ANALYTICAL TORQUE AND HARMONIC TORQUE OF SWITCHED RELUCTANCE MOTOR 6/4 BASED ON STATOR/ROTOR POLE ARCS

Linh Dinh Hai
Thuyloi University
Hanoi, Vietnam
linhdh@tlu.edu.vn

Thai Le Van
Thanh Dong University
Hai Duong, Vietnam
Lethaikhh59@gmail.com

Loan Dang Thi To
Vietnam National University of Forestry
Hanoi, Vietnam
Toloan852000@gmail.com

Trung Nguyen Thanh
Vietnam National University of Forestry
Hanoi, Vietnam
trungnt@vfu.edu.vn

ABSTRACT

A Switched Reluctance Motor (SRM) is a type of electric motor that operates on the principle of reluctance. Unlike conventional electric motors, SRMs do not have a permanent magnet or a wound field on the rotor. However, SRMs also have some disadvantages as high torque ripple and big acoustic noise. The three-phase SRM torque ripple is affected by electromagnetic torque harmonic orders which is difficult to estimate due to a highly nonlinear flux-linkage magnetic characteristic. Flux-linkage characteristic is a key SRM torque performance that depends on the stator/rotor pole structure. In this paper, torque and harmonic torque of SRM 6/4, 30kW is considered by the change of the stator/rotor pole arcs. The variation of average torque and harmonic amplitude torque with rotor pole arc in each SRM configuration is different. The finite element analysis results indicate the harmonic torque of SRM three phase 6/4 has mainly harmonic 3^{rd}, 6^{th}, 9^{th}, 12^{th}, and 15^{th} which is a multiple of three phases of the stator winding. When gradually increasing the stator/rotor pole arcs, the torque does not increase linearly but exhibits nonlinearity. The torque reaches its maximum value at a stator pole arc of 30^0 and a rotor pole arc of 32^0. Additionally, at this pair of stator and rotor pole angles, the amplitude of torque harmonic waves is minimized, resulting in the least amount of torque ripple.

Keywords: *Harmonic torque, Switched reluctance motor (SRM 6/4), Stator/rotor pole arcs, torque.*

1. INTRODUCTION

A Switched Reluctance Motor (SRM) is a type of electric motor that operates on the principle of reluctance, It do not have a permanent magnet or a wound field on the rotor. SRMs find applications in various industries, including automotive, aerospace, and industrial automation. They are particularly suitable for high-speed applications where their unique characteristics can be advantageous.

An analytical modelling and harmonic calculation of the three-phase SRM 6/4 electromagnetic torque is defined to obtain the non-linear relationship between inductance, flux-linkage, and phase torque vs position rotor to minimize the torque ripple. The harmonic orders of torque were not yet determined by a function of inductance, flux linkage, and the number of stator phases in SRM 6/4. The effects of stator and rotor geometry parameters on the instantaneous torque were implemented by developing the equations of electromagnetic energy. Application of FEM combined with optimization algorithms have been used to investigate electromagnetic torque by adjusting the rotor and stator pole shape such as tapped and pair pole in [1–4] and but those studies have been not yet analyzed in effects of stator and rotor geometry parameters on the harmonic torque of SRM 6/4. When the stator has a fixed number of poles, but the rotor has a different number of poles, the optimal rotor pole angle for maximum average torque and minimum torque ripple is different [5]. The electromagnetic torque analysis of SRM 12/8 by rotor/stator pole angle in [6]. The rotor poles projected slightly outward on one side with corresponding decrease in pole heights in the stator pole will create airgap with varying mean diameters under a pole pair to maximum torque [7]. The torque ripple minimization in SRM 8/6 by pole arcs design and the length of pole arc is limited by turn-on angle, turn-off angle as [8]. The design of a T-pole for stator and an L-pole for rotor is proposed to improve the output torque quality of an outer-rotor type SRM in [9]. The influence of stator and rotor pole embraces on the torque of an in-wheel SRM (IWSRM) using numerical modeling based on the finite element method (FEM) in [10]. This paper will figure out the effects of stator and rotor pole to average torque and harmonic torque of the SRM three phase 6/4 30kW, 3000 rpm. Based on this analysis, it is possible to choose the stator pole angle and rotor pole angle values that best suit the SRM 6/4 structure to minimize torque undulation and maximize the average moment.

2.ANALYTICAL THE TORQUE IN SRM

Normally, electromagnetic energy of the SRMs can be calculated by electromagnetic energy and co-energy as figure. 1. With the number of winding coils (N) in a pair of the stator teeth, the electromagnetic energy W_e is indefinited by phase voltage, current and flux as [3][4]:

$$W_e = W_f + W_m \quad (1)$$

$$\Rightarrow dW_e = dW_f + dW_m \quad (2)$$

Where :

$$dW_e = u.idt = \frac{d\psi}{dt}.idt = id\psi ;$$

$dW_m = T_k.d\theta$; *T_k is electromagnetic of one phase.*

Thus :

$$dW_f = dW_e - dW_m = id\Psi - T_k d\theta \quad (3)$$

In the other, $dW_f + dW_C = i.\Psi$

$$did\Psi - Td\theta + dW_C = id\Psi + \Psi di \quad (4)$$

The result:

$$dW_C(\theta, i) = \frac{\partial W_C}{\partial \theta} d\theta + \frac{\partial W_C}{\partial i} di \quad (5)$$

With constant current, each torque phase can be obtained:

$$T_k = \left. \frac{\partial W_C}{\partial \theta} \right|_{i=\text{const}} \quad (6)$$

$$\Rightarrow T_k = \frac{i^2}{2} \frac{dL(\theta,i)}{d\theta} \quad (7)$$

The total torque of the SRM is summaried of each phase:

$$T = \sum_{m=1}^{k} T_k \quad (8)$$

3. ANALYZING TORQUE AS A FUNCTION OF CHANGES IN STATOR/ ROTOR POLE ARC.

The stator/ rotor pole arc of the SRM 6/4 is defined by one stator phase winding **(figure 1).**

$$\beta_s = 2\, arcsin \frac{t_s}{D_{\text{is}}} ; \quad \beta_r = 2\, arcsin \frac{t_r}{D_{\text{is}}} \quad (9)$$

Where: βr, βs are rotor and stator pole arcs, Dis is inner diameter stator.

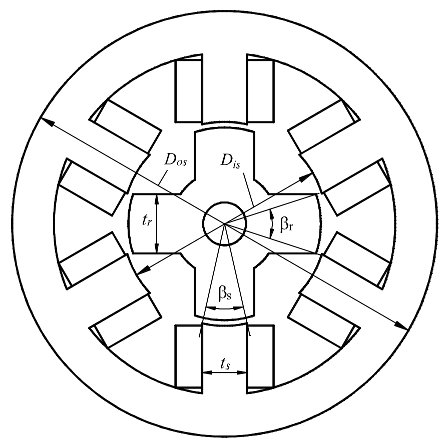

Fig. 1. *Structure of SRM 6/4*

The torque in SRM is highly dependent on the rotor position and the magnetic reluctance of the motor. Here's how torque varies with changes in stator/ rotor pole arc. To assess the variation of average torque with changes in stator pole arc and rotor pole arc, you can consider the change in average torque when altering the ratio of stator pole arc to stator pole pitch and the ratio of rotor pole arc to rotor pole pitch. The ratio of stator pole arcs to stator pole pitch is typically denoted as "α_s" and is defined as the arc between adjacent stator poles divided by the pitch of the stator poles and the ratio of rotor as "α_r".

$$\alpha_s = \frac{\beta_s}{\tau_s};\ \alpha_r = \frac{\beta_r}{\tau_r} \qquad (10)$$

Where: τ_s , τ_r : stator and rotor pitch.

And τ_s, τ_r are calculated as:

$$\tau_s = \frac{2\pi}{N_s};\ \tau_r = \frac{2\pi}{N_r} \qquad (11)$$

Where: Ns is number of stator poles; Nr is number of rotor poles

This paper analyzes the variation of average torque when the stator pole arc varies from 12^0 to 36^0 degrees and the rotor pole arc varies from 9^0 to 63^0 degrees, subject to the boundary condition where the sum of the stator pole arc and rotor pole arc is less than the rotor pole pitch ($2\pi/Nr = 90^0$). That are corresponding α_s from 0.2 to 0.6 with step 0.05 and α_r from 0. 2 to 0.7 with step 0.05 as Table 1.

Table 1. *Caculation of α_s, α_r*

$\beta s(^0)$	α_s	$\beta r\ (^0)$	α_r
12	0.2	18	0.2
15	0.25	22.5	0.25
18	0.30	27	0.30
21	0.35	31.5	0.35
24	0.40	36	0.40
27	0.45	40.5	0.45
30	0.50	45	0.50
33	0.55	49.5	0.55
36	0.60	54	0.60
-	-	58.5	0.65
-	-	63	0.70

The result of FEM on the SRM 6/4 model when the stator/rotor pole arc changed as figure 2, 3.

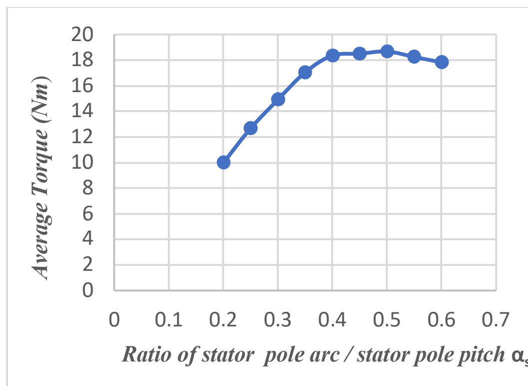

Fig. 2. *Average Torque vs the ratio of pole arc /pole pitch stator SRM 6/4.*

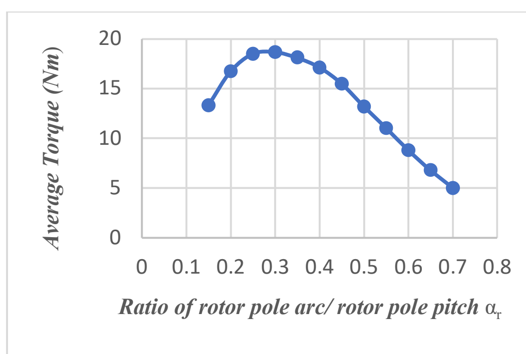

Fig. 3. *Average Torque vs the ratio*

of pole arc /pole pitch rotor SRM 6/4

The results shown in Figure 3 indicate that within the range of $\alpha_s = 0.4$ to 0.55, the average torque is relatively high. Furthermore, at the point $\alpha_s = 0.5$, the average torque reaches its maximum value, making it the peak torque in the analyzed range.

The result in Figure 4 shows that when changing the rotor pole arc, the maximum value of the average torque initially increases gradually with the increasing ratio of rotor pole arc to rotor pole pitch (α_r). Within the range of $\alpha_r =$ 0.25 to 0.40, the average torque remains relatively stable. After that, as you continue to increase the rotor pole arc, the average torque begins to decrease and decreases rapidly from $\alpha_r = 0.5$ and at the point $\alpha_r = 0.35$, the average torque is maximum. With the SRM 6/4 configuration, it has been analyzed and concluded that in various cases of stator pole arcs, the ratio of stator pole arc to stator pole pitch $\alpha_s = 0.5$ (or stator pole arc $\beta_s = 30^0$ degrees) achieves the highest average torque. This refers to optimizing the stator pole arc to achieve optimal torque performance. Combining the condition that the rotor pole arc must be greater than the stator pole arc, you have determined that the pair of values $\alpha_s = 0.5$ and $\alpha_r = 0.35$ results in the highest average torque for the SRM 6/4. This can help you optimize the SRM 6/4 motor for the best torque performance in your specific application.

4. ANALYTICAL HARMONIC TORQUE

The model 2D SRM 6/4 with I = 200A is analyzed by FEM base on Motor Cad software as Figure 4.

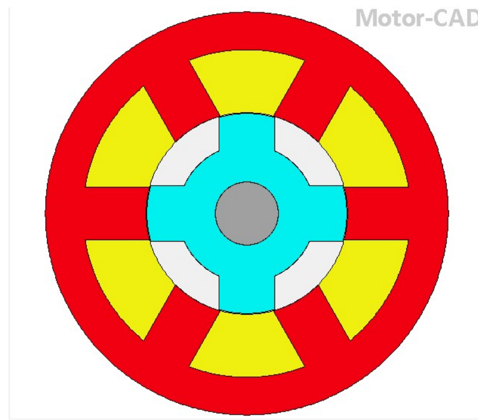

Fig. 4. *Model 2D SRM 6/4*

In the process of designing for a Switched Reluctance Motor (SRM), it's not only necessary to meet the requirements for average torque but also to achieve torque ripple minimization. Maximizing average torque while simultaneously minimizing torque ripple is essential. As the analysis results indicate, when the rotor pole arc changes, the average torque also changes; however, this change is nonlinear. The rotor pole arc may increase, but the average torque only increases up to a certain point. If you continue to increase the rotor pole arc beyond that point, the average torque starts to decrease. Torque ripple is expressed as the high-order harmonics of the the electromagnetic torque curves. When the amplitude of the torque harmonics is reduced or eliminated the torque ripple is decreased. The results of simulating the change in amplitude of the harmonic moments when varying the rotor polar arc are illustrated in Figure 5-11.

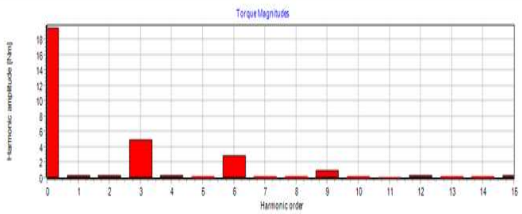

Fig. 5. *The amplitude harmonic torque of SRM 6/4 when* $\beta_S = 30^0$, $\beta_R = 30^0$

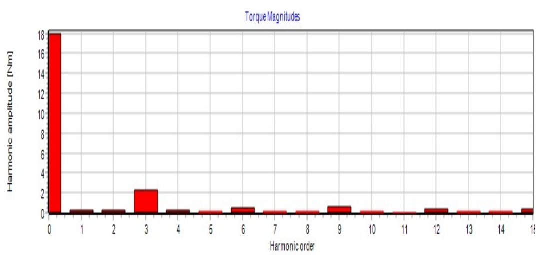

Fig. 6. *The amplitude harmonictorque of SRM 6/4 when* $\beta_S = 30^0$, $\beta_R = 31^0$

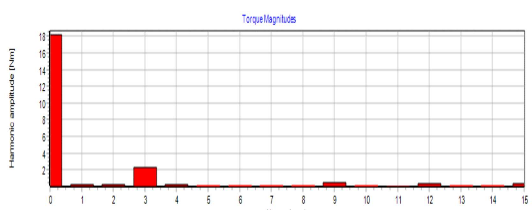

Fig. 7. *The amplitude harmonic torque of SRM 6/4 when* $\beta_S = 30^0$, $\beta_R = 32^0$

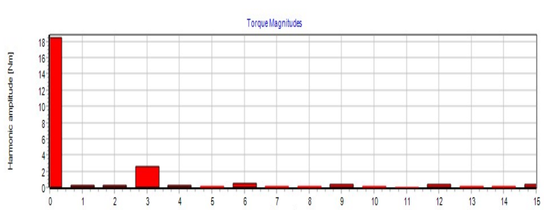

Fig. 8. *The amplitude harmonictorque of SRM 6/4 when* $\beta_S = 30^0$, $\beta_R = 33^0$

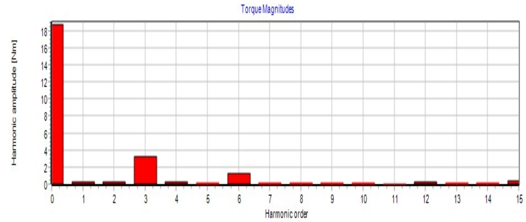

Fig. 9. *The amplitude harmonictorque of SRM 6/4 when* $\beta_S = 30^0$, $\beta_R = 34^0$

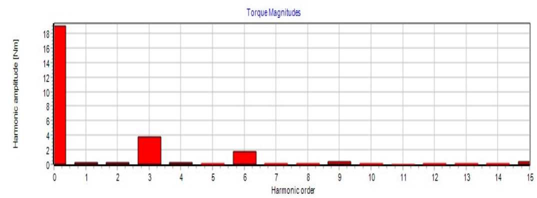

Fig. 10. *The amplitude harmonic torque of SRM 6/4 when* $\beta_S = 30^0$, $\beta_R = 35^0$

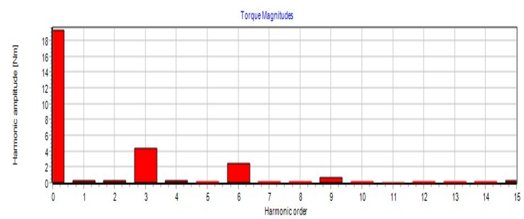

Fig. 11. *The amplitude harmonictorque of SRM 6/4 when* $\beta_S = 30^0$, $\beta_R = 36^0$

The result shows that the harmonics of SRM 6/4 are mainly multiples of three: order 3, 6, 9, 12 in the three-phase of SRM's. The amplitude of these harmonic waves also exhibits nonlinear variation with the increasing rotor polar arc: in the case where the rotor polar arc aligns with the stator polar arc, the amplitudes of the 3rd and 6th harmonic waves are the highest, with the highest percentage of torque ripple.

The calculation results for torque, torque ripple, and efficiency of the SRM 6/4 in table 2.

Table 2. *Result of performance when change the rotor pole arc*

Parameter	Unit	$\beta_s = 30^0$ $\beta_r = 30^0$	$\beta_s = 30^0$ $\beta_r = 31^0$	$\beta_s = 30^0$ $\beta_r = 32^0$	$\beta_s = 30^0$ $\beta_r = 33^0$	$\beta_s = 30^0$ $\beta_r = 34^0$	$\beta_s = 30^0$ $\beta_r = 35^0$	$\beta_s = 30^0$ $\beta_r = 36^0$
Average Torque	Nm	19,441	18,759	18,342	18,465	18,003	19,052	19,281
Torque ripple	Nm	12,463	6,999	5,182	5,363	5,915	8,942	10,772
% Torque ripple	%	64,176	37,374	28,281	29,065	32,962	47,037	55,953
Input Power	W	31591	30502	30462	30071	29275	30947	31326
Output Power	W	29804	28773	28712	28356	27578	29203	29563
Efficiency	%	94,344	94,330	94,256	94,298	94,204	94,365	94,371
Shaft Torque	Nm	18,974	18,317	18,279	18,052	17,557	18,591	18,820

The table 2 shows that the 6/4 SRM exhibits significant torque ripple. When the rotor pole arc aligns with the stator pole arc ($\beta_S = 30^0$, $\beta_R = 30^0$), the maximum torque ripple occurs, at 64.17%. Continuing to increase the rotor pole arc results in both average torque and torque ripple decreasing compared to the case where the stator and rotor pole arcs are equal. Initially, as the rotor pole arc increases from 31^0 to 34^0, the average torque decreases by 3.5% to 7.4%, and the torque ripple also decreases significantly, by up to 35.9%. However, when the rotor pole arc continues to increase, $\beta r = 35^0$ and beyond, the average torque increases by about 5% but the torque ripple increases substantially, reaching 27% compared to the cases with rotor pole arcs of $\beta r = 32^0$, 33^0, and 34^0. Among these cases, the scenario with a rotor pole arc of $\beta r = 32^0$ has the lowest torque ripple.

Torque speed characteristics of SRM 6/4 when $\beta_r = 30^0$, $\beta_r = 32^0$ với I = 200A as Fig 12.

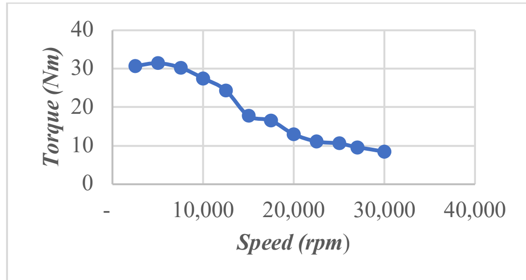

Fig. 12. *Torque speed characteristics*

CONCLUSIONS

The research article has analyzed the variations in average torque as the stator and rotor pole angles change. Additionally, the article has investigated the harmonic waves of torque in the 6/4 Switched Reluctance Motor (SRM) and the changes in the amplitude of torque harmonic waves with varying rotor pole angles. Based on the analysis results, the article has determined the optimal stator and rotor pole angles to maximize the average torque and minimize torque ripple in the design of the electromagnetic configuration for the 6/4 SRM.

REFERENCES

[1] Sanada, M., Morimoto, S., Takeda, Y., et al.: 'Novel rotor pole design of switched reluctance motors to reduce the acoustic noise'. 2000 IEEE Industry Applications Conf., Rome, Italy, 2000, pp. 107–113

[2] Lee, D.H., Pham, T.H., Ahn, J.W.: 'Design and operation characteristics of four-two pole high-speed SRM for torque ripple reduction', IEEE Tran. Ind. Electron., 2013, 60, (9), pp. 3637–3643

[3] T. J. E. Miller, *Switched Reluctance Motors and Their Control*. Hillsboro, OH : Magna Physics Publishing ; Oxford : Clarendon Press, 2001.

[4] R. Krishnan, *Switched reluctance motor drives: Modeling, simulation, analysis, design, and applications*, 2017.

[5] J. T. Shi, X. Liu, D. Wu, and Z. Q. Zhu, Fellow, Influence of Stator and Rotor Pole Arcs on Electromagnetic Torque of Variable Flux Reluctance Machines, IEEE Transactions on magnetics, vol. 50, no. 11, November 2014.

[6] Bui Minh Dinh, Dinh Hai Linh, Dang Quoc Vuong, *Electromagnetic Torque Analysis of SRM 12/8 by Rotor/Stator Pole Angle,* Engineering, Technology & Applied Sciece Research, Vol 11, No3, pp 7187 -7190, 2021.

[7] N. K. Sheth and K. R. Rajagopal, "*Torque profiles of a switched reluctance motor having special pole face shapes and asymmetric stator poles*," *IEEE Transactions on Magnetics*, vol. 40, no. 4 II, pp. 2035–2037, 2004, doi: 10.1109/TMAG.2004.829841.

[8] Bingni Qu, Jiancheng Song, Jianbin Zheng, *Torque Ripple Minimization in Switched Reluctance Machine by Pole Arcs Design*, 4th IEEE Conference on Industrial Electronics and Applications, DOI: 10.1109/ICIEA.2009.5138610, 2009.

[9] Grace Firsta Lukman; Kwang-Il Jeong; Jin-Woo Ahn *, Design and Analysis of T-L Stator-Rotor Pole Combination of Outer-Rotor SRM Considering Dynamic Torque Characteristic*, 25th International Conference on Electrical Machines and Systems (ICEMS), doi:10.1109/ICEMS56177.2022.9983004, 2022.

[10] M. Yildirim and H. Kurum, "*Influence of Poles Embrace on In-Wheel Switched Reluctance Motor Design*," Proc. - 2018 IEEE 18th Int. Conf. Power Electron. Motion Control. PEMC 2018, pp. 562–567, 2018.