{pmatrix} = \begin{pmatrix} \sin\beta & \cos\beta \\ \cos\beta & -\sin\beta \end{pmatrix} \begin{pmatrix} I_\chi^1 \\ I_\eta^3 \end{pmatrix} \tag{21}$$

.

Next, in the base (I_χ^3, I_η^1, I_ρ, I_φ) we have square mass matrix

$$M_{4odd} = \frac{L}{2} \begin{pmatrix} \frac{1}{\upsilon_\chi^2} & \frac{1}{\upsilon_\chi \upsilon_\eta} & \frac{1}{\upsilon_\chi \upsilon_\rho} & \frac{1}{\upsilon_\chi \upsilon_\phi} \\ & \frac{1}{\upsilon_\eta^2} & \frac{1}{\upsilon_\rho \upsilon_\eta} & \frac{1}{\upsilon_\phi \upsilon_\eta} \\ & & \frac{1}{\upsilon_\rho^2} & \frac{1}{\upsilon_\rho \upsilon_\phi} \\ & & & \frac{1}{\upsilon_\phi^2} \end{pmatrix}. \tag{22}$$

Let us diagonalize the matrix in (22). For this aim, we denote

$$N = \frac{1}{\upsilon_\chi}, \quad B = \frac{1}{\upsilon_\eta}, \quad C = \frac{1}{\upsilon_\rho}, \quad D = \frac{1}{\upsilon_\phi}. \tag{23}$$

The the matrix in (22) is rewritten as

$$M_{4o} = \frac{L}{2} \begin{pmatrix} N^2 & NB & NC & ND \\ NB & B^2 & BC & BD \\ NC & BC & C^2 & CD \\ ND & BD & CD & D^2 \end{pmatrix}. \tag{24}$$

The above matrix has three massless states and one massive with the following eigenvectors

$$U=\begin{pmatrix} -\frac{D}{N}I_\chi^3 & -\frac{C}{N}I_\eta^1 & -\frac{B}{N}I_\rho & \frac{C}{N}I_\phi \\ 0 & 0 & I_\rho & \frac{B}{D}I_\phi \\ 0 & I_\eta^1 & 0 & \frac{C}{D}I_\phi \\ I_\chi^3 & & & I_\phi \end{pmatrix}. \tag{25}$$

Taking eigenvector in the first column of (25) and write rotation matrix

$$C_{43}=\begin{pmatrix} \frac{D}{C_2} & 0 & 0 & -\frac{N}{C_2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{N}{C_2} & 0 & 0 & \frac{D}{C_2} \end{pmatrix}, \tag{26}$$

where we denote

$$C_2=\sqrt{N^2+D^2}\Rightarrow \frac{D}{C_2}=\sin\theta_1\,\frac{N}{C_2}=\cos\theta_1\,,\tan\theta_1=\frac{D}{N}=\frac{v_\chi}{v_\phi}. \tag{27}$$

For the limit $v_\varphi \gg v_\chi$ we have

$$C_2\approx 1/v_x$$

We can check out that

$$M_{4di}\quad = C43\times M_{4odd}\times C43^\dagger=\frac{L}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & B^2 & BC & BC_2 \\ 0 & BC & C^2 & CC_2 \\ 0 & BC_2 & CC_2 & C_2^2 \end{pmatrix}. \tag{28}$$

Here we get one massless state a which is identified to axion and one massive state A_2

$$\begin{aligned} a&=\cos\theta_1 I_\phi-\sin\theta_1 I_\chi^3,\\ A_2&=\sin\theta_1 I_\phi+\cos\theta_1 I_\chi^3. \end{aligned} \tag{29}$$

From (29), it follows that in the limit $v_\varphi \gg v_\chi$

$$a=I_\varphi. \tag{30}$$

Summarising the first step

$$\begin{pmatrix} I_\chi^3 \\ I_\eta^1 \\ I_\rho \\ I_\phi \end{pmatrix} = C43 \begin{pmatrix} a \\ \mathrm{I}_\eta^1 \\ \mathrm{I}_\rho \\ \mathrm{A}_2 \end{pmatrix}, \tag{31}$$

In the basis $a,\ I_\eta^1, A_2$, we have square mass matrix given in (28). The 3×3 matrix in right-bottom here has two massless states and one massive as follows

$$\left\{\left\{-\frac{C_2}{B},0,1\right\},\left\{-\frac{C}{B},1,0\right\},\left\{\frac{B}{C_2},\frac{C}{C_2},1\right\}\right\} \tag{32}$$

Using the second solution, we have rotation matrix

$$C32 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\mathrm{C}}{\mathrm{C}_3} & -\frac{\mathrm{B}}{\mathrm{C}_3} & 0 \\ 0 & \frac{\mathrm{B}}{\mathrm{C}_3} & \frac{\mathrm{C}}{\mathrm{C}_3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{33}$$

where

$$C_3 = \sqrt{C^2 + B^2} \sim \mathcal{O}\left(\frac{1}{v_\eta},\frac{1}{v_\rho}\right)$$

. Therefore:

$$\tan\theta_2 = \frac{C}{B} \sim \frac{v_\eta}{v_\rho} \Rightarrow \sin\theta_2 = \frac{C_3}{B}, \cos\theta_2 = \frac{C_3}{C}. \tag{34}$$

Then

$$M_{3diag} = C32 \times M_{4diag} \times C32^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \mathrm{C}_3^2 & \mathrm{C}_2\mathrm{C}_3 \\ 0 & 0 & \mathrm{C}_2\mathrm{C}_3 & \mathrm{C}_2^2 \end{pmatrix} \tag{35}$$

Here we have one massless G_2 and one massive fields $A3$

$$G_2 = \sin_{\theta_2 I_\rho} - \cos_{\theta_2 I_\eta^1},$$
$$A_3 = cos_{\theta_2 I_\rho} + \sin_{\theta_2 I_\eta^1}. \qquad (36)$$

Here, G_2 is Goldstone boson for the Z boson. We have

$$\begin{pmatrix} I_\chi^3 \\ I_\eta^1 \\ I_\rho \\ I_\phi \end{pmatrix} = C43.C32 \begin{pmatrix} a \\ G_Z \\ A_3 \\ A_2 \end{pmatrix}, \qquad (37)$$

The 2 × 2 matrix in right-bottom of (35) is easily diagonalized. Let define

$$\tan\theta_3 = \frac{C_2}{C_3} \sim \frac{v_\eta}{v_\chi}. \qquad (38)$$

Then we have one massless G_3 which is identified as $G_{Z'}$ and one massive A_4 fields

$$G_3 = \sin\theta_3 A_3 - \cos\theta_3 A_2, \qquad (39)$$
$$A_4 = \cos\theta_3 A_3 + \sin\theta_2 A_2, \qquad (40)$$

where mass of A_4 is given as

$$m_{A_4}^2 = L \times \frac{C_3^2}{cos^2\theta_3}. \qquad (41)$$

Let us write

$$C21 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta_3 & -\sin\theta_3 \\ 0 & 0 & \sin\theta_3 & \cos\theta_3 \end{pmatrix} \qquad (42)$$

then
$$\begin{pmatrix} I_\chi^3 \\ I_\eta^1 \\ I_\rho \\ I_\phi \end{pmatrix} = C43.C32.C21 = \tag{43}$$
$$\begin{pmatrix} \sin\theta_1 & 0 & -\cos\theta_1\cos\theta_3 & -\cos\theta_1\sin\theta_3 \\ 0 & \sin\theta_2 & -\cos\theta_2\sin\theta_3 & \cos\theta_2\cos\theta_3 \\ 0 & \cos\theta_2 & \sin\theta_2\sin\theta_3 & -\cos\theta_3\sin\theta_2 \\ \cos\theta_1 & 0 & \cos\theta_3\sin\theta_2 & \cos\theta_1\sin\theta_3 \end{pmatrix} \begin{pmatrix} a \\ G_Z \\ G_{Z'} \\ A_4 \end{pmatrix}.$$

For practical analysis
$$\begin{pmatrix} a \\ G_Z \\ G_{Z'} \\ A_4 \end{pmatrix} = \begin{pmatrix} \sin\theta_1 & 0 & 0 & \cos\theta_1 \\ 0 & \sin\theta_2 & \cos\theta_2 & 0 \\ -\cos\theta_1\cos\theta_3 & -\cos\theta_2\sin\theta_3 & \sin\theta_2\sin\theta_3 & \cos\theta_3\sin\theta_2 \\ -\cos\theta_1\sin\theta_3 & \cos\theta_2\cos\theta_3 & -\cos\theta_3\sin\theta_2 & \cos\theta_1\sin\theta_3 \end{pmatrix} \begin{pmatrix} I_\chi^3 \\ I_\eta^1 \\ I_\rho \\ I_\phi \end{pmatrix}. \tag{44}$$

Note that here we do not have massive states PS_1 and PS_2 as in Ref. [?].

From (44), it follows that in the limit $v_\varphi \gg v_\chi \gg v_\rho \gg v_\eta$
$$\begin{aligned} a &\backsimeq I_\phi, \\ G_Z &\backsimeq I_\rho, \\ G_{Z'} &\backsimeq I_\chi^3, \\ A_4 &\backsimeq I_\eta^1 . \end{aligned} \tag{45}$$

Substituting related values into (41) yields
$$m_{A_4}^2 = L\left(\frac{1}{v_\phi^2} + \frac{1}{v_\chi^2} + \frac{1}{v_\rho^2} + \frac{1}{v_\eta^2}\right) \tag{46}$$
$$\approx \lambda_\phi v_\phi v_\chi \left(\frac{v_\rho}{v_\eta}\right). \tag{47}$$

From (46) it follows
$$\lambda_\varphi > 0. \tag{48}$$

Hence, if $\lambda_\varphi \sim O(1)$ then A_4 is very heavy with mass in the range of v_φ.

Summary: In the CP-odd sector we have 6 fields: two Goldstone bosons for Z and Z$'$, one axion a, one massless field G_1 and two massive pseudoscalars A_1 and A_4.

2.4. CP-EVEN sector

There are six CP-even scalars and they separate into two square mass matrices.

Within the constraint conditions in (5), ones get square mass matrix of CP-even scalars written in the basis of (R_χ^1, R_η^3) as

$$M_R\left(R_\chi^1, R_\eta^3\right) = \begin{pmatrix} \frac{\lambda_7}{4}v_\eta^2 - \frac{L}{2v_\chi^2} & \frac{\lambda_7}{4}v_\chi v_\eta - \frac{L}{2v_\chi v_\eta} \\ \frac{\lambda_7}{4}v_\chi v_\eta - \frac{L}{2v_\chi v_\eta} & \frac{\lambda_7}{4}v_\chi^2 - \frac{L}{2v_\eta^2} \end{pmatrix} \quad (49)$$

Diagonalization of matrix in (5) yields one massless scalar G4 and one massive field H1 with masse as follows

$$m_{H_1}^2 = -\frac{\left(L - \lambda_7 v_\eta^2 v_\chi^2\right)}{2} \cdot \frac{\left(v_\eta^2 + v_\chi^2\right)}{v_\eta^2 v_\chi^2}. \quad (50)$$

The physical fields are

$$\begin{pmatrix} G_4 \\ H_1 \end{pmatrix} = \begin{pmatrix} -\sin\beta & \cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} R_\chi^1 \\ R_\eta^3 \end{pmatrix}. \quad (51)$$

In the limit $v_\eta \ll v_\chi$, we have $R_\eta{}^3 = G_4, R_\chi{}^1 = H_1, G_1 = I_\eta{}^3, I_\eta{}^1 = A_1$, hence

$$\eta_3^0 \equiv G_{X^0}. \quad (52)$$

Here

$$G_{X^0} = \frac{1}{\sqrt{2}}(G_4 + iG_1) = \frac{1}{\sqrt{2}}\left(R_\eta^3 + iI_\eta^3\right),$$

is the Goldstone boson for the X^0 boson.

Looking at Eqs (41) and (50) we realize that A_1 and H_1 have the same mass and they are component of χ_1^0. Hence we can compose them to new massive complex scalar ϕ^0

$$\varphi^0 = \frac{1}{\sqrt{2}}\left(R_\chi^1 + iI_\chi^1\right),$$

with mass given in (50).

In the limit $v_\phi \gg v_\chi \gg v_\rho \gg v_\eta$, one has

$$\chi \simeq \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi^0 \\ G_{Y^-} \\ \frac{1}{\sqrt{2}}(\upsilon_\chi + R_\chi^3 + iG_{Z'}) \end{pmatrix}, \eta \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(u + R_\eta^1 + iA_4) \\ H_1^- \\ G_{X^0} \end{pmatrix}, \rho \simeq \frac{1}{\sqrt{2}}\begin{pmatrix} G_{W^+} \\ \frac{1}{\sqrt{2}}(\upsilon + R_\rho + iG_Z) \\ H_2^+ \end{pmatrix}. \quad (53)$$

Thus, at this step we have already determined Goldstone bosons for Z, Z' and neutral bilepton X^0 and one massive complex scalar φ^0.

Next let us consider the second part of CP-even scalars. In the basis $(R_\chi^3, R_\eta^1, R_\rho, R_\phi)$, one has

$$\begin{pmatrix} 2\lambda_1\upsilon_\chi^2 - \frac{L}{2\upsilon_\chi^2} & \frac{\lambda_4\upsilon_\chi\upsilon_\eta}{2} + \frac{L}{2\upsilon_\eta\upsilon_\chi} & \frac{\lambda_5\upsilon_\chi\upsilon_\rho}{2} + \frac{L}{2\upsilon_\rho\upsilon_\chi} & \frac{L}{2\upsilon_\phi\upsilon_\chi} \\ \frac{\lambda_4\upsilon_\chi\upsilon_\eta}{2} + \frac{L}{2\upsilon_\eta\upsilon_\chi} & 2\lambda_2\upsilon_\eta^2 - \frac{L}{2\upsilon_\eta^2} & \frac{\lambda_6\upsilon_\eta\upsilon_\rho}{2} + \frac{L}{2\upsilon_\rho\upsilon_\eta} & \frac{L}{2\upsilon_\eta\upsilon_\phi} \\ \frac{\lambda_5\upsilon_\chi\upsilon_\rho}{2} + \frac{L}{2\upsilon_\rho\upsilon_\chi} & \frac{\lambda_6\upsilon_\eta\upsilon_\rho}{2} + \frac{L}{2\upsilon_\rho\upsilon_\eta} & 2\lambda_3\upsilon_\rho^2 - \frac{L}{2\upsilon_\rho^2} & \frac{L}{2\upsilon_\rho\upsilon_\phi} \\ \frac{L}{2\upsilon_\phi\upsilon_\chi} & \frac{L}{2\upsilon_\eta\upsilon_\phi} & \frac{L}{2\upsilon_\rho\upsilon_\phi} & 2\lambda_{10}\upsilon_\phi^2 - \frac{L}{2\upsilon_\phi^2} \end{pmatrix} \quad (54)$$

In the case of $\upsilon_\phi \gg \upsilon_\rho, \upsilon_\eta, \upsilon_\chi$, we have that R_φ decouples and its mass is predicted to be

$$m_{R_\phi}^2 \simeq 2\lambda_{10}\upsilon_\phi^2. \quad (55)$$

From (55) it follows

$$\lambda_{10} > 0. \quad (56)$$

For the future studies, we will identify R_ϕ to inflaton.

Keeping the next term of order $v_\varphi v_\chi$ yields

$$M_{4R} = \begin{pmatrix} 2\lambda_1\upsilon_\chi^2 & 0 & 0 & 0 \\ 0 & -\frac{L}{2\upsilon_\eta^2} & \frac{L}{2\upsilon_\rho\upsilon_\eta} & 0 \\ 0 & \frac{L}{2\upsilon_\rho\upsilon_\eta} & -\frac{L}{2\upsilon_\rho^2} & 0 \\ 0 & 0 & 0 & 2\lambda_{10}\upsilon_\phi^2 \end{pmatrix} \quad (57)$$

Hence at this step one has one massive state R_χ^3 with mass

$$m^2_{R^3_\chi} \simeq \lambda_1 v_\chi^2. \tag{58}$$

From (58) it follows

$$\lambda_1 > 0. \tag{59}$$

We will identify $R^3_\chi \equiv H_5$. This is heavy scalar.

It is easily diagonalize matrix in (57) and there are two solutions: one massless G_5 and one massive H_2

$$\begin{aligned} G_5 &= -\cos\theta_4 R_\eta - \sin\theta_4 R_\rho, \\ H_2 &= +\sin\theta_4 R_\eta - \cos\theta_4 R_\rho, \end{aligned} \tag{60}$$

where $\tan\theta_4 = v_\eta v_\rho$ and the H_2 mass is given by

$$m^2_{H_2} = \frac{L v_\rho^2}{2v_\eta^2 cos^2\theta_4} = \frac{L(v_\rho^2 + v_\eta^2)}{2v_\eta^2} \tag{61}$$

To avoid massless state G_5, let us diagonalize 2 × 2 matrix in central part of (54), e.g.,

$$M_{R2} = \begin{pmatrix} 2\lambda_2 v_\eta^2 - \dfrac{L}{2v_\eta^2} & \dfrac{\lambda_6 v_\eta v_\rho}{2} + \dfrac{L}{2v_\rho v_\eta} \\ \dfrac{\lambda_6 v_\eta v_\rho}{2} + \dfrac{L}{2v_\rho v_\eta} & 2\lambda_3 v_\rho^2 - \dfrac{L}{2v_\rho^2} \end{pmatrix} \equiv - \begin{pmatrix} \mathrm{b} & -\mathrm{d} \\ -\mathrm{d} & \mathrm{c} \end{pmatrix} \tag{62}$$

□

Then we have two massive states H_3 and H_4

$$\begin{aligned} H_3 &= -\sin\theta_5\, R^1_\eta + \cos\theta_5\, R_\rho, \\ H_4 &= -\cos\theta_5\, R^1_\eta - \sin\theta_5\, R_\rho, \end{aligned} \tag{63}$$

where

$$\tan 2\theta_5 = \frac{2d}{c-b}. \tag{64}$$

with masses given by

$$2m^2_{H_3} = (b+c) - \sqrt{(c-b)^2 + 4d^2}, \tag{65}$$

$$2m^2_{H_4} = (b+c) - \sqrt{(c-b)^2 + 4d^2}. \tag{66}$$

We can identify H_3 as the SM-like Higgs boson h.

Let us consider the limit $v_\varphi \gg v_\chi \gg v_\rho \gg v_\eta$, then

$$b = -2\lambda_2 v_\eta^2 + \frac{L}{2v_\eta^2} \approx \lambda_\phi v_\phi v_\chi \frac{v_\rho}{v_\eta},$$
$$c = -2\lambda_3 v_\rho^2 + \frac{L}{2v_\rho^2} \approx -2\lambda_3 v_\rho^2 + \lambda_\phi v_\phi v_\chi \frac{v_\rho}{v_\eta}, \tag{67}$$
$$d = \frac{\lambda_6 v_\eta v_\rho}{2} + \frac{L}{2v_\rho v_\eta} \approx \frac{\lambda_6 v_\eta v_\rho}{2} + \frac{1}{2}\lambda_\phi v_\phi v_\chi.$$

Hence

$$c - b \approx -2\lambda_3 v_\rho^2 - \lambda_\phi v_\phi v_\chi \frac{v_\rho}{v_\eta},$$
$$(c+b) \approx -2\lambda_3 v_\rho^2 + \lambda_\phi v_\phi v_\chi \frac{v_\rho}{v_\eta}, \tag{68}$$
$$\tan 2\theta_5 = \frac{2d}{c-b} \approx \frac{v_\eta}{v_\rho} \Rightarrow \tan\theta_5 \approx \frac{v_\eta}{2v_\rho}.$$

$$\Delta = \left(2\lambda_3 v_\rho^2 + \lambda_\phi v_\phi v_\chi \frac{v_\rho}{v_\eta}\right)^2 + 4\left(\frac{\lambda_6 v_\eta v_\rho}{2} + \frac{1}{2}\lambda_\phi v_\phi v_\chi\right)^2$$
$$\approx \left(\lambda_\phi v_\phi v_\chi \frac{v_\rho}{v_\eta}\right)^2 \Rightarrow \sqrt{\Delta} \approx \lambda_\phi v_\phi v_\chi \frac{v_\rho}{v_\eta} \tag{69}$$

Substituting (69) into (65) and (66) yields

$$m_{H_3}^2 = -\lambda_3 v_\rho^2, \tag{70}$$
$$m_{H_4}^2 = \lambda_\phi v_\phi v_\chi \frac{v_\rho}{v_\eta} - \lambda_3 v_\rho^2. \tag{71}$$

From (70) it follows

$$\lambda_3 < 0\,. \tag{72}$$

From (70) it follows that $\lambda_3 < 0$ and H_3 can be identified to SM Higgs boson h, while H_4 is heavy scalar and $\lambda_\varphi > 0$. Note that

$$h \approx R_\rho, \qquad H_4 \approx -R_\eta^1. \tag{73}$$

Hence

$$\chi \simeq \begin{pmatrix} \varphi^0 \\ G_{Y^-} \\ \frac{1}{\sqrt{2}}(v_\chi + H_5 + iG_{Z'} \end{pmatrix}, \eta \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}(u - H_4 + iA_4 \\ H_1^- \\ G_{\chi^0} \end{pmatrix}, \rho \simeq \begin{pmatrix} G_{W^+} \\ \frac{1}{\sqrt{2}}(v_\chi + H_5 + iG_{Z'} \\ H_2^- \end{pmatrix},$$
$$\phi \simeq \frac{1}{\sqrt{2}}(v_\phi + \Phi + ia). \tag{74}$$