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OPTIMUM DESIGN OF THE TUNED MASS DAMPER TO REDUCE THE TORSIONAL VIBRATION OF THE MACHINE SHAFT SUBJECTED TO RANDOM EXCITATION

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ABSTRACT

In practice, torsional vibration plays an important role in degrading the safety and stability of structures under the effects of torsional torque such as machine shafts, turbine shafts, etc. However, the study on the design of a tuned mass damper (TMD) for shafts is very limited in the literature. In case of the shaft is excited by random excitation, there has been no study to reduce the torsional vibration of the shaft. This paper presents an analytical method to determine optimal parameters of the tuned mass damper (TMD), such as the ratio between natural frequency of TMD and the shaft (tuning ratio), the ratio of the viscous coefficient of TMD (damping ratio). Two novel findings of the present study are summarized as follows. First, the optimal parameters of TMD for the shafts are given by using the minimum quadratic torque method. Next, a numerical simulation is done for an example of the machine shaft to validate the effectiveness of the results obtained in this work.

Keywords: *Tuned mass damper, torsional vibration, pendulum, machine shaft, minimum quadratic torque, random excitation.*

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THIẾT KẾ TỐI ƯU BỘ HẤP THỤ DAO ĐỘNG ĐỂ GIẢM DAO ĐỘNG XOẮN CHO TRỤC MÁY CHỊU TÁC DỤNG CỦA LỰC KÍCH THÍCH NGẪU NHIÊN

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TÓM TẮT

Trong thực tế, dao động xoắn đóng vai trò quan trọng trong việc làm giảm sự an toàn và ổn định của các cơ cấu dưới tác động của mô-men xoắn, ví dụ như trục máy, trục tuabin,... Tuy nhiên, nghiên cứu về thiết kế bộ hấp thụ dao động (TMD) cho trục lại rất hạn chế trong các tài liệu. Trong trường hợp trục chịu tác dụng bởi lực kích thích ngẫu nhiên, chưa có nghiên cứu nào giảm dao động xoắn cho trục. Bài báo này trình bày một phương pháp phân tích để xác định các tham số tối ưu của bộ hấp thụ dao động (TMD), chẳng hạn như tỷ số giữa tần số tự nhiên của bộ TMD và trục (tỷ số điều chỉnh), tỷ số cản nhớt của TMD (tỷ lệ giảm chấn). Hai phát hiện mới của nghiên cứu này được tóm tắt như sau. Đầu tiên, các tham số tối ưu của TMD cho các trục được đưa ra bằng cách sử dụng phương pháp cực tiểu mô men bậc hai. Tiếp theo, một mô phỏng số được thực hiện cho một ví dụ về trục máy để xác nhận tính hiệu quả của các kết quả thu được trong nghiên cứu này.

Từ khóa: *Bộ hấp thụ dao động, dao động xoắn, con lắc, trục máy, cực tiểu mô men bậc hai, kích thích ngẫu nhiên*

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1. Introduction

The study to reduction of shaft vibration is an important and timely task [1-15]. From the researches in [1-6], the author finds out that there are many studies on the reduction of torsional vibration with or without CPVA (centrifugal pendulum vibration absorber), CDR (centrifugal delay resonant) and DVA (dynamic vibration absorbers). But these studies just focus on the stability and motion control of oscillating absorber systems, and it has no research that uses the optimum arithmetic calculations to calculate the optimal parameters of absorbers for the main system under torsional vibration. There are some studies to reduce the torsional vibration of the shaft by setting an absorber in different forms. In these studies, authors also focused on determining optimal parameters for the DVA (or TMD) design. In [7, 8] have determined the optimal parameters of the absorbers set in the form of expressions, reduce the torsional vibration for the shaft from the effects of different excitation. Vu et al. [7] have determined the optimal parameters of the dynamic vibration absorber (DVA) in case the shaft is subject to harmonic excitation, under the harmonic excitation, the fixed point method is used to determine the optimal parameters. In case the shaft is subject to impact excitation, Chinh [8] has determined the optimal parameters of the tuned mass damper (TMD) to reduce the torsional vibration of the shaft by using the principle of minimum kinetic energy. The results were given by

$$\alpha_{opt}^{MKE} = \frac{1}{1+2\mu\gamma^2} \quad ; \; \xi_{opt}^{MKE} = \gamma\sqrt{\frac{\mu}{2(1+2\mu\gamma^2)}} \tag{1}$$

In case of the shaft is excited by random excitation. To the best knowledge of the author, there has been no study on the TMD using minimum quadratic torque method for the shaft. Perhaps a reason is that the calculations in this case is too complicated. This paper presents minimization of quadratic torque to determine the optimal parameters of the passive mass-spring-pendulum-type tuned mass dampers (TMD) such as tuning ratio and damping ratio. The results indicate that the effectiveness in torsional vibration reduction in case of the shaft is excited by random excitation. The minimum quadratic torque method in Reference [9] is used for determining the optimal parameters of the TMD.

2. Shaft modelling and equations of vibration

Fig. 1 illustrates a pendulum type TMD attached to a shaft. The shaft has the torsion spring coefficient is k_t. A pendulum type TMD has a concentrated mass $2m$ at the top, spring constant k_m and damping constant c, the length of beam is $2L$ and the length mass is $2m_t$. The TMD is installed in the shaft through a mass rotor (the rotor is mounted rigidly to the shaft), with radius ρ, mass M. By considering the whole system, one can conclude that the system is completely determined if two coordinates φ_1 and φ_2 are given. Thus, independent generalized coordinates are absolute angle of rotation of the rotor φ_1 and relatively angel of rotation of the TMD to the rotor φ_2.

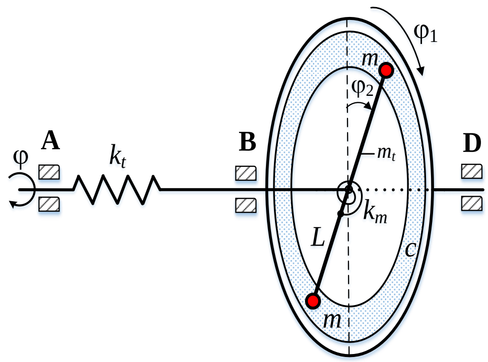

Figure 1. *Shaft Model with Installed TMD*

By applying the second-order Lagrange equation, the differential equations for the vibration system in Fig. 1 is established as [8]

$$\begin{cases} (M\rho^2 + \frac{2}{3}m_t L^2 + 2mL^2)\ddot{\theta} + 2(\frac{1}{3}m_t L^2 + mL^2)\ddot{\varphi}_2 = M(t) - k_t\theta \\ 2(\frac{1}{3}m_t L^2 + mL^2)\ddot{\theta} + 2(\frac{1}{3}m_t L^2 + mL^2)\ddot{\varphi}_2 = -k_m\varphi_2 - 2cL^2\dot{\varphi}_2 \end{cases} \quad (2)$$

where: $\varphi_1 - \varphi = \theta$ (3)

In which: θ is torsional angle of the shaft, $\dot{\theta}$ is angular velocity of the shaft, $\ddot{\theta}$ is angular acceleration of the shaft.

Eq. (2) can be used in the design of the TMD

3. Determine optimal parameters of the TMD

The minimization of quadratic torque (MQT) applied to the impactor of the random excitation moment with white noise $M(t)$ has the spectral density S_f.

We introduce [8]

$$\mu = \frac{m + m_t/3}{M}, \omega_D = \sqrt{\frac{k_t}{M\rho^2}}, \omega_d = \sqrt{\frac{k_m}{2(m + m_t/3)L^2}},$$
$$\xi^{MQT} = \frac{c}{2(m + \frac{m_t}{3})\omega_d}, \quad \alpha^{MQT} = \frac{\omega_d}{\omega_D}, \quad \gamma = \frac{L}{\rho} \quad (4)$$

In which, ω_D is the natural frequency of the shaft, ω_d and ξ^{MQT} respectively are the natural frequency and the viscous damping ratio of the TMD, μ is the TMD mass ratio, α^{MQT} is the tuning ratio of TMD, γ is ratio between length of pendulum and radius of gyration of rotor.

The matrix equations (1, 2) can be rewritten as

$$\mathbf{M}^{MQT}\ddot{\mathbf{q}} + \mathbf{C}^{MQT}\dot{\mathbf{q}} + \mathbf{K}^{MQT}\mathbf{q} = \mathbf{F}^{MQT} \quad (5)$$

Where $\mathbf{q} = \{\theta \quad \varphi_2\}^T$ (6)

The mass matrix, viscous matrix, stiffness matrix and excitation force vector can be derived as

$$\mathbf{M}^{MQT} = \begin{bmatrix} 1+2\mu\gamma^2 & 2\mu\gamma^2 \\ 1 & 1 \end{bmatrix}; \mathbf{C}^{MQT} = \begin{bmatrix} 0 & 0 \\ 0 & 2\xi^{MQT}\alpha^{MQT}\omega_D \end{bmatrix};$$
$$\mathbf{K}^{MQT} = \begin{bmatrix} \omega_D^2 & 0 \\ 0 & \omega_D^2(\alpha^{MQT})^2 \end{bmatrix}; \quad \mathbf{F}^{MQT} = \begin{Bmatrix} \frac{M(t)}{M\rho^2} \\ 0 \end{Bmatrix} \quad (7)$$

From the oscillator equation in matrix form (5), the equation of state is constructed:

$$\dot{\mathbf{y}}(t) = \mathbf{B}\mathbf{y}(t) + \mathbf{H}_f M(t) \quad (8)$$

Where: $\mathbf{y(t)}$ is the state vector corresponding to the response of the system and is defined as follows:

$$\mathbf{y} = \{\theta \quad \varphi_2 \quad \dot{\theta} \quad \dot{\varphi}_2\}^T \quad (9)$$

The system matrix $\mathbf{B}$ has the form

$$\mathbf{B}=\begin{bmatrix} \mathbf{0} & \mathbf{E} \\ -(\mathbf{M}^{MQT})^{-1}\mathbf{K}^{MQT} & -(\mathbf{M}^{MQT})^{-1}\mathbf{C}^{MQT} \end{bmatrix} \tag{10}$$

where $\mathbf{E}$ is the matrix unit

Hence the $\mathbf{B}$ matrix can be obtained as

$$\mathbf{B}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_D^2 & 2\mu\gamma^2(\alpha^{MQT})^2\omega_D^2 & 0 & 4\mu\gamma^2(\xi^{MQT})(\alpha^{MQT})\omega_D \\ \omega_D^2 & -(1+2\mu\gamma^2)(\alpha^{MQT})^2\omega_D^2 & 0 & -2(1+2\mu\gamma^2)(\xi^{MQT})(\alpha^{MQT})\omega_D \end{bmatrix} \tag{11}$$

The matrix of excitation force is obtained as [12]

$$\mathbf{H}_f M(t)=\begin{bmatrix} 0 \\ (\mathbf{M}^{MQT})^{-1}\mathbf{F}^{MQT} \end{bmatrix} \quad \rightarrow \mathbf{H}_f=\begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{M\rho^2} \\ -\dfrac{1}{M\rho^2} \end{bmatrix} \tag{12}$$

The quadratic torque matrix $\mathbf{P}$ is a solution of the Lyapunov equation [9]

$$\mathbf{BP}+\mathbf{PB}^T+S_f\mathbf{H}_\mathrm{f}\mathbf{H}_\mathrm{f}^T=\mathbf{0} \tag{13}$$

Substituting Eqs. (11) and (12) into Eq.(13), The matrix $\mathbf{P}$ can be determined as:

$$\mathbf{P}=\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \\ P_{41} & P_{42} & P_{43} & P_{44} \end{bmatrix} \tag{14}$$

where

$$P_{11}=\frac{1}{2}\frac{S_f\left[\frac{1}{4}+(\alpha^{MQT})^4(\frac{1}{2}+\mu\gamma^2)^2+(\mu(2(\xi^{MQT})^2-\frac{1}{2})\gamma^2+(\xi^{MQT})^2-\frac{1}{2})(\alpha^{MQT})^2\right]}{\mu\gamma^2(\xi^{MQT})(\alpha^{MQT})\Omega_D^3M^2\rho^4} \tag{15}$$

$$P_{12}=\frac{\left[2(\alpha^{MQT})^2\gamma^2\mu+(\alpha^{MQT})^2-1\right]S_f}{8\mu\gamma^2\xi^{MQT}\alpha^{MQT}\omega_D^3M^2\rho^4}\;;\; P_{13}=0\;;\; P_{14}=\frac{S_f}{4\omega_D^2M^2\rho^4\mu\gamma^2} \tag{16}$$

$$P_{21}=\frac{\left[2(\alpha^{MQT})^2\gamma^2\mu+(\alpha^{MQT})^2-1\right]S_f}{8\mu\gamma^2\xi^{MQT}\alpha^{MQT}\omega_D^3M^2\rho^4}\;;\quad P_{22}=\frac{S_f}{8\alpha^{MQT}\mu\gamma^2\xi^{MQT}\omega_D^3M^2\rho^4} \tag{17}$$

$$P_{23}=-\frac{S_f}{4\omega_D^2M^2\rho^4\mu\gamma^2}\;;\; P_{24}=0\;;\; P_{31}=0\;;\; P_{32}=-\frac{S_f}{4\omega_D^2M^2\rho^4\mu\gamma^2} \tag{18}$$

$$P_{33}=\frac{1}{8}\frac{S_f\left[2(\alpha^{MQT})^4\gamma^2\mu+(\alpha^{MQT})^4+4(\alpha^{MQT})^2(\xi^{MQT})^2-2(\alpha^{MQT})^2+1\right]}{\mu\gamma^2\xi^{MQT}\alpha^{MQT}\omega_DM^2\rho^4} \tag{19}$$

$$P_{34}=\frac{1}{8}\frac{S_f\left[(\alpha^{MQT})^2-1\right]}{\mu\gamma^2\xi^{MQT}\alpha^{MQT}\omega_D M^2\rho^4}\quad;\quad P_{41}=\frac{S_f}{4\omega_D^2 M^2\rho^4\mu\gamma^2}\quad;\quad P_{42}=0 \tag{20}$$

$$P_{43}=\frac{1}{8}\frac{S_f\left[(\alpha^{MQT})^2-1\right]}{\mu\gamma^2\xi^{MQT}\alpha^{MQT}\omega_D M^2\rho^4}\quad;\quad P_{44}=\frac{1}{8}\frac{S_f}{\omega_D\mu\gamma^2\xi^{MQT}\alpha^{MQT}M^2\rho^4} \tag{21}$$

Minimum conditions are expressed as [9]

$$\left.\frac{\partial P_{11}}{\partial\alpha^{MQT}}\right|_{\alpha_{opt}^{MQT}=\alpha^{MQT}}=0\quad;\quad\left.\frac{\partial P_{11}}{\partial\xi^{MQT}}\right|_{\xi_{opt}^{MQT}=\xi^{MQT}}=0 \tag{22}$$

The optimal parameters of the TMD were determined by solving the Eqs. (15,22)

$$\alpha_{opt}^{MQT}=\alpha^{MQT}=\frac{\sqrt{1+\mu\gamma^2}}{1+2\mu\gamma^2} \tag{23}$$

$$\xi_{opt}^{MQT}=\xi^{MQT}=\frac{\gamma}{2}\sqrt{\frac{\mu(2+3\mu\gamma^2)}{(1+\mu\gamma^2)(1+2\mu\gamma^2)}} \tag{24}$$

Table 1. *The optimal parameters of the tuned mass damper for various mass ratios and ratio between the length of pendulum and radius of gyration of the rotor.*

μ	γ	α_{opt}^{MKE}	ξ_{opt}^{MKE}	α_{opt}^{MQT}	ξ_{opt}^{MQT}
0.01	0.1	0.9998	0.0070	0.9981	0.0071
0.02	0.2	0.9984	0.0196	0.9925	0.0200
0.03	0.3	0.9946	0.0352	0.9836	0.0367
0.04	0.4	0.9874	0.0525	0.9721	0.0563
0.05	0.5	0.9756	0.0707	0.9583	0.0783
0.06	0.6	0.9586	0.0891	0.9429	0.1023
0.07	0.7	0.9358	0.1073	0.9262	0.1277
0.08	0.8	0.9071	0.1249	0.9089	0.1542
0.09	0.9	0.8728	0.1419	0.8914	0.1814
0.10	1.0	0.8333	0.1581	0.8740	0.2087

From equations (23, 24), we obtain the optimal parameters of the TMD to reduce the torsional vibration of the shaft by using the minimum quadratic torque method, which is different from the optimal parameters of the TMD to reduce the torsional vibration of the shaft by using the principle of minimum kinetic energy in the reference [8]. This asserts with a shaft model with installed TMD, but applying different methods to find optimal parameters gives different analytical results.

Table 1 presents the optimal parameters obtained by the two methods according to the various mass ratios and ratio between the length of pendulum and radius of gyration of the rotor. We see that the tuning ratio of TMD is approximately 1, indicating that the optimized TMD has the natural frequency is approximately the natural frequency of the shaft. With the design of this TMD will reduce the vibration of the shaft in the best way.

From table 1, we again assert that the same shaft model with installed TMD is the same with the values of the various mass ratios and ratio between the length of pendulum and radius of gyration of rotor, the optimal parameter is obtained by two methods of the principle of minimum kinetic energy and the minimum quadratic torque method is different. Therefore, when applying the optimum parameters to reduce the vibration of the shaft, we must see the machine shaft is subject to the force of any excitation to apply the method of optimal parameters for the appropriate.

4. Numerical simulation study

In this section, numerical simulation is employed for the system by using the achieved optimal parameters of the TMD, as shown in Eq. (23) and Eq. (24). To demonstrate the above analysis, computations will be performed for a system with parameters given in Table 2 [8].

Table 2. *The input parameters for shaft and TMD*

Parameters	M	ρ	k_t	m_t	M	L
Value	*500kg*	*1.0 m*	*10^5Nm/rad*	*15kg*	*10kg*	*0.9m*

The dimensionless parameters can be calculated and shown in Table 3

Table 3. *Value of the dimensionless parameters*

Parameters	μ	γ
Value	0.03	0.9

Table 4 shows the optimization results calculated by the present method.

Table 4. *The optimal parameters of the TMD*

Optimal Parameters	α_{opt}^{MQT}	ξ_{opt}^{MQT}	c	k_m
Value	0.965	0.108	44.34 Ns/m	4527.35Nm/rad

* Simulation Results

Numerical simulations for torsional vibration of the machine shaft using the Maple are implemented in different operating conditions in case of the shaft is excited by random excitation

$$M(t) = 15.10^8 \frac{\sqrt{2}e^{-\frac{1}{2}\frac{(t-a)^2}{b^2}}}{\sqrt{\pi}b} \text{ (N)}; \; a = 10^{11}; \; b = 10^{10} \quad (25)$$

Table 5 shows the different operating conditions of the machine shaft. In the case 1, simulation is implemented with initial torsional angle of $\theta_0 = 0.002(rad)$. Secondly, simulation results of initial torsional angle $\theta_0 = 0.0(rad)$ and initial angular velocity of $\dot{\theta}_0 = 0.05(rad/s)$ is shown. Finally, simulation study presents the simulation with initial torsional angle $\theta_0 = 0.002(rad)$ and initial angular velocity of $\dot{\theta}_0 = 0.05(rad/s)$.

Table 5. *The different operating conditions of the machine shaft*

Cases	1	2	3
θ_0	$2\times10^{-3}(rad)$	$0.0(rad)$	$2\times10^{-3}(rad)$
$\dot{\theta}_0$	$0.0(rad/s)$	$5\times10^{-2}(rad/s)$	$5\times10^{-2}(rad/s)$

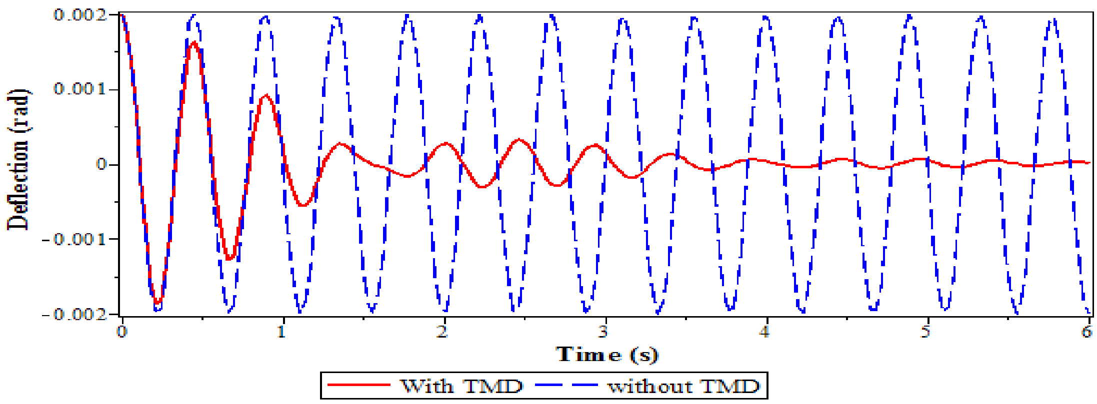

Figure 2. *Vibration of the machine shaft with $\theta_0 = 2\times10^{-3}$ (rad) in the case of random excitation M(t)*

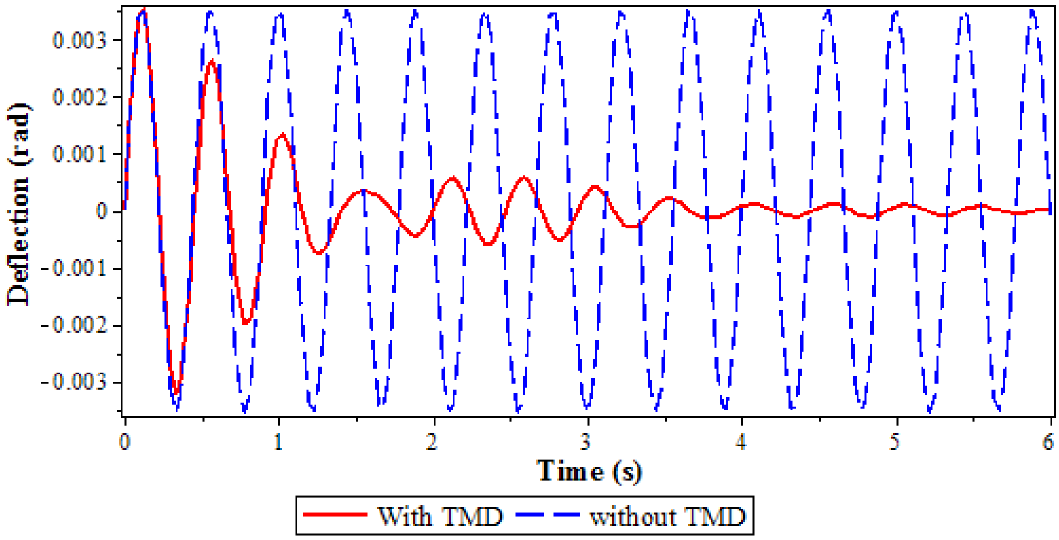

Figure 3. *The vibration of the TMD with $\theta_0 = 2\times10^{-3}(rad)$ and $\dot{\theta}_0 = 5\times10^{-2}(rad/s)$ in the case of random excitation M(t)*

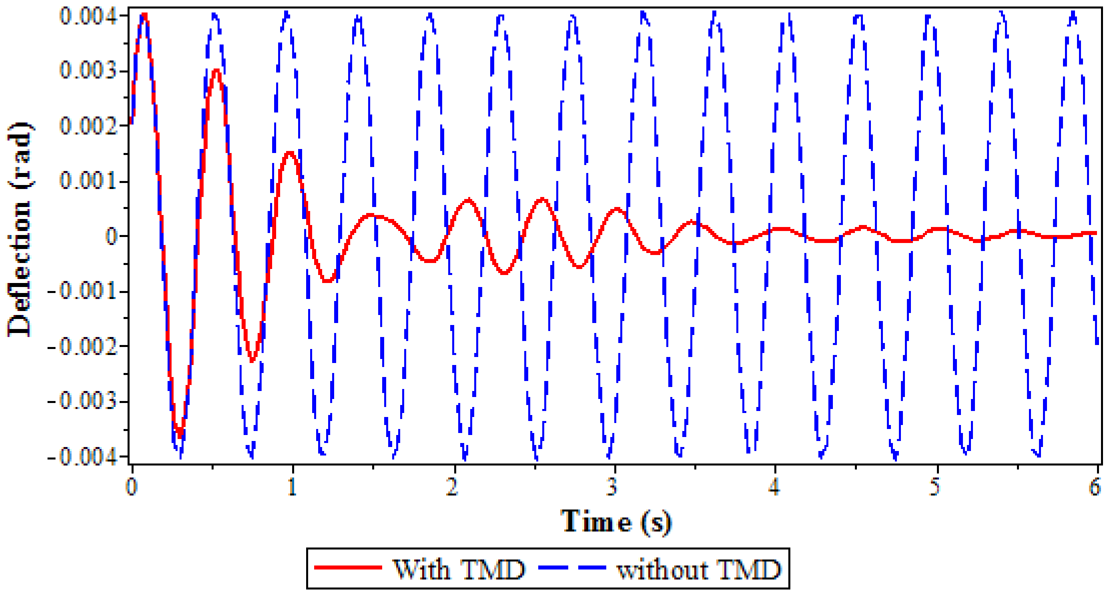

Figure 4. *The vibration of the machine shaft with $\theta_0 = 2\times10^{-3}(rad)$ and $\dot{\theta}_0 = 5\times10^{-2}(rad/s)$ in the case of random excitation M(t)*

The responses of the shart are shown in Figs 2, 3 and 4. The results show that the TMD can reduce the torsional vibration of the shaft in all case.

5. Conclusions

In this paper, the minimization of quadratic torque (MQT) has been examined for a shaft model. The same procedure as in the conventional MQT theory has been used to derive the optimum tuning and damping ratios of the device. It was found that the optimum tuning and damping ratios have an analytical form. Research results are verified by numerical simulation with high reliability. The optimal parameters were determined in analytical form and furthermore lead to the simple explicit formulas in Eqs. (23, 24). Paper has studied, analyzed and evaluated the effect of reducing the vibration of the shaft in the case of without and with TMD is mounted oscillating with the optimal analysis solution found the TMD. From the simulation of the vibration amplitude over time in case of the shaft is excited by random excitation, it was found that the torsional vibration amplitude of the machine shaft when the TMD was installed according to the optimal parameters found by equations (23, 24) was effective in reducing vibration for the machine shaft.

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