FROM 1D TO 3D MODELING OF MAGNETIC CIRCUITS BY A SUBPROBLEM FINITE ELEMENT TECHNIQUE

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ABSTRACT
In this paper, the subproblem finite element technique is developed for model refinements of magnetic circuits in electrical machines. The method allows a complete problem composed of local and global fields to split into lower dimensions with independent meshes. Sub models are performed from 1-D to 2-D as well as 3-D models, linear to nonlinear problems, without depending on the meshes of previous subproblems. The subproblems are contrained via interface and boundary conditions. Each subproblem is independently solved on its own domain and mesh without depending on the meshes of previous subproblems, which facilitates meshing and may increase computational efficiency on both local fields and global quantities. The complete solution is then defined as the sum of the subproblem solutions by a superposition method.
Keywords: *Eddy current; mangetic fields; finite element method; subproblem method; magnetic circuits.*

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MÔ HÌNH HOÁ MẠCH TỪ BÀI TOÁN 1D ĐẾN 3D BẰNG PHƯƠNG PHÁP MIỀN NHỎ HỮU HẠN

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TÓM TẮT
Trong bài báo này, phương pháp miền nhỏ hữu hạn được phát triển cho mô hình của mạch từ trong máy điện. Phương pháp cho phép chia một bài toán hoàn chỉnh bao gồm các trường cục bộ và toàn cục thành các bài toán nhỏ có kích thước nhỏ hơn với các lưới độc lập. Do đó, các mô hình nhỏ có thể được thực hiện từ bài toán 1-D đến 2-D đến 3-D, từ bài toán tuyến tính đến bài toán phi tuyến mà không phụ thuộc vào lưới của các bài toán nhỏ trước đó. Các bài toán nhỏ được ràng buộc thông qua các điều kiện biên và điều kiện liên kết bề mặt. Mỗi một bài toán nhỏ được giải trên miền và lưới riêng của nó mà không ảnh hưởng tới miền khác hoặc trước đó, điều này giúp cho việc chia lưới thuận lơi hơn cũng như làm tăng hiệu quả tính toán cho cả các đại lượng trường cục bộ và trường toàn cục. Sau đó, nghiệm của bài toán hoàn chỉnh được xác định như là tập hợp nghiệm của các bài toán nhỏ thông qua phương pháp xếp chồng nghiệm.
Keywords: *Dòng điện xoáy; từ trường; phương pháp phần tử hữu hạn; phương pháp miền nhỏ hữu hạn; mạch từ.*

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1. Introduction

The methodology of supproblem method (SPM) has been developed by many authors and, up to now, only applied for actual problems [1]–[8]. In this paper, the step-by-step SPM is extended for the efficient numerical modeling of magnetic circuits, with defining model refinements: change from 1-D to 2-D as well as 3-D models, change from linear to nonlinear of materials, change from perfect to real materials, and change from statics to dynamics. The method allows to benefit from previous computations instead of starting a new complete finite elemento (FE) solution for any geometrical, physical or model variation. It also allows different problem - adapted meshes and computational efficiency due to the reduced size of each subproblem (SP). Each SP can be defined via combinations of surface sources (SSs) and volume sources (VSs). SSs express changes of interface conditions (ICs) and boundary conditions (BCs), and VSs express changes of material properties from this problem to others [1]-[9]. The method is validated on a test problem. Its main advantages are pointed out.

2. Subproblem apporach

2.1 Methodology

A complete problem is split into a series of SPs that define a sequence of changes, with the complete solution being replaced by the sum of the SP solutions. Each SP is defined in its particular domain, generally distinct from the complete one and usually overlapping those of the other SPs. At the discrete level, this aims at descreting the problem complexity and at allowing distinct meshes with suitable refinements. No remeshing is necessary when adding some regions.

2.2 Canonical Magnetodynamic Problem

A canonical magnetodynamic problem i, to be solved at step i of the SPM, is defined in a domain Ω_i, with boundary $\partial\Omega_i = \Gamma_i = \Gamma_{h,i} \cup \Gamma_{b,i}$. The eddy current conducting part of Ω_i is denoted $\Omega_{c,i}$ and the non-conducting one $\Omega_{c,i}^C$, with $\Omega_i = \Omega_{c,i} \cup \Omega_{c,i}^C$. Stranded inductors belong to $\Omega_{c,i}^C$, whereas massive inductors belong to $\Omega_{c,i}$. The equations, material relations and BCs of problem i are [8] - [11]

$$\text{curl}\, \boldsymbol{h}_i = \boldsymbol{j}_i,\ \text{div}\, \boldsymbol{b}_i = 0\ ,\ \text{curl}\, \boldsymbol{e}_i = -\partial t\, \boldsymbol{b}_i, \quad \text{(1a-b-c)}$$

$$\boldsymbol{h}_i = \mu_i^{-1}\boldsymbol{b}_i + \boldsymbol{h}_{s,i},\ \boldsymbol{j}_i = \sigma_i \boldsymbol{e}_i + \boldsymbol{j}_{s,i} \quad \text{(2a-b)}$$

where $\boldsymbol{h}_i$ is the magnetic field, $\boldsymbol{b}_i$ is the magnetic flux density, $\boldsymbol{e}_i$ is the electric field, $\boldsymbol{j}_i$ is the electric current density, μ_i is the magnetic permeability, σ_i is the electric conductivity and $\boldsymbol{n}$ is the unit normal exterior to Ω_i.

The fields $\boldsymbol{h}_{s,i}$ and $\boldsymbol{j}_{s,i}$ in (2a-b) are VSs. With the SPM, $\boldsymbol{h}_{s,i}$ is also used for expressing changes of permeability and $\boldsymbol{j}_{s,i}$ for changes of conductivity. For changes in a region, from μ_q and σ_q for problem ($i = q$) to μ_k and σ_k for problem ($i = p$), the associated VSs $\boldsymbol{h}_{s,i}$ and $\boldsymbol{j}_{s,i}$ are [2-5]

$$\boldsymbol{h}_{s,p} = (\mu_p^{-1} - \mu_q^{-1})\boldsymbol{b}_q, \quad (3)$$

$$\boldsymbol{j}_{s,p} = (\sigma_p - \sigma_q)\boldsymbol{e}_q, \quad (4)$$

for the total fields to be related by $\boldsymbol{h}_q + \boldsymbol{h}_p = (\mu_p^{-1}(\boldsymbol{b}_q + \boldsymbol{b}_p)$ and $\boldsymbol{j}_q + \boldsymbol{j}_p = \sigma_p(\boldsymbol{e}_q + \boldsymbol{e}_p)$.

Equations (1b-c) are fulfilled via the definition of a magnetic vector potential $\boldsymbol{a}_i$ and an electric scalar potential v_i, leading to the $\boldsymbol{a}_i$-formulation, with

$$\text{curl}\, \boldsymbol{a}_i = \boldsymbol{b}_i,\ \boldsymbol{e}_i = -\partial_t \boldsymbol{a}_i - \text{grad}\, v_i = \partial_t \boldsymbol{a}_i - \boldsymbol{u}_i. \quad \text{(5a-b)}$$

The Gauss and Faraday equations are strongly satisfied. The $\boldsymbol{a}_i$ weak formulation of the magnetodynamic problem is then obtained from the weak form of the Ampere equation, i.e. [1] - [9]

$$\left(\mu_i^{-1}\text{curl}\, \boldsymbol{a}_i, \text{curl}\, \boldsymbol{a}'\right)_{\Omega_i} + (\sigma_i \partial_t \boldsymbol{a}_i, \boldsymbol{a}')_{\Omega_{c,i}} + (\sigma_i \boldsymbol{u}_i, \boldsymbol{a}')_{\Omega_{c,i}} + \langle \boldsymbol{n} \times \boldsymbol{h}_i, \boldsymbol{a}' \rangle_{\Gamma_{h,i}-\gamma_i} + \left(\boldsymbol{h}_{s,i}, \text{curl}\, \boldsymbol{a}'\right)_{\Omega_{c,i}} + \langle [\boldsymbol{n} \times \boldsymbol{h}_i]_{\gamma_i}, \boldsymbol{a}' \rangle_{\gamma_i} = \left(\boldsymbol{j}_{s,i}, \boldsymbol{a}'\right)_{\Omega_{s,i}},$$

$\forall\, \boldsymbol{a}' \in F_i^1(\Omega_i)$, (6)

where $F_i^1(\Omega_i)$ is a curl-conform function space defined on Ω_i, gauged in $\Omega_{c,i}^C$, and containing the basis functions for $\boldsymbol{a}_i$ as well as for the test function $\boldsymbol{a}'$ (at the discrete level, this space is defined by edge FEs; the gauge is based on the tree-co-tree technique); $(\cdot,\cdot)$ and $<\cdot,\cdot>$ respectively denote a volume integral Ω_i in and a surface integral on Γ_i of the product of their vector field arguments.

The term $\langle \boldsymbol{n} \times \boldsymbol{h}_i, \boldsymbol{a}' \rangle_{\Gamma_{h,i}-\gamma_i}$ in (6) is generally zero for classical homogenous BC. If nonzero, it defines a possible SS that account for particular phenomena occurring in the thin region between γ_i^+ and γ_i^-[2]- [5]. The trace $[\boldsymbol{n} \times \boldsymbol{h}_i]_{\gamma_i}$ in (6) is fixed as a discontinuity on the both side of γ_i, i.e.,

$$[\boldsymbol{n} \times \boldsymbol{h}_i]_{\gamma_i} = \boldsymbol{n}_{\gamma_i} \times \boldsymbol{h}_{\gamma_i}|_{\gamma_i^+} - \boldsymbol{n}_{\gamma_i} \times \boldsymbol{h}_{\gamma_i}|_{\gamma_i^-}. \quad (7)$$

This is the case when some field traces in a SP_p ($i = p$) are forced to be discontinuous. The continuity has to be recovered after a correction via a SP_k ($i = k$). The SSs in SP_k are thus to be fixed as the opposite of the trace solution of SP_p.

Each SP_p is to be constrained via the so defined VSs and SSs from parts of solutions of other SPs. This is a key element of the SPM, offering a wide variety of possible corretions, as shown hereafter.

2.3 Projections of Solutions between Meshes

As presented in the previous part, some parts of a previous solution $\boldsymbol{a}_p$ serve as sources in a subdomain $\Omega_{s,k} \subset \Omega_k$ of the current problem SP_k. At the discrete level, this means that this source quantity $\boldsymbol{a}_p$ has to be expressed in the mesh of problem SP_k, while initially given in the mesh of problem SP_p. This can be done via a projection method [2-4] of its curl limited to $\Omega_{s,k}$, i.e.

$$\left(\mathrm{curl}\, \boldsymbol{a}_{p,k-proj}, \mathrm{curl}\, \boldsymbol{a}_k'\right)_{\Omega_k} = \left(\mathrm{curl}\, \boldsymbol{a}_p, \mathrm{curl}\, \boldsymbol{a}_k'\right)_{\Omega_k}, \forall\, \boldsymbol{a}_k' \in F_k^1(\Omega_k) \quad (8)$$

where $F_k^1(\Omega_{s,k})$ is a gauged curl-conform function space for the k-projected source $\boldsymbol{a}_{p,k-proj}$ (the projection of $\boldsymbol{a}_p$ on mesh SP_k) and the test function $\boldsymbol{a}_k'$. Directly projecting $\boldsymbol{a}_p$ (not its curl) would result in significant numerical inaccuracies when evaluating its curl.

2.4 SSs for Changes of ICs

As for IC in (7), it is to be weakly expressed via the last integral in (4), with $\gamma_i = \Gamma_p = \Gamma_k$. The so involved trace $\boldsymbol{n}_{\gamma_p} \times \boldsymbol{h}_{\gamma_p}|_{\gamma_p^+}$ gains at being kept in a surface integral, that originally appears in (6) for SP_p on Γ_p now restricted to $\Gamma_p = \Gamma_k$. It can then be naturally expressed via the other (volume) integrals in (6), i.e.

$$\langle [\boldsymbol{n} \times \boldsymbol{h}_p]_{\gamma_k=\Gamma_k}, \boldsymbol{a}' \rangle_{\gamma_k=\Gamma_k} = \langle \boldsymbol{n} \times \boldsymbol{h}_p, \boldsymbol{a}' \rangle_{\Gamma_p^+} = \left(\mu_p^{-1} \mathrm{curl}\, \boldsymbol{a}_p, \mathrm{curl}\, \boldsymbol{a}'\right)_{\Omega_k=\Omega_p}. \quad (9)$$

At the discrete level, the volume integral in (8) is limited to one single layer of Fes touching Γ_p^+, because it involves only the assoiciated traces $\boldsymbol{n} \times \boldsymbol{h}_p|_{\gamma_k^+}$. The source $\boldsymbol{a}_p$, initially in mesh of SP_p, has to be projected in mesh of SP_k via a (8), with $\Omega_{s,k}$ limited to the FE layer, which thus decreases the computational effort of the projection process.

2.5 VSs for Changes of Material Properties

A change of material properties from SP_q to SP_p is taken into account in (3) and (4) via the volume integrals $\left(\boldsymbol{h}_{s,i}, \mathrm{curl}\, \boldsymbol{a}'\right)_{\Omega_{c,i}}$ and $\left(\boldsymbol{j}_{s,i}, \boldsymbol{a}'\right)_{\Omega_{s,i}}$ in (6). The VSs $\boldsymbol{h}_{s,i}$ and $\boldsymbol{j}_{s,i}$ are respectively given by (3) and (4). At the discrete level, the source primal quantity of SP_q, initially given in mesh of SP_q, is projected in the mesh of SP_p via (8), with $\Omega_{s,i}$ limited to the modified regions.

3. Application test

The SPM can be applied for coupling soulutions of various dimensions, starting from simplied models, based on ideal flux tubes defining 1-D models, that evolve towards 2-D and 3-D accurated models.

Series connections of models of lower dimensions are direct applications requiring such changes. A violation of ICs when connecting two models can be corrected via SSs in opposition to the unwanted discontinuitities.

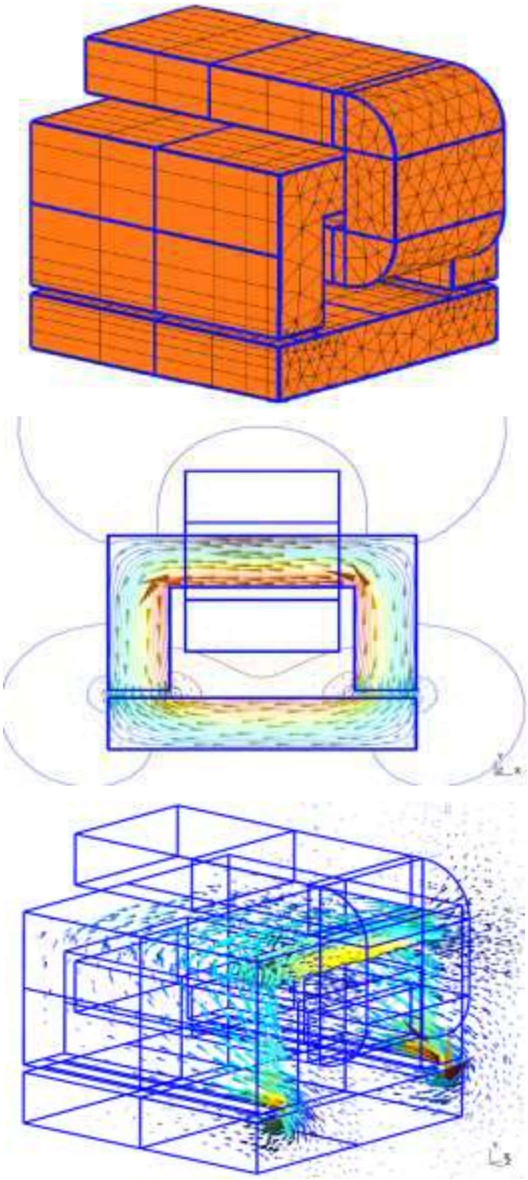

Figure 1. *3-D model of an electromagnet (top), 2-D cross section and solution (magnetic flux density and field lines) (middle), 3-D correction of the magnetic flux density (bottom)*

The first test is shown in Figure 1. Change from ideal to real flux tubes can be presented in a dimension change, e.g. from 2-D to 3-D: a 2-D solution is first considered as limited to a certain thickness in the third dimension, with a zero field outside; on the other side, another independente SP is solved. Changes of ICs corrections of the flux linkage, from 1-D to 3-D, are shown in Figure 2.

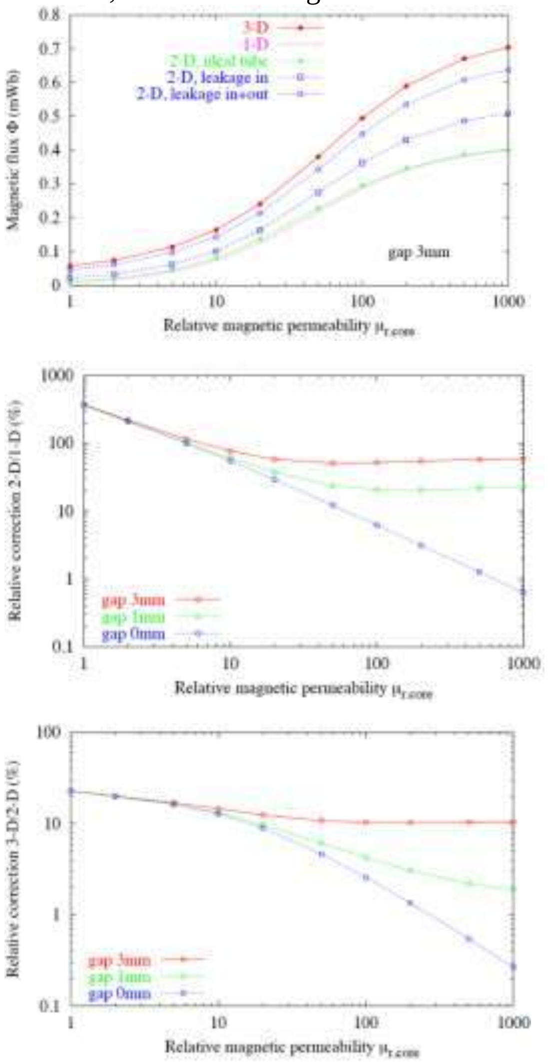

Figure 2. *Inductor flux linkage versus the core magnetic permeability (air gap thickness of 3 mm) updated after each model refinement (top); flux linkage relative correction from 1-D to 2-D models (middle) and from 2-D to 3-D models (bottom) versus the core magnetic permeability for different air gap thickness*

The second test is considered with the changes from ideal to real flux tubes to real materials (Figure 3). A SP_1 (i =1) can first consider ideal tubes [5], i.e. surrounded by perfect flux walls through which BC is zero and $\boldsymbol{b}_1$ and $\boldsymbol{h}_1$ outside are zero. The complementary trace $\boldsymbol{n} \times \boldsymbol{h}_1|_{\gamma_1}$ is unknown and non-zero. Consequntly, a change to permeable fulx wall defines a SP_2 (i =2) with SSs opposed to this non-zero trace. This change can be done simultaneously with a material change (Figure 4): a leakage flux solution $\boldsymbol{b}_3$ can complete an ideal distribution $\boldsymbol{b}_1$ while knowing the source $\boldsymbol{b}_2$ proper to the inductor; this allows independent overlapping meshes for both source and reaction fields.

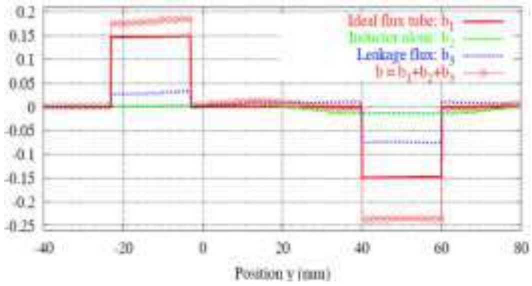

Figure 3. *Field lines in the ideal flux tube ($\boldsymbol{b}_1$, $\mu_{1,core} = 100$), for the inductor alone ($\boldsymbol{b}_2$), for the leakage flux ($\boldsymbol{b}_3$) and for the total field ($\boldsymbol{b} = \boldsymbol{b}_1 + \boldsymbol{b}_2 + \boldsymbol{b}_3$) (left to right)*

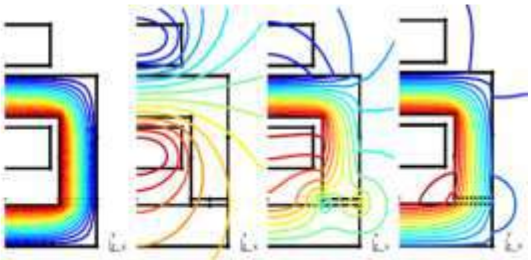

Figure 4. *Magnetic flux density through the horizontal legs of the electromagnet for the ideal flux tube ($\boldsymbol{b}_1$), for the inductor alone ($\boldsymbol{b}_2$), for the leakage flux ($\boldsymbol{b}_3$) and for the total field ($\boldsymbol{b} = \boldsymbol{b}_1 + \boldsymbol{b}_2 + \boldsymbol{b}_3$)*

4. Conclusions

The developed SP FE method splits magnetic problems into SPs of lower complexity with regard to meshing operations and computational aspects. This allows a natural propression from simple to more elaborate models, from 1-D to 3-D geometries, is thus possilble, while quantifying the gain given by each model refinement and justifying its utility. It can be also a good step to help in education with a progessive understanding of the various aspects of magnetic circuit design for the future work.

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