MODELLING OF EDDY CURRENT LOSSES BASED ON THE DEVELOPMENT OF MAGNETIC VECTOR POTENTIAL FORMULATIONS

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ABSTRACT

Eddy currents always appear in any conduction regions of all types electrical devices, such as electrical apparatus, the cores of transformers, rotating electrical machines, which are subjected to a time-varying magnetic field variation. This leads to an increase of losses and a descrease of efficiency for electrical devices. However, in other fields, it has the effect of detecting break-downs, such as cracks occuring in metal pipes under the seabed or through mountains, or magnetic inductions, or induction furnaces. In the recent years, many papers have been devoted to the finite element method for solving this problem related to eddy currents in conducting regions. In this litterature, a new finite element method is presented with the development of magnetic vector potential formulations to compute and simulate local fields, such as the magnetic field, magnetic flux density and eddy current losses of magnetodynamic problems. The developed method is validated via a practical problem.

XÂY DỰNG MÔ HÌNH TỔN HAO DÒNG ĐIỆN XOÁY DỰA VÀO SỰ PHÁT TRIỂN CÔNG THỨC VÉC TƠ TỪ THẾ

Bùi Đức Hùng, Đặng Quốc Vương
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TỪ KHÓA

Dòng điện xoáy
Từ trường
Véc tơ từ thế
Bài toán từ động
Phương pháp phần tử hữu hạn

TÓM TẮT

Dòng điện xoáy luôn luôn tồn tại và xuất hiện hầu hết trong các vùng dẫn của các thiết bị điện (cụ thể như: trong khí cụ điện, lõi của máy biến áp, máy điện quay,...) do sự biến thiên của từ trường theo thời gian. Điều này dẫn đến làm tăng tổn hao và giảm hiệu suất vận hành của các thiết bị điện. Tuy nhiên, trong một số lĩnh vực khác, dòng điện xoáy lại có tác dụng nhằm phát hiện các sự cố như các vết nứt của các ống kim loại dưới đáy biển hoặc xuyên qua núi, hoặc có tác dụng đối với bếp từ, lò luyện cao tần… Trong những năm gần đây, một số bài báo đã áp dụng phương pháp phần tử hữu hạn để giải quyết các bài toán từ động liên quan đến dòng điện xoáy trong các vùng dẫn. Trong bài báo này, phương pháp phần tử hữu hạn được đề xuất với sự phát triển của công thức véc tơ từ thế để tính toán và mô phỏng các đại lượng trường như: sự phân bố của từ trường trong miền dẫn và không dẫn từ, sự phân bố của dòng điện xoáy và tổn hao gây trong miền dẫn từ. Sự phát triển của phương pháp được kiểm nghiệm/ứng dụng thông qua bài toán thực tế.

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1. Introduction

Nowadays, eddy current losses are one of the main parts of the total power losses in electrical devices. It seems to be appeared in any conduction regions of all types of electrical apparatus, cores of transformers and rotating electrical machines, which are subjected to a time-varying magnetic field variation. This leads to an increase of losses and a decrease of efficiency for electrical devices [1], [2]. In order to reduce joule power losses due to the eddy currents, many authors have recently used finite element technique to calculate the eddy currents already in the design phase of transformers [3]. This paper is developed based on a two steps to compute eddy current losses and magnetic flux distribution for 2D model. In [4], the **H**-conformal formulations are proposed with edge elements via a subproblem method for solving magnetodynamic problems. Or in [5], authors have used a technique to couple finite element method (FEM) and scalar boundary element method formulations to calculate eddy current losses.

In this research, an expended FEM is presented for computing local fields, such eddy current losses and magnetic flux distributions appearing in electromagnetic problems. The method is herein performed with the weak magnetic vector potential formulations (**A**), where a magnetic flux density (**B**) is in terms of **A.** The method allows to solve directly local fields without taking a magnetic scalar potential quantiy into account as pointed out in [4]. The develoment of method is also validated on a practical problem with the frequency domain.

2. Magnetodynamic finite element problems

2.1. Maxwell's equations

A model of a canonical magnetodynamic problem with a simple connected domain Ω (with boundary $\partial\Omega = \Gamma = \Gamma_h \cup \Gamma_e$) is shown in Figure 1. Where Ω_0 is the non conducting region, parameters μ and σ are the permeability and conductivity, respectively. The excitation magnetic field is generated by the fixed current $\mathbf{J_s}$ in stranded inductors.

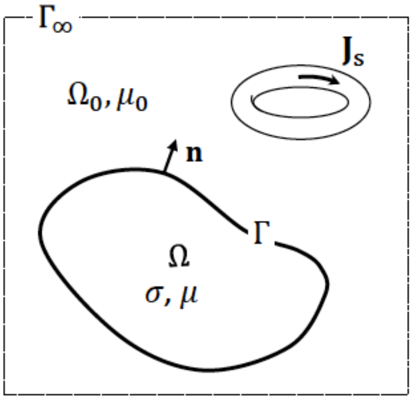

Figure 1. *Eddy current problems*

The Maxwell's equations considered in the frequency domain and behavior laws are written in Euclidean space $\mathbb{R}^3$[6], [7].

$$\text{curl}\,\mathbf{H} = \mathbf{J}_s, \text{curl}\,\mathbf{E} = -j\omega\,\mathbf{B}, \ \ \text{div}\,\mathbf{B} = 0. \tag{1a-b-c}$$

$$\mathbf{B} = \mu\mathbf{H}, \ \ \mathbf{J} = \sigma\mathbf{E}, \tag{2a-b}$$

where **B** is magnetic flux density (T), **H** is the magnetic field (A/m), **E** is the electric field (V/m), $\mathbf{J}_s$ is the current density (A/m2), μ and σ are respectively the relative permeability and electric conductivity (S/m).

The boundary conditions (BCs) defined on Γ are expressed as

$$\boldsymbol{n} \times \mathbf{H}|_{\Gamma_h} = \boldsymbol{j}_f, \ \ \boldsymbol{n} \cdot \mathbf{B}|_{\Gamma_e} = \boldsymbol{b}_f, \tag{3a-b}$$

where $\boldsymbol{n}$ is the unit normal exterior to Ω, with $\Omega = \Omega_c \cup \Omega_C^c$. Where domains Ω_c and Ω_C^c are respectively the conducting non-conducting regions. The equations (1 a) and (1 b) are solved with BCs taken the tangential component of **H** in (3 a) and the normal component of **B** in (3 b) into account.

The fields **H, B, E, J** are defined to satisfy Tonti's diagram [9]. This means that $\mathbf{H} \in \mathbf{H}_\mathrm{h}\,(\mathrm{curl};\Omega)$, $\mathbf{E} \in \mathbf{H}_\mathrm{e}\,(\mathrm{curl};\Omega)$, $\mathbf{J} \in \mathbf{H}\,(\mathrm{div};\Omega)$ and $\mathbf{B} \in \mathbf{H}_\mathrm{e}\,(\mathrm{div};\Omega)$, where $\mathbf{H}_\mathrm{h}\,(\mathrm{curl};\Omega)$ and $\mathbf{H}_\mathrm{e}\,(\mathrm{div};\Omega)$ are function spaces containing BCs and the fields defined on Γ_h and Γ_e of studied domain Ω.

The fields of $\boldsymbol{j}_f$ and $\boldsymbol{b}_f$ in (3a-b) are generally equal zero for classical homogeneous BCs. The field **B** in (1 c) is obtained a vector potential **A** such that [8], [9].

$$\mathbf{B} = \mathrm{curl}\,\mathbf{A}. \tag{4}$$

By substituting (4) into (1 b), one has curl $(\mathbf{E} + \partial_t\mathbf{A}) = 0$, this leads to the definition of an electric scalar potential ν such that

$$\mathbf{E} = -\partial_t\mathbf{A} - \mathrm{grad}\,v. \tag{5}$$

2.2. *Magnetic vector potential weak formulations*

The magnetic vector potential **A** is of great use and applicability when dealing with two or three- dimensional problems. Based on the weak form of Ampere's law (1 a), the weak formulations in study domain Ω is presented as [6], [7].

$$\frac{1}{\mu}\oint_{\Omega}(\mathbf{B}\cdot\ \mathrm{curl}\,\mathbf{w})\mathrm{d}\Omega - \sigma\oint_{\Omega_c}(\mathbf{E}\cdot\ \mathrm{curl}\,\mathbf{w})\mathrm{d}\Omega_c + \oint_{\Gamma}(n\times\mathbf{H})\cdot\mathbf{w}\mathrm{d}\Gamma = \oint_{\Omega_s}(\mathbf{J}\cdot\mathbf{w})\mathrm{d}\Omega_s,$$
$$\forall\,\mathbf{w}\in\mathbf{H}_e^0(\mathrm{curl},\ \Omega). \tag{6}$$

By introducting the definition of the vector potential ($\mathbf{B} = \mathrm{curl}\,\mathbf{A}$) given in (4) and the electrical field $\boldsymbol{E}$ in (5) into the equation (6), one has

$$\frac{1}{\mu}\oint_{\Omega}(\mathrm{curl}\,\mathbf{A}\cdot\ \mathrm{curl}\,\mathbf{w})\mathrm{d}\Omega - \sigma\oint_{\Omega_c}(\partial_t\boldsymbol{A}\cdot\ \mathrm{curl}\,\mathbf{w})\mathrm{d}\Omega_c + \sigma\oint_{\Omega_c}(\mathrm{grad}\,\nu\cdot\ \mathrm{curl}\,\mathbf{w})\mathrm{d}\Omega_c + \oint_{\Gamma}(n\times\mathbf{H})$$
$$\cdot\,\mathbf{w}\mathrm{d}\Gamma = \oint_{\Omega_s}(\mathbf{J}\cdot\mathbf{w})\mathrm{d}\Omega_s$$
$$\forall\,\mathbf{w}\in\mathbf{H}_e^0(\mathrm{curl},\ \Omega), \tag{7}$$

where $\mathbf{H}_\mathrm{e}^0(\mathrm{curl},\ \Omega)$ is a function space containing the interpolation functions for **A** as well as for the shape function **w**. The term Γ_h is surface integral term considering as a natural BC. This is the case for a homogeneous Neumann BC, e.g. imposing a symmetry condition of "zero crossing current", i.e.

$$\boldsymbol{n}\times\mathbf{H}|_{\Gamma_h} = 0 \Rightarrow \boldsymbol{n}\cdot\mathbf{B}|_{\Gamma_h} = 0 \iff \boldsymbol{n}\cdot\mathbf{J}|_{\Gamma_h} = 0. \tag{8}$$

2.3. *Computation of Joule power losses via a post-processing*

After solving the weak formulation (7), the magnetic vector potential **A** is obtained in the study domain Ω, which makes possible the calculation of the eddy currents with:

$$\mathbf{J} = \sigma\mathbf{E} = -j\omega\sigma\mathbf{A}. \tag{9}$$

Thus, the Joules losses are computed with:

$$P_{losses} = \frac{1}{2}\int_{\Omega}\frac{\mathbf{J}\bar{\mathbf{J}}}{\sigma}d\Omega, \tag{10}$$

where $\bar{\mathbf{J}}$ is the conjugate of **J**. An alternative to the volume integration is to use the Poynting theorem associated to the surface integral term of degrees of freedom of A and ν located on the border

$$P_{losses} = \frac{1}{2} Re\left(\oint_{\Gamma} (n \times \mathbf{H}) \cdot \bar{\mathbf{E}} \mathrm{d}\Gamma \right) = Re\left(-\frac{j\omega}{2} \oint_{\Gamma} (n \times \mathbf{H}_s - n \times \operatorname{grad} \nu) . \bar{\mathbf{A}} \mathrm{d}\Gamma \right), \tag{11}$$

where **E** and **A** denotes the complex conjugate of electric field $\bar{\mathbf{E}}$ and magnetic vector $\bar{\mathbf{A}}$ respectively.

3. Numerical validation

In order to validate the developed method, the practical test problem based on the IEEE of Japan [10] is introduced (Fig. 2). It consists of two aluminium plates, with the conductivity of 3x215.10^7 S/m. A relative permeability of the ferrite core is 3000. An alternating current value of the excitation coil is 1000A, and frequency of 50Hz. The problem at hand is considered in 3D case.

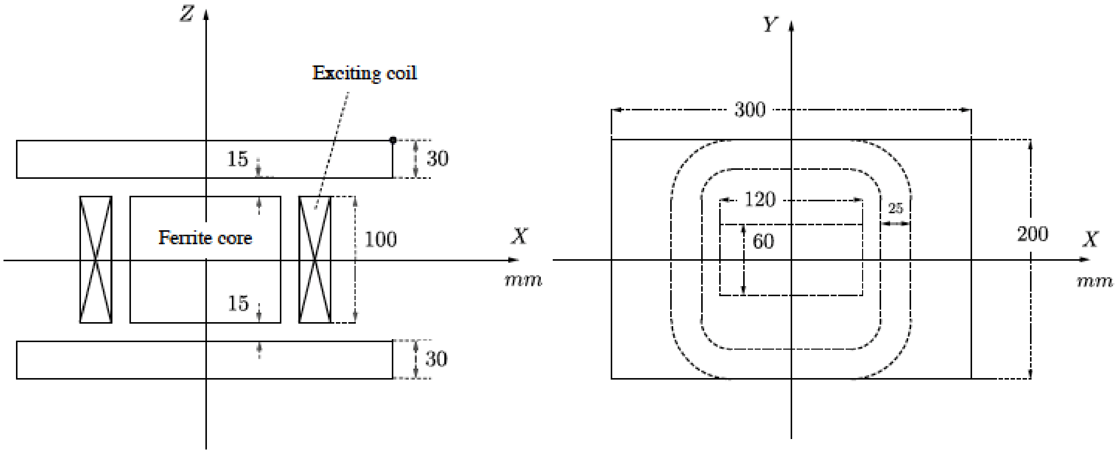

Figure 2. *Practical problem prosoed by IEEE* [8]

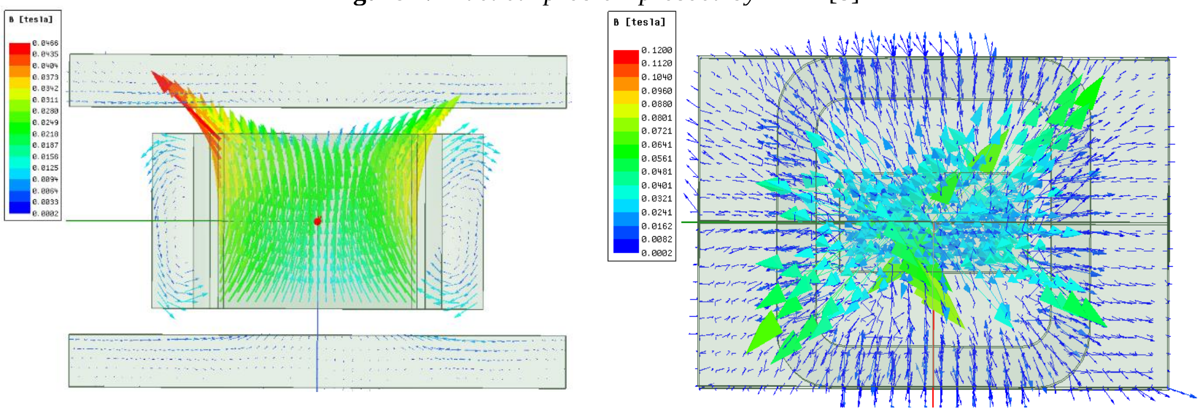

Figure 3. *Magnetic flux denisty distributions*

The magnetic flux density distribution due to "**B** = curl **A**"is presented in Figure 3. It can be seen that the fields almost focus on the edges of the ferrite core region due to the skin effect with properties of the conductivity and permeability. Significant magnetic fields along the two alumimum plates (*top* and *bottom*) are pointed in Figure 4, with effects of different properties. The skin-depth (δ) is equal to 12.5 mm with f= 50Hz, for μ_r = 1, σ = 3,215x10^7 S/m.

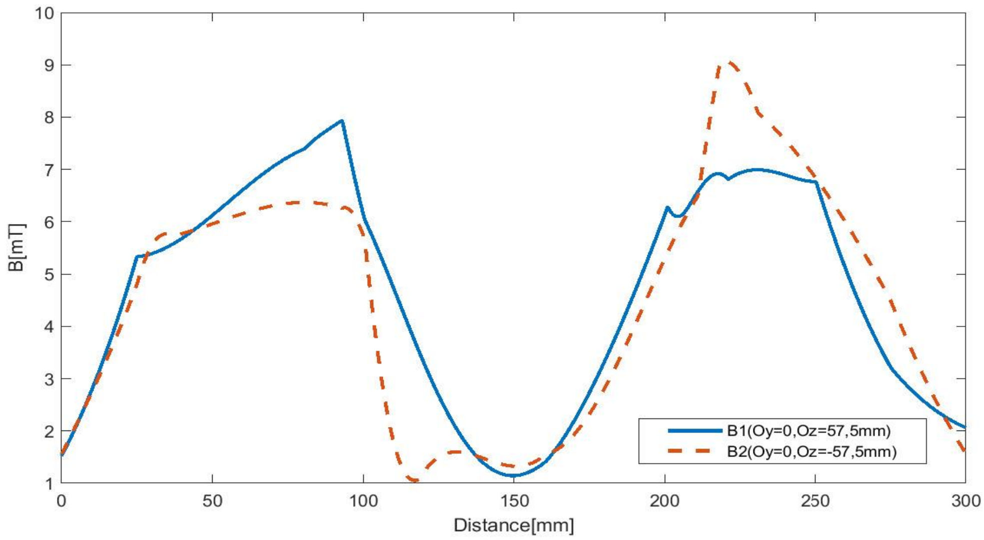

Figure 4. *Distributions of magnetic flux density in both the aluminum plates (top and botom)*

Significant eddy current values along the border of a top plate with effects of μ_r = 1, σ = 3.215x10^7 MS/m and f = 50 Hz, are shown in Figure 5. It mainly focuses on the surface of the plate near edges and corner. It can reach 2.8x10^5 A/m2 along the *x*-axis, at the position of y = 0 and z =65 mm, and for 2x10^5 A/m2 along *x*-axis at position of z = 0 and z =65 mm.

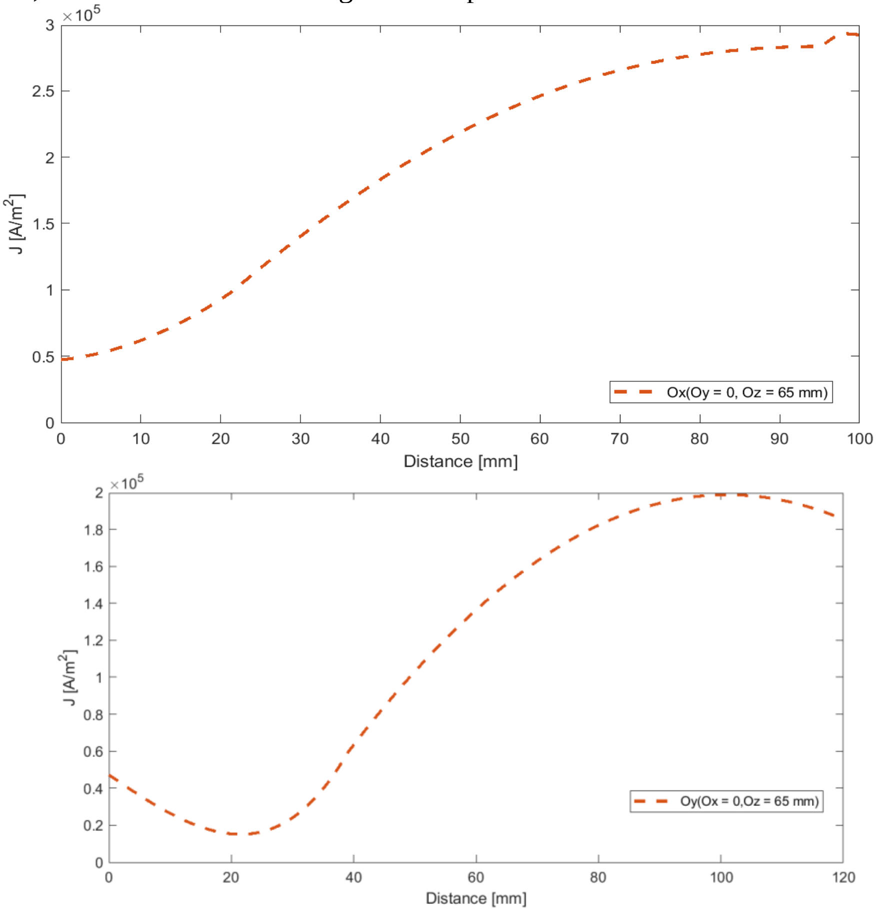

Figure 5. *Distributions of eddy current density on the border of the top aluminum plate*

4. Conclusions

The proposed method has been successfully presented with the weak magnetic vector potential formulations. The extended formulations allow to compute and simulate local fields (magnetic fields, magnetic flux density and eddy current losses) due to the alternating current following in the coil with effects of different properties. The obtained results have been shown that there is a very good validation of the development formulations in the computation of local fields taking skin effects into account. In particular, the validation of the presented method has been also successfully applied to the practical test problem.

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