



Research Article

RESEARCHING AND COMPARING VIETNAMESE AND AMERICAN
TEXTBOOKS ON TEACHING THE FUNCTIONS $y = ax^2 (a \neq 0)$
FROM THE APPROACH OF REALISTIC MATHEMATICS EDUCATION

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ABSTRACT

In the context of current educational innovation with an increase in the use of mathematics to solve real life problems, the application of the theory of Realistic Mathematics Education (RME) in teaching mathematics is appropriate and feasible. This article provides an overview of the RME theory and its application in teaching mathematics. Based on this theoretical framework, we compare the ways of the introduction of the function $y = ax^2 (a \neq 0)$ in the Vietnamese and American textbooks. The results of the analysis show that with the use of the RME approach in teaching this function in the American textbooks, students are involved in the process of re-inventing knowledge, recognizing the role of mathematics in real life, and have opportunities to develop critical thinking and problem-solving capacities. We then propose further studies so that we can deploy the quadratic function teaching in Vietnam using the RME approach.

Keywords: realistic Mathematic Education; realistic mathematical problems; $y = ax^2 (a \neq 0)$ functions

1. Introduction

Mathematics is derived from life. Keeping up with the demand of lives is the basic fundamental for mathematical development. Math plays an important part in solving practical problems and orienting the development of science and technology. Nowadays Vietnamese Education has shifted to competency based approach to replace the previous content based approach. Consequently, this shift creates more opportunities for students to

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experience and apply mathematics into reality. Indeed, it creates a connection between mathematical ideas, mathematics, and practical application.

Since the second half of the twentieth century, modern educational institutions in the world - such as the United States, the United Kingdom, Germany, France, Australia, and the Netherlands have applied new teaching theories to their educational programs. *Théorie des Situations* in France, *American Intelligence* in the US are some examples. While studying new teaching theories, we have opportunities to know about the Realistic Mathematics Education (RME), which was developed by Hans Freudental (1905-1990), a German mathematician at Utrecht University in the Netherlands. RME theory recognizes that learning mathematics can start with situations where students engage in active learning, and it is an interesting approach. Therefore, students can use existing mathematical tools to discover the knowledge to be learned. In RME, a mathematical relationship with reality is not only recognizable at the end of a learning process, such as applying math to reality, but also serving as a source for teaching and learning mathematics.

After many years of development, the current RME has become the main foundation for mathematics education in the Netherlands with 100% of books written from the ideology of RME. Not only is it popular in the windmill country, but the RME ideology also affects the education of many other countries such as the formation of *Recontextualization in Mathematics Education* in UK, the *Mathematics in Context (MiC)* in US. In Vietnam, RME was introduced by Le Tuan Anh, Nguyen Thanh Thuy and some other researchers.

This article will clarify the basic characteristics of RME, and make a comparison of the differences in the way the concept of $y = ax^2$ ($a \neq 0$) function is formed between Vietnamese textbooks and the US MiC teaching materials. Furthermore, we want to propose a situation to teach $y = ax^2$ ($a \neq 0$) function concept applying RME.

In the next section, we will briefly provide an outline of the views and principles of RME. In addition, we analyze and compare how to form the concept of $y = ax^2$ ($a \neq 0$) functions in Vietnamese and American textbooks.

2.1. RME

2.1.1. Mathematics as a human activity

Freudenthal (1991) said that the process of learning mathematics as a connected process, the reflective exchange between mathematical reality and mathematical mathematics. For example, he recommends that students have an opportunity to handle problems in informal situations before they learn a formal method. Freudenthal said that this is a natural and normal way to prepare students to receive new knowledge.

According to Freudenthal (1973), mathematics as a human activity. Mathematics arises from human needs, also from those increasingly diverse and evolving needs or from internally mathematics. It advocates for self-examination as well as self-improvement and development, and so it generates new knowledge. In other words, mathematics is the product of man, invented by man, and serves human life.

2.1.2. Teaching mathematics means guiding students to “re-invent” knowledge

According to the RME theory, students should be given an opportunity to experience a process similar to the way mathematicians formed itself. That means that by performing a number of activities to solve problems placed in the context, students will judge what mathematics will be real. They can use their ‘unofficial’ knowledge to reinvent the mathematical knowledge.

However, the role of a teacher in the re-invention process is very important because students cannot independently perform this process. They need sufficient guidance from a textbook and teacher. The way to discover mathematics is not covered with roses. Instead, they go through a complicated and twisted process, sometimes it takes thousands of years, with many mathematicians working with each other to find out. Therefore, because teachers cannot completely faithfully reproduce that process in the classroom like in history, students cannot repeat the process of re-verification completely that mathematicians have experienced. That process must be recreated appropriately, close to the students' knowledge. Besides, because the process of reinventing knowledge is done based on the personal knowledge of students, teachers need to build and lead students on the path with the main foundation based on that idea.

2.1.3. Mathematization

Freudenthal (1971) believes that mathematical education focuses on helping students "reinvent" knowledge through establishing a mathematical model and using mathematical tools to solve problems placed in a context where there is meaning from reality instead of directly providing students with knowledge.

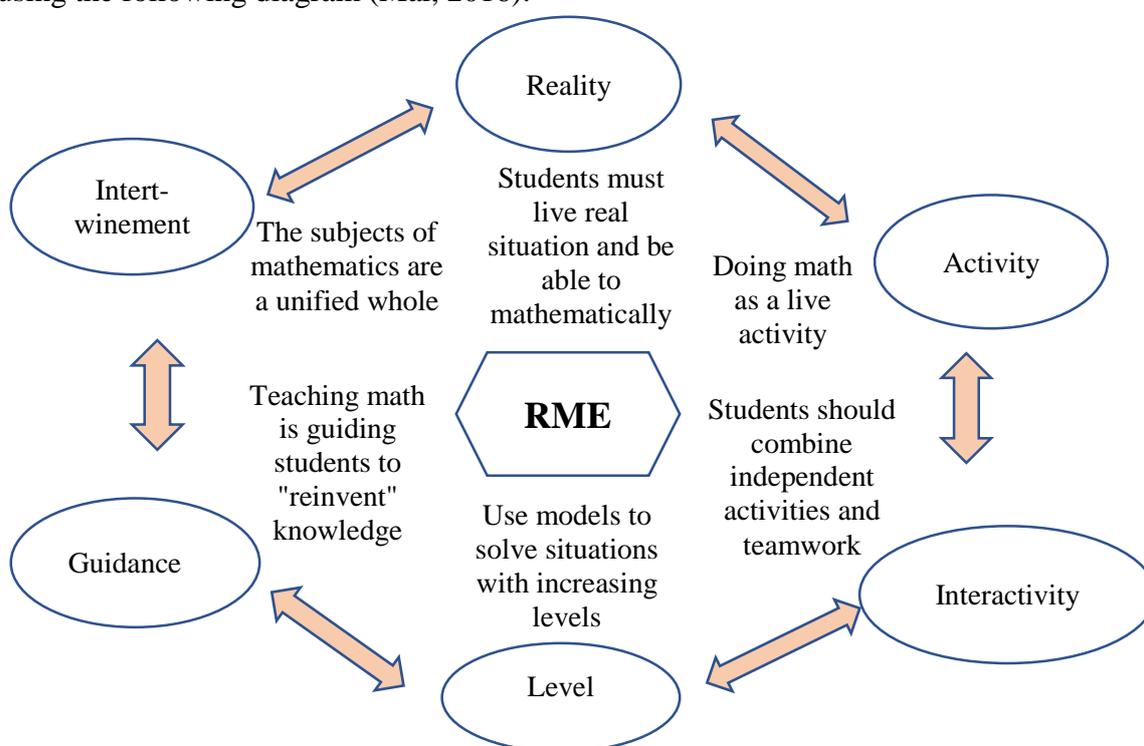
Besides, Lange (1996) thinks that context is really very important, as a starting point in learning mathematics. De Lange said that the process of developing mathematical concepts and ideas starts from the real world, and finally, we need to reflect the solution back to the real world. So what we do in math education is to take things from the real world, mathemize them, and then bring them back to the real world. All this process leads to conceptual mathematics.

2.2. The core teaching principles of RME

During the process of designing teaching materials based on RME's theory, these following questions emerged: how the teaching process using a textbook should be conducted; how teachers should present the classroom curriculum; and how do students learn from teaching materials. Concerning these questions, Treffers (1978, 1991) proposed

five principles of learning and teaching. These principles help to formulate and to concretize level and model, reflection and special exercises, social contexts, in interaction, structure, and intertwining. These principles have been developed and reformed over the years, including Treffers himself. It is undeniable that RME is a product of the times and inseparable from the global reform movement in mathematical education that has occurred over the past decades. Therefore, RME has many similarities with current approaches to mathematics education in other countries, but RME itself has developed very specific principles.

After reviewing the related documents, we describe the six principles of teaching using the following diagram (Mai, 2016):



Picture 1. The principles of RME

The activity principle: in RME, students are treated as active participants in the learning process. It also emphasizes that mathematics is best learned by doing mathematics, which is strongly reflected in Freudenthal's mathematical interpretation, as a living activity of man. The result of student's operating process is the decisive factor for the effectiveness of the teaching process.

The reality principle: according to this principle, mathematics should be connected to reality and be meaningful to students or be suitable for common life. First, it expresses the importance in which Mathematics is attached to the goal of mathematics education including students' ability to apply mathematics in solving "real-life" problems. Second, it means that mathematics education should start from problem situations that are meaningful

to students, which offers them opportunities to attach meaning to the mathematics. Therefore, it constructs their development while solving problems. Rather than beginning with teaching abstractions or definitions, in RME, teaching starts with problems in rich contexts that require mathematical organization. Therefore, the selected problem situations must be both familiar and intimate, interesting for students and at the same time suitable for the students' knowledge and abilities.

The level principle underlines that learning mathematics means students pass various levels of understanding: from informal context-related solutions, through creating various levels of shortcuts and schematizations to acquiring insight into how concepts and strategies are related. Models are important for bridging the gap between the informal, context-related mathematics and the more formal mathematics.

The intertwinement principle discusses the relationship between mathematical disciplines. Teaching according to the RME trend will not focus on the boundaries such as mathematics available between the subjects Algebra, Geometry, Statistical Probability... but see them as a unified whole, interwoven, support and tie together. This principle also emphasizes that teachers should create situations in order for students to be placed in diverse situations in which they may have to perform many different types of tasks intermittently (deduction, calculation). Mathematics, statistics, conducting algorithms... enables students to look at the problem from the perspective of each subject, using a lot of knowledge, tools, math from different subjects, even other sciences.

The interactivity principle signifies that learning mathematics is not only an individual activity but also a social activity. Therefore, RME favors whole-class discussions and group work which offer students opportunities to share their strategies and inventions with others. In this way students can get ideas for improving their strategies. Moreover, interaction evokes reflection, which enables students to reach a higher level of understanding. However, the "collective" in this principle does not all work together, not in a small group to complete a job. But each individual student works independently with his own ideas, combined with the combined results from the group work, and cannot ignore the process of interacting with teachers and working with documents, to complete his product.

The guide principle proposed by Freudenthal himself from the idea of the process of reinventing knowledge re-teaching of teachers (guides re-invention) in teaching mathematics: "The mathematical knowledge gained through rediscovering will help children understand better and remember more easily." In particular, the teacher is the person who plays the role of a pioneer on the path of rich potential activities. It implies that within RME teachers should have an active role for students that conducting such activities will create meaningful cognitive leaps for learners. In order to realize this principle, it should be noted that RME prioritizes long-term teaching projects, rather than traditional single lessons.

2.3. The concept of quadratic function in US and Vietnamese textbooks from the RME approach

2.3.1. The situation leads to the concept of $y = ax^2 (a \neq 0)$ functions in Vietnam's secondary school mathematics textbook

The situation leading up to the concept of $y = ax^2 (a \neq 0)$ functions is in Lesson 1: "Functions $y = ax^2 (a \neq 0)$ " in Mathematics 9 textbook, volume 2, p. 28. The concept of quadratic functions is introduced through an opening example by showing distance traveled freely in time.

At the top of the Leaning Tower of Pisa in Italy, Gallie dropped two lead orbs of different weights to experiment on the motion of a free-falling object. He asserts that, when an object falls freely (regardless of air resistance), its velocity increases and does not depend on the weight of the object. Its movement distance s is represented by the formula

$$s = 5t^2$$

with t is the time in seconds, s in meters.

According to this formula, each value of t determines a unique corresponding value of s .

The following table shows several pairs of corresponding values of t and s .

T	1	2	3	4
S	5	20	45	80

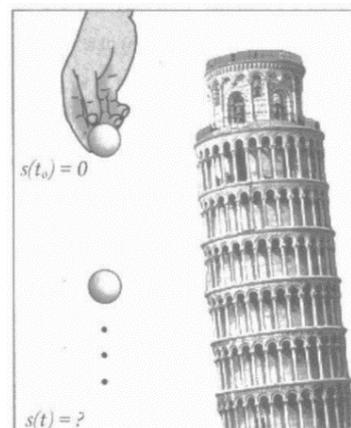
The formula $s = 5t^2$ represents a form function $y = ax^2 (a \neq 0)$.

After introducing the formula to represent the distance traveled by time $s = 5t^2$, the authors wrote: "According to this formula, each value of t determines a unique corresponding value of s ". Because students have approached the concept of "Function" in the textbook of Mathematics 9 volume 1, they can understand the above formula is a function. At the same time, the textbook also provides some value pairs $(t; s)$ to illustrate the above affirmation. After that, the textbook concludes "The formula $s = 5t^2$ express a function has a form $y = ax^2 (a \neq 0)$ ".

Comment:

The situation leading to the $y = ax^2 (a \neq 0)$ function is merely derived from the formula available through an example related to reality.

Clearly, the conceptual approach to the $y = ax^2 (a \neq 0)$ function has never been thought of by RME. Vietnamese textbook still uses a practical example, not rigidly given



the concept immediately. However, we have not appreciated this situation because in fact students are still passive in receiving knowledge, and textbook do not give students the opportunity to self-explore, work in groups... Also, students have not been exposed to many situations in order to work with models.

Next, we will analyze the exercises with the appearance of Mathematization based on RME theory.

Exercise 2 (p. 31): *An object falls at an altitude of 100m above the ground. The movement distance s (meters) of the falling object depends on the time t (seconds) by the formula: $s = 4t^2$*

a) *After 1 second, how far in meter is this object from the ground? Similarly, after 2 seconds?*

b) *How long does it take for this object being landed?*

Exercise 3 (p. 31): *Wind force F when it blows perpendicular to the sail, which is proportional to the square of the velocity v of the wind, that is $F = av^2$ (a is constant). Knowing that when wind speed is 2 m/s , the force acting on the sail of a boat is 120N .*

a) *Calculate a .*

b) *When $v = 10\text{m/s}$ then how much is F force? Give answer for the same question when $v = 20\text{m/s}$?*

c) *Knowing that sail bears maximum force of 1200N , can the ship get through the storm with speed at 90km/h ?*

Comments:

The two above exercises are real problems but like an introductory problem, the subject given the formula of the function $y = ax^2$ ($a \neq 0$), students only need to use mathematical tools to calculate but not find the form of the function themselves. These two exercises only focus on applying mathematical tools without figuring out mathematical function. Hence, we realize these mathematics exercises can satisfy the reality principle. However, the activity principle is not promoted. It just appears in some student's calculation activities. Besides, the level principle was just built a bit with a few difficult questions, whereas students are not working on models. We definitely do not find any ideas in building up the interactivity principle and the guide principle.

2.3.2. *The situation leads to the concept of quadratic functions in American textbooks*

The US education system also lasts 12 years and has similar levels of education in Vietnam. In the US, the Principles and Standards for School Mathematics document is designed as a framework for mathematics at the national level. Based on that, each state will publish its own standards of knowledge and math skills to guide specific content for each grade. These documents are called Common Core State Standards.

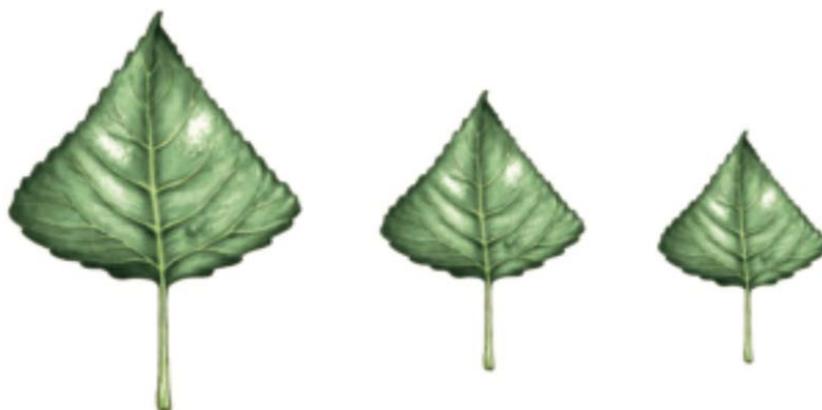
The content of quadratic functions is taught for grade 8 students in the US in Algebra. The set of materials for grade 8 for this subject includes 4 books: Ups and Downs, Graphing Equations, Patterns and Figures, and Algebra Rules. The situations that form the concept of quadratic functions are in the “C: Differences in Growth” section of the Ups and Downs book, including the situations of Leaf Area, Water Lily, Aquatic Weeds, and Double Trouble. In the following section, we will analyze the Leaf Area situation.

The main function of leaves is to create food for the entire plant. Each leaf absorbs light energy and uses it to decompose the water in the leaf into its elements — hydrogen and oxygen. The oxygen is released into the atmosphere. The hydrogen is combined with carbon dioxide from the atmosphere to create sugars that feed the plant. This process is called photosynthesis.

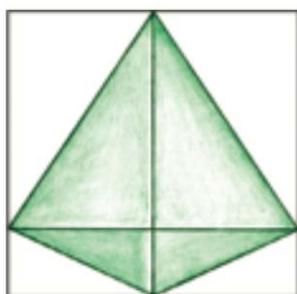
1. a. *Why do you think a leaf’s ability to manufacture plant food might depend on its surface area?*

b. *Describe a way to find the surface area of any of the leaves shown on the left.*

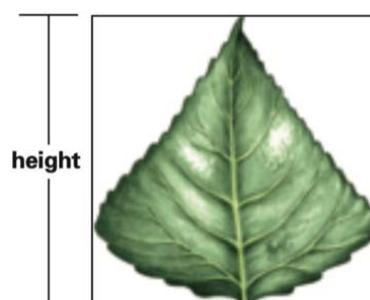
The picture below shows three poplar leaves. Marsha states, “These leaves are similar. Each leaf is a reduction of the previous one.”



2. *Measure the height and width of each of the leaves to determine whether Marsha is right*



One way to estimate the surface area of a poplar leaf is to draw a square around it as shown in the diagram on the right. The kite-shaped model on the left covers about the same portion of the square as the actual leaf on the left.



3. a. *Approximately what portion of the square does the leaf cover? Explain your reasoning.*

b. If you know the height (h) of such a leaf, write a direct formula that you can use to calculate its area (A).

c. If his measured in centimeters, what units should be used to express A ?

d. The formula that you created in part b finds the area of poplar leaves that are symmetrical. Draw a picture of a leaf that is not symmetrical for which the formula will still work.

Area Differences

You can use this formula for the area of a poplar leaf when the height (h) is known:

$$A = \frac{1}{2}h^2$$

You can rewrite the formula using arrow language:

$$h \xrightarrow{\text{ squared }} \dots \xrightarrow{\times \frac{1}{2}} A$$

The table shows the areas of two poplar leaves.

Height (in cm)	6	7	8	9	10	11	12
Area (in cm²)	18	24.5					

4. a. Verify that the areas for heights of 6 cm and 7 cm are correct in the table.
 b. On **Student Activity Sheet 7**, fill in the remaining area values in the table. Describe any patterns that you see.

c. **Reflect.** How do you know that the relationship between area and height is not linear?

The diagram below shows the differences between the areas of the first three leaves in the table.

Height (in cm)	6	7	8	9	10	11	12
Area (in cm²)	18	24.5	32	?	?	?	?

First Difference



5. a. On **Student Activity Sheet 7**, fill in the remaining “first difference” values. Do you see any patterns in the differences?

b. The first “first difference” value (6.5) is plotted on the graph on **Student Activity Sheet 8**. Plot the rest of the differences that you found in part a on this graph.

c. Describe your graph.

Height (in cm)	6	7	8	9	10	11	12
Area (in cm²)	18	24.5	32	?	?	?	?

First Difference



Second Difference



As shown in the diagram, you can find one more row of differences, called the **second differences**.

6. a. Finish filling in the row of second differences in the diagram on **Student Activity Sheet 7**.

b. What do you notice about the second differences? If the diagram were continued to the right, find the next two second differences.

c. How can you use the patterns of the second differences and first differences to find the areas of leaves that have heights of 13 cm and 14 cm? Continue the diagram on **Student Activity Sheet 7** for these new values.

d. Use the area formula for poplar leaves $\left(A = \frac{1}{2}h^2 \right)$ to verify your work in part c.

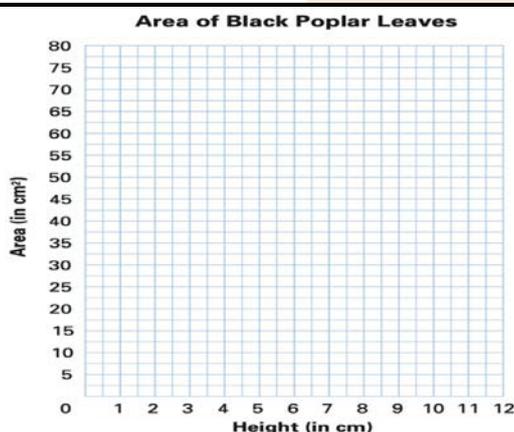
7. a. What is the value for $A \left(A = \frac{1}{2}h^2 \right)$ if $h = 2\frac{1}{2}$?

b. How does the value of A for a poplar leaf change when you double the value of h? Use some specific examples to support your answer.

8. If the area of one poplar leaf is about 65 square centimeters (cm^2), what is its height? Explain how you found your answer.

The table shown below is also printed on **Student Activity Sheet 9**.

Hieght (in cm)	1	2	3	4	5	6	7	8
Area (in cm^2)	0.5	2	4.5					



9. a. Use Student Activity Sheet 9 to fill in the remaining area values in the table. Use this formula:

$$A = \frac{1}{2}h^2$$

b. Graph the formula on the grid. Why do you think the graph curves upward?

c. Using your graph, estimate the areas of poplar leaves with the following heights: 5.5 cm, 9.3 cm and 11.7 cm.

d. Check your answers to part c, using the formula for the area of a poplar leaf. Which method do you prefer for finding the area of a poplar leaf given its height: the graph or the formula? Explain.

If the second differences in the table are equal, the growth is **quadratic**.

Comments:

The concept of function $y = ax^2$ ($a \neq 0$) is formed as in American textbook by a question in reality. From that a student can imagine, then use his/her own mathematics knowledge and directed questions in the textbook to find an answer. The American textbook gives several relevant questions: showing the first difference and the second difference to shape the function. This is also seen as a sign that students can identify whether the function is linear or quadratic.

2.4. Results and discussion

Based on the analysis results from the Vietnamese and American textbooks, from the perspective of RME, we found that:

- Although there are real situations in the Vietnamese textbook, the extent that student can live and feel it in the situation is almost none but passive knowledge. The principles of RME are almost not expressed, students only have mathematical operations and use formulas to perform exercises.

- The American textbooks apply most of RME's principles to their teaching situations:
 - + *The reality principle*: put students closely in a real-world problem: "Why do you think a leaf's ability to manufacture plant food might depend on its surface area?"

- + *The guide principle*: guide students to find the function $y = ax^2$ ($a \neq 0$) with a series of small questions.

- + *The activity principle*: students solve questions with many activities such as calculating, checking, drawing and describing graph, commenting...

- + *The level principle*: the questions are designed in an ascending order, from simple to complex, from specific to general.

- + *The intertwinement principle*: Students switch from real problems to geometric problems by drawing a square around them and finally using the algebra tool to calculate.

- + *The interactivity principle*: this principle can be expressed more clearly in the actual teaching process via students' discussion and group activities.

- Guiding students to find knowledge by approaching RME will help them realize the role of mathematics in life (linking mathematics with real life), understanding and remembering knowledge more through re-inventing knowledge. Besides, it also helps students improve critical thinking through working with a series of questions or models in progressive levels.

3. Conclusion

Vietnam education in recent years has also focused on the development of competencies of learners by linking theoretical learning with practical problem-solving skills. If we want to change the teaching methods from teacher-centered to student-centered, we must force teachers to change their perceptions about teaching. We recognize that the RME theory is well suited in the educational goals of the country.

In addition, the differences in forming the concept of function $y = ax^2$ ($a \neq 0$) between the Vietnamese textbooks and American textbooks show that instead of teaching using passive methods such as introducing the form of the function $y = ax^2$ ($a \neq 0$), we can build up the concept of functions towards mathematics by letting students solve a real problem and direct them to use mathematical tools to solve them. Using a series of guiding questions, teachers will help students re-invent knowledge.

The function $y = ax^2$ ($a \neq 0$) is a special case of quadratic functions. Therefore, from the research results, we will build a teaching circumstance based on the RME approach to help students reinvent the quadratic function knowledge in the future research.

❖ **Conflict of Interest:** Authors have no conflict of interest to declare.

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NGHIÊN CỨU VÀ SO SÁNH CÁCH DẠY HỌC HÀM SỐ $y = ax^2$ ($a \neq 0$)
GIỮA SÁCH GIÁO KHOA VIỆT NAM VÀ SÁCH GIÁO KHOA MỸ
THEO HƯỚNG TIẾP CẬN LÍ THUYẾT TOÁN HỌC TRONG NGỮ CẢNH

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TÓM TẮT

Trong bối cảnh đổi mới giáo dục hiện nay với việc tăng cường vận dụng toán học vào giải quyết các vấn đề thực tiễn thì việc vận dụng lý thuyết Toán học trong ngữ cảnh (RME) vào dạy học Toán là hoàn toàn phù hợp và khả thi. Bài viết này trình bày tổng quan về Lý thuyết RME cũng như cơ sở của việc áp dụng lý thuyết này vào quá trình dạy học Toán. Trên cơ sở khung lý thuyết này, chúng tôi so sánh cách trình bày hàm số $y = ax^2$ ($a \neq 0$) giữa sách giáo khoa Việt Nam và sách giáo khoa Mỹ. Kết quả phân tích cho thấy với cách tiếp cận RME trong dạy học hàm số này ở Mỹ, học sinh được tự mình tham gia vào quá trình khám phá lại kiến thức, nhận ra vai trò của toán học trong thực tế cuộc sống, có cơ hội phát triển tư duy phản biện và năng lực giải quyết vấn đề. Từ đó, chúng tôi đề xuất những nghiên cứu tiếp theo để có thể triển khai dạy học hàm số bậc hai ở Việt Nam theo hướng tiếp cận RME.

Từ khóa: lý thuyết Toán học trong ngữ cảnh; tình huống toán thực tế; hàm số $y = ax^2$ ($a \neq 0$)