SYNTHESIS OF ADAPTIVE SLIDING MODE CONTROL SYSTEMS FOR CONTINUOUS MIXING TECHNOLOGIES

Le Van Chuong[1], Ngo Tri Nam Cuong[2*]
[1]Vinh University, [2]Systemtec JSC

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ABSTRACT

This paper presents a controller synthesis method for continuous mixing technology commonly encountered in industry. The kinematic model of the control object is described in the form of a system of nonlinear equations and is affected by unknown external disturbance. The control law for the system is built based on adaptive control theory, RBF neural network, and sliding mode control method. The obtained results are the identification law of nonlinear functions, the disturbance update adaptive rule, and the sliding mode controller. As a result, the control system of continuous mixing technology has high control quality, good adaptability, and anti-interference ability. Research results are simulated using Matlab Simulink software to demonstrate the correctness and effectiveness of the proposed method.

TỔNG HỢP BỘ ĐIỀU KHIỂN TRƯỢT THÍCH NGHI CHO CÔNG NGHỆ TRỘN LIÊN TỤC

Lê Văn Chương[1], Ngô Trí Nam Cường[2*]
[1]Trường Đại học Vinh, [2]Công ty Cổ phần Systemtec

THÔNG TIN BÀI BÁO

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TỪ KHÓA

Điều khiển tự động
Điều khiển thích nghi
Điều khiển trượt
Mạng nơron RBF
Công nghệ trộn liên tục

TÓM TẮT

Bài báo trình bày một phương pháp tổng hợp bộ điều khiển cho công nghệ trộn liên tục thường gặp trong công nghiệp. Trong đó, mô hình động học của đối tượng điều khiển được mô tả dưới dạng hệ phương trình phi tuyến và chịu tác động của nhiễu ngoài không biết trước. Luật điều khiển cho hệ thống được xây dựng trên cơ sở lý thuyết điều khiển thích nghi, mạng nơron RBF và phương pháp điều khiển trượt. Kết quả thu được là luật nhận dạng các hàm phi tuyến, luật thích nghi cập nhật nhiễu và bộ điều khiển trượt. Nhờ đó hệ thống điều khiển công nghệ trộn liên tục có chất lượng điều khiển cao, có khả năng thích nghi và kháng nhiễu tốt. Kết quả nghiên cứu được mô phỏng bằng phần mềm Matlab Simulink để minh chứng tính đúng đắn và hiệu quả của phương pháp mà bài báo đề xuất.

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[*] Corresponding author. *Email:*

1. Introduction

Continuous mixing technology plays an essential role in many industrial fields such as chemical, food, pharmaceutical, etc. In the face of increasing requirements for product quality, the research on synthesizing high-quality control algorithms for the above plant continues to be an urgent issue. The studies [1], [2] used a classical PID controller; the system is stable when the uncertainty components vary in a small range. In the articles [3], [4], using a PID controller combined with fuzzy logic, however, the quality of the fuzzy controller depends on expert knowledge, so the application area is limited. The adaptive control method using a neural network is presented in [5], [6]. The multilayer feedforward neural network approximates the unknown nonlinear functions; the neural network's weights are updated by the gradient descent method to minimize the objective function. However, the gradient method has some limitations, such as the local minima problem and the algorithm's convergence speed. In addition, the studies mentioned above have not mentioned the impact of disturbance. The articles [7]-[9] synthesized a control system based on the sliding mode control principle. The system ensures stability when the nonlinear characteristics and the impact of disturbance change within a specific range. The existence of this approach is that chattering causes disadvantages to the system, especially in the case the control plant contains uncertain nonlinear characteristics and unmeasured disturbances. This paper presents a synthesizing adaptive sliding mode control system for continuous mixing technology to overcome some of the remaining problems mentioned above. The control plant has nonlinear characteristics and is affected by unmeasured external disturbances and changes unpredictably over time.

2. Mathematical modeling of continuous mixing technology

There will be a mathematical model describing different plants in continuous mixing technology, depending on technology requirements, production scale, and specific conditions. In this section, the paper presents the kinetics of the technology of continuous mixing of two input streams with first-order reactions under isothermal conditions [10], [11] with the diagram described in Fig. 1.

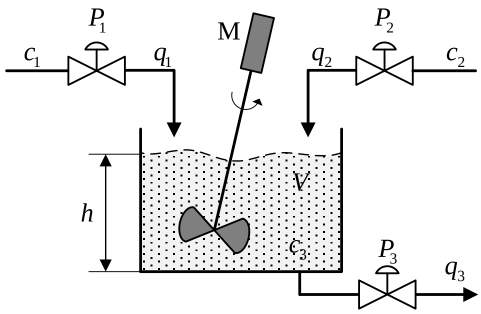

Fig 1. *Schematic diagram of the technology of continuous mixing of two components*

Two input streams of concentration c_1 and c_2 with flow q_1 and q_2 are put into the mixing tank through valves P_1 and P_2, respectively. Product solution with concentration c_3 is led out of the tank with flow q_3 through valve P_3; V and h are the volume of liquid and is the liquid level in the tank. The stirring process is carried out by the electric motor M with a constant speed.

This process can be considered a multivariable system with two control inputs denoted as q_1 and q_2 and two control outputs denoted as h and c_3. The control system is expected to set the liquid level in the tank h, and the product concentration extracted at the bottom of the tank c_3, to the desired reference values. The final concentration c_3 is obtained by mixing two input streams

q_1 (with concentration c_1) and q_2 (with concentration c_2). It is also assumed that the liquid level determines the output flow rate q_3 in the tank.

For the mixing tank, the volume balance equation takes the form:

$$q_1 + q_2 - q_3 = \frac{dV}{dt} = S\frac{dh}{dt}, \tag{1}$$

where S is the cross sectional area of the tank and is a constant.

The instantaneous flow of output stream: $q_3 = C_v\sqrt{\rho g h}$, (2)

where C_v is the valve constant; ρ is the density of the liquid inside the tank (assumed constant here); g is the acceleration of the gravity of earth.

Substitute (2) into (1): $$q_1 + q_2 - C_v\sqrt{\rho g h} = S\frac{dh}{dt}. \tag{3}$$

Thus, the problem of stabilizing the product flow q_3 is transferred to stabilizing the liquid level h in the mixing tank. Without any loss of generality, the scientific papers which study the control of continuous mixing technology often use the following simplified equation instead of (3) [10], [11]:

$$\frac{dh}{dt} = q_1 + q_2 - k_1\sqrt{h}, \tag{4}$$

where again k_1 is roughly called the valve constant.

Similarly, the mole balance equation is generally in the form of [10], [11]:

$$\frac{dc_3}{dt} = \left[c_1 - c_3\right]\frac{q_1}{h} + \left[c_2 - c_3\right]\frac{q_2}{h} - \frac{k_2 c_3}{\left(1 + c_3\right)^2}, \tag{5}$$

where k_2 is a kinetic constant.

From (4) and (5), we obtain a model of two-component continuous mixing technology:

$$\begin{cases} \dfrac{dh}{dt} = q_1 + q_2 - k_1\sqrt{h} \\ \dfrac{dc_3}{dt} = \left[c_1 - c_3\right]\dfrac{q_1}{h} + \left[c_2 - c_3\right]\dfrac{q_2}{h} - \dfrac{k_2 c_3}{\left(1 + c_3\right)^2} \end{cases}, \tag{6}$$

We set: $\mathbf{x} = \left[x_1, x_2\right]^T$, $\mathbf{u} = \left[u_1, u_2\right]^T$, where $x_1 = h$, $x_2 = c_3$, $u_1 = q_1$, $u_2 = q_2$. The system of equations (6) is reduced to the form:

$$\dot{\mathbf{x}} = \boldsymbol{\psi}\left(\mathbf{x}, \mathbf{u}\right), \tag{7}$$

Perform Taylor expansion of equation (7) at the equilibrium point $\left(\mathbf{x}_0, \mathbf{u}_0\right) = \left(\left[h_0, c_{30}\right]^T, \left[q_{10}, q_{20}\right]^T\right)$, we have [12]-[14]:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{f}\left(\mathbf{x}\right), \tag{8}$$

where $\mathbf{A}$, $\mathbf{B}$ are Jacobian matrices:

$$\mathbf{A} = \left.\frac{\partial\boldsymbol{\psi}}{\partial\mathbf{x}}\right|_{\left(\mathbf{x}_0, \mathbf{u}_0\right)}; \tag{9}$$ $$\mathbf{B} = \left.\frac{\partial\boldsymbol{\psi}}{\partial\mathbf{u}}\right|_{\left(\mathbf{x}_0, \mathbf{u}_0\right)}; \tag{10}$$

$\mathbf{f}\left(\mathbf{x}\right) = \left[f_1, f_2\right]^T$ is a higher order terms of the Taylor expansion.

Continuous mixing technology may be affected by external disturbance during operation, which is unknown and may change over time. Therefore, equation (8) can be rewritten as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{f}\left(\mathbf{x}\right) + \mathbf{d}\left(t\right), \tag{11}$$

where $\mathbf{d}\left(t\right) = \left[d_1, d_2\right]^T$ is unmeasured external disturbance vector, changes unpredictably over time.

Thus, the continuous mixing technology (11) has a nonlinear kinetic model and is affected by unmeasured external disturbance. The following section presents a synthetic solution of the control system for the plant (11).

3. Synthesis of a control system of continuous mixing technology

The control structure diagram of the continuous mixing technology system proposed by the article is shown in Fig. 2. In which: The plant is a continuous mixing technology system; Reference Model is the identification model; Adaptive Mechanism is the adaptive control block; Compensation is the reciprocal block that compensates for uncertain components; SMC is a sliding mode controller.

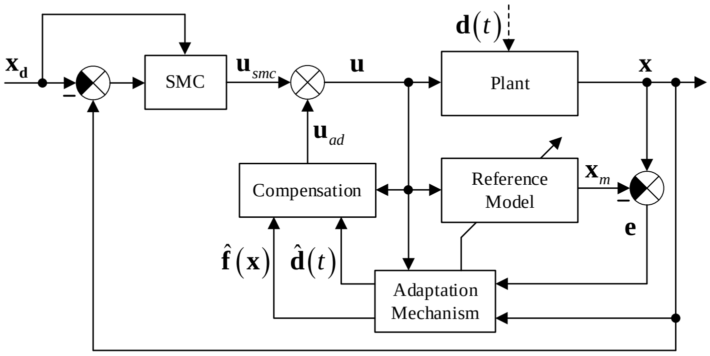

Fig. 2. *The control structure diagram of the continuous mixing technology system*

Plant with the model (11) will follow the desired signal vector $\mathbf{x}_d$ if the control law $\mathbf{u}$ is selected in the form:

$$\mathbf{u} = \mathbf{u}_{smc} + \mathbf{u}_{ad}, \tag{12}$$

where $\mathbf{u}_{smc}$ is the sliding mode control law; $\mathbf{u}_{ad}$ is an adaptive control law.

3.1. Synthesis of the adaptive control law

We rewrite equation (11) as:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} + \mathbf{If}_{\Sigma}, \tag{13}$$

where $\mathbf{f}_{\Sigma} = \mathbf{f}(\mathbf{x}) + \mathbf{d}(t) = \left[f_{\Sigma 1}, f_{\Sigma 2}\right]^T$; $\mathbf{I}$ is identity matrix. Substitute (12) into (13):

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}_{smc} + \mathbf{Bu}_{ad} + \mathbf{If}_{\Sigma}. \tag{14}$$

From (14), we can see that uncertain elements will be compensated with the condition:

$$\mathbf{Bu}_{ad} + \mathbf{If}_{\Sigma} = 0. \tag{15}$$

To satisfy equation (15), we choose: $\mathbf{u}_{ad} = -\mathbf{Hf}_{\Sigma}$, (16)

where $\mathbf{H} = \mathbf{B}^T\left[\mathbf{BB}^T\right]^{-1}$ is the gain matrix; $\det\left(\mathbf{BB}^T\right) \neq 0$.

In order to synthesize the control law (16), it is necessary to identify the nonlinear components $\mathbf{f}(\mathbf{x})$ and the external disturbance $\mathbf{d}(t)$ present in $\mathbf{f}_{\Sigma}$.

The identification model for uncertain parameters in (11) can be written:

$$\dot{\mathbf{x}}_m = \mathbf{Ax} + \mathbf{Bu} + \hat{\mathbf{f}}(\mathbf{x}) + \hat{\mathbf{d}}(t), \tag{17}$$

where $\mathbf{x}_m = \left[x_{m1}, x_{m2}\right]^T$ is state vector of the model; $\hat{\mathbf{f}}(\mathbf{x}) = \left[\hat{f}_1(\mathbf{x}), \hat{f}_2(\mathbf{x})\right]^T$ is the estimated vector of $\mathbf{f}(\mathbf{x})$; $\hat{\mathbf{d}}(t) = \left[\hat{d}_1(t), \hat{d}_2(t)\right]^T$ is the estimated vector of $\mathbf{d}(t)$.

From (11) and (17), we have: $\dot{\mathbf{e}} = \mathbf{Ae} + \tilde{\mathbf{f}}(\mathbf{x}) + \tilde{\mathbf{d}}(t)$, (18)

where: $\mathbf{e} = \mathbf{x} - \mathbf{x}_m$, (19) $\tilde{\mathbf{f}}(\mathbf{x}) = \mathbf{f}(\mathbf{x}) - \hat{\mathbf{f}}(\mathbf{x})$ (20) $\tilde{\mathbf{d}}(t) = \mathbf{d}(t) - \hat{\mathbf{d}}(t)$ (21)

Identification progress will be converging when $\tilde{\mathbf{f}}(\mathbf{x}) \to 0$, $\tilde{\mathbf{d}}(t) \to 0$. With the assumption $\mathbf{A}$ is a Hurwitz matrix, so $\mathbf{e} \to 0$, and (18) is stability.

With $\mathbf{f}(\mathbf{x})$ is a smooth function vector, by using a RBF neural network for the approximation [15]. The elements of $\mathbf{f}(\mathbf{x})$ can be written:

$$f_i(\mathbf{x}) = \sum_{j=1}^{L} w_{ij}^* \phi_{ij}(\mathbf{x}) + \varepsilon_i \ , \tag{22}$$

$i = \overline{1,2}$; $j = \overline{1,L}$ where L is number of basis function with a large enough number to guarantee the error $|\varepsilon_i| < \varepsilon_i^m$, $\varepsilon_i^m = const$; $w_{ij}^* = const$ is the ideal weights. The basis functions are selected by the following form:

$$\phi_{ij}(\mathbf{x}) = \exp\left(\left\| \mathbf{x} - \mathbf{c}_{ij} \right\|^2 \Big/ 2\sigma_{ij}^2 \right), i = \overline{1,2}\ , \ \ j = \overline{1,L}\ , \tag{23}$$

where $\mathbf{c}_{ij}$ are the position of the center of the basis functions $\phi_{ij}(\mathbf{x})$, and σ_{ij} are the standard deviation of the basis functions.

The estimated vector $\hat{\mathbf{f}}(\mathbf{x})$ is defined by (23) with adjusted weights $\hat{w}_{ij}$:

$$\hat{f}_i(\mathbf{x}) = \sum_{j=1}^{L} \hat{w}_{ij} \phi_{ij}(\mathbf{x})\ , \ \ i = \overline{1,2}\ , \ \ j = \overline{1,L}\ . \tag{24}$$

Training of the RBF neural network is implemented by adjustment of the weights $\hat{w}_{ij}$ in comparison with the ideal weights w_{ij}^*:

$$\tilde{w}_{ij} = w_{ij}^* - \hat{w}_{ij}\ . \tag{25}$$

From (22), (24), with attention to (25), we have:

$$f_i(\mathbf{x}) = \hat{f}_i(\mathbf{x}) + \varepsilon_i \qquad \to \qquad \tilde{f}(\mathbf{x}) = \sum_{j=1}^{L} \tilde{w}_{ij} \phi_{ij}(\mathbf{x}) + \varepsilon_i\ . \tag{26}$$

For equations (18), the Lyapunov function is selected as follows:

$$V = \mathbf{e}^T \mathbf{P} \mathbf{e} + \sum_{i=1}^{2} \sum_{j=1}^{L} \tilde{w}_{ij}^2 + \sum_{i=1}^{2} \tilde{d}_i^2\ . \tag{27}$$

where $\mathbf{P}$ is a positive definite symmetric matrix. The equations (18) will be stable if the derivative (27) $\dot{V} < 0$. Take the derivative of both sides of (27):

$$\dot{V} = \dot{\mathbf{e}} \mathbf{P} \mathbf{e} + \mathbf{e}^T \mathbf{P} \dot{\mathbf{e}} + 2\sum_{i=1}^{2} \sum_{j=1}^{L} \dot{\tilde{w}}_{ij} \tilde{w}_{ij} + 2\sum_{i=1}^{2} \dot{\tilde{d}}_i \tilde{d}_i\ . \tag{28}$$

Substitute (18) into (28):

$$\dot{V} = \mathbf{e}^T \left(\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} \right) \mathbf{e} + 2\mathbf{e}^T \mathbf{P} \tilde{\mathbf{f}}(\mathbf{x}) + 2\mathbf{e}^T \mathbf{P} \tilde{\mathbf{d}}(t) + 2\sum_{i=1}^{2} \sum_{j=1}^{L} \dot{\tilde{w}}_{ij} \tilde{w}_{ij} + 2\sum_{i=1}^{2} \dot{\tilde{d}}_i \tilde{d}_i. \tag{29}$$

From (29) and (26), we have:

$$\dot{V} = \mathbf{e}^T \left(\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} \right) \mathbf{e} + 2\mathbf{e}^T \mathbf{P} \boldsymbol{\varepsilon} + 2\left(\mathbf{e}^T \mathbf{P} \begin{bmatrix} \sum_{j=1}^{L} \tilde{w}_{1j} \phi_{ij}(\mathbf{x}) \\ \sum_{j=1}^{L} \tilde{w}_{2j} \phi_{ij}(\mathbf{x}) \end{bmatrix} + \sum_{i=1}^{2} \sum_{j=1}^{L} \dot{\tilde{w}}_{ij} \tilde{w}_{ij} \right) + 2\left(\mathbf{e}^T \mathbf{P} \tilde{\mathbf{d}}(t) + 2\sum_{i=1}^{2} \dot{\tilde{d}}_i \tilde{d}_i \right). \tag{30}$$

The condition for $\dot{V} < 0$ is as follows:

$$\mathbf{e}^T \left(\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} \right) \mathbf{e} + 2\mathbf{e}^T \mathbf{P} \boldsymbol{\varepsilon} < 0\ ; \tag{31}$$

$$2\left(\mathbf{e}^T\mathbf{P}\begin{bmatrix}\sum_{j=1}^{L}\tilde{w}_{1j}\phi_{ij}\left(\mathbf{x}\right)\\ \sum_{j=1}^{L}\tilde{w}_{2j}\phi_{ij}\left(\mathbf{x}\right)\end{bmatrix}+\sum_{i=1}^{2}\sum_{j=1}^{L}\dot{\tilde{w}}_{ij}\tilde{w}_{ij}\right)=0 \quad (32)$$

$$2\left(\mathbf{e}^T\mathbf{P}\tilde{\mathbf{d}}\left(t\right)+2\sum_{i=1}^{2}\dot{\tilde{d}}_i\tilde{d}_i\right)=0 \quad (33)$$

Transform the left side of the inequality (31): $-\mathbf{e}^T\mathbf{Q}\mathbf{e}+2\sum_{i=1}^{2}\varepsilon_i\bar{\mathbf{P}}_i\mathbf{e}<0$, (34)

$\mathbf{Q}=-\left(\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}\right)$, $\bar{\mathbf{P}}_i$ is the i -th row of the matrix $\mathbf{P}$.

Using inequality transformations [16], the equation (31) can be written:

$$-\mathbf{e}^T\mathbf{Q}\mathbf{e}+2\sum_{i=1}^{2}\varepsilon_i\bar{\mathbf{P}}_i\mathbf{e}<-r_{\min}(\mathbf{Q})\left\|\mathbf{e}\right\|^2+2\sum_{i=1}^{2}\varepsilon_i\left\|\bar{\mathbf{P}}_i\right\|\left\|\mathbf{e}\right\|<0\,, \quad (35)$$

$r_{\min}(\mathbf{Q})$ is the smallest eigenvalue of the matrix $\mathbf{Q}$.

Thus, to satisfy the inequality (31) from (35), we must have:

$$\left\|\mathbf{e}\right\|>2\sum_{i=1}^{2}\varepsilon_i\left\|\bar{\mathbf{P}}_i\right\|/r_{\min}(\mathbf{Q})\,. \quad (36)$$

Solving equations (32), (33), we have:

$$\dot{\tilde{w}}_{ij}=-\bar{\mathbf{P}}_i\mathbf{e}\phi_{ij}\left(\mathbf{x}\right),\ i=\overline{1,2}\,,\ j=\overline{1,...L}\,; \quad (37)$$

$$\dot{\tilde{d}}_i=-\bar{\mathbf{P}}_i\mathbf{e}\,,\ i=\overline{1,2}\,. \quad (38)$$

If simultaneous (36-38) is satisfied, then $\dot{V}<0$, so the system (18) is stable. The stability domain of (18) defined at (36) is the entire state space except the neighborhood of the origin. The stability domain of (18) defined at (36) is the entire state space except for the neighborhood of the origin. The radius of this region depends on the approximate error of the RBF neural network, where ε_i is arbitrarily tiny and can be ignored. Thus, the stability domain is the entire state space except for the origin region with a radius close to zero.

With the identification results (24), (37), (38), we replace $\mathbf{f}_\Sigma$ with $\hat{\mathbf{f}}_\Sigma$ as follows:

$$\hat{\mathbf{f}}_\Sigma=\hat{\mathbf{f}}\left(\mathbf{x}\right)+\hat{\mathbf{d}}\left(t\right)=\left[\hat{f}_{\Sigma 1},\hat{f}_{\Sigma 2}\right]^T. \quad (39)$$

The control law $\mathbf{u}_{ad}$ (16) is rewritten as follows: $\mathbf{u}_{ad}=-\mathbf{H}\hat{\mathbf{f}}_\Sigma$, (40)

then (14) becomes: $\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}_{smc}$. (41)

For (41), the control law is synthesized using the sliding mode control method.

3.2. Synthesis of the sliding mode control law

The error vector between the state vector $\mathbf{x}$ and the desired state vector $\mathbf{x}_d$:

$$\tilde{\mathbf{x}}=\mathbf{x}-\mathbf{x}_d\rightarrow\mathbf{x}=\tilde{\mathbf{x}}-\mathbf{x}_d\,. \quad (42)$$

Substitute (42) into (41): $\dot{\tilde{\mathbf{x}}}=\mathbf{A}\tilde{\mathbf{x}}+\mathbf{B}\mathbf{u}_{smc}+\mathbf{A}\mathbf{x}_d-\dot{\mathbf{x}}_d$. (43)

For (43), the hyper sliding surface is chosen as follows [17]: $\mathbf{s}=\mathbf{C}\tilde{\mathbf{x}}$, (44)

where $\mathbf{C}$ is the parameter matrix of hyper sliding surface, $\det(\mathbf{CB})\neq 0$, $\mathbf{s}=\left[s_1,s_2\right]^T$.

The next problem is defining $\mathbf{u}_{smc}$, which ensures system (43) movement towards the hyper sliding surface (44) and keeps it there.

The control signal $\mathbf{u}_{smc}$ can be written by: $\mathbf{u}_{smc}=\begin{cases}\mathbf{u}_s & khi\quad \mathbf{s}\neq 0\\ \mathbf{u}_{eq} & khi\quad \mathbf{s}=0\end{cases}$, (45)

$\mathbf{u}_s$ is the control signal that moves the system (43) towards the hyper sliding surface (44); $\mathbf{u}_{eq}$ is the equivalent control signal that keeps the system (43) on the hyper sliding surface (44).

The equation (45) can be rewritten as: $\mathbf{u}_{smc} = \mathbf{u}_{eq} + \mathbf{u}_s$. (46)

$\mathbf{u}_{eq}$ is defined in [17]: $\mathbf{s} = \mathbf{C}\tilde{\mathbf{x}} = 0$. (47)

From (43) and (47), we have: $\mathbf{u}_{eq} = -[\mathbf{CB}]^{-1}[\mathbf{CA}\tilde{\mathbf{x}} + \mathbf{CAx}_d - \mathbf{C}\dot{\mathbf{x}}_d]$. (48)

Next, we define the control signal $\mathbf{u}_s$ that moves the system (43) towards the hyper sliding surface (44). For the hyper sliding surface (44), the Lyapunov function can be selected by:

$$V = \frac{1}{2}\mathbf{s}^T\mathbf{s}. \quad (49)$$

Condition for the existence of slip mode can be written: $\dot{V} = \mathbf{s}^T\dot{\mathbf{s}} < 0$. (50)

Substitute (43) and (46) into (50), with attention to (47), (48), we have:

$$\dot{V} = \mathbf{s}^T\left[\mathbf{C}\left(\mathbf{A}\tilde{\mathbf{x}} + \mathbf{Bu}_{smc} + \mathbf{Ax}_d - \dot{\mathbf{x}}_d\right) + \mathbf{CBu}_s\right] < 0. \quad (51)$$

The inequality (51) is equivalent to: $\mathbf{s}^T[\mathbf{CBu}_s] < 0$. (52)

To satisfy (50) from (52), we have: $\mathbf{u}_s = -[\mathbf{CB}]^{-1}\left[\delta\,\mathrm{sgn}(s_1), \delta\,\mathrm{sgn}(s_2)\right]^T$, (53)

δ is a small positive coefficient.

Substitute (48) and (53) into (45), we have:

$$\mathbf{u}_{smc} = \begin{cases} -[\mathbf{CB}]^{-1}\left[\delta\,\mathrm{sgn}(s_1), \delta\,\mathrm{sgn}(s_2)\right]^T & khi\ \mathbf{s} \neq 0 \\ -[\mathbf{CB}]^{-1}\left[\mathbf{CA}\tilde{\mathbf{x}} + \mathbf{CAx}_d - \mathbf{C}\dot{\mathbf{x}}_d\right] & khi\ \mathbf{s} = 0 \end{cases}. \quad (54)$$

Finally, the control signals (40) and (54) are used for (11). Thus, the paper has synthesized the control law for continuous mixing technology.

4. Results and discussion

Continuous mixing technology is described in (6) with parameters shown in Table 1 [10], [11].

Table 1. *Continuous mixing technology parameters and nominal values* [10]

Parameter	Meaning	Nominal Value
c_1	Concentration in the inlet flow q_1	24.9 kmol/m^3
c_2	Concentration in the inlet flow q_2	0.1 kmol/m^3
k_1	Valve constant	0.2 m$^{1/2}$/s
k_2	Kinetic constant	1.0 mol^2/m^{6}s

Perform Taylor expansion of equation (6) at the equilibrium point $(\mathbf{x}_0, \mathbf{u}_0) = \left([h_0, c_{30}]^T, [q_{10}, q_{20}]^T\right)$ with $h_0 = 1.0$ m, $c_{30} = 12.15$ kmol/m^3, $q_{10} = 0.1$ m/s, $q_{20} = 0.1$ m/s. The matrices (9), (10) are obtained as follows:

$$\mathbf{A} = \begin{bmatrix} -0.1 & 0 \\ -0.07 & -0.195 \end{bmatrix}, \quad (55) \qquad \mathbf{B} = \begin{bmatrix} 1 & 1 \\ 12.75 & -12.05 \end{bmatrix}. \quad (56)$$

Nonlinear function vectors and disturbance vectors in (11) are defined as follows:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} 0.025x_1^2 - 0.0125x_1^3 \\ 0,2x_1x_1 + 0,001x_2^2 \end{bmatrix}, \quad (57) \qquad \mathbf{d}(t) = \begin{bmatrix} 0 \\ 0,2\sin(0.05t) + 0,25\cos(0,03t) \end{bmatrix}. \quad (58)$$

Simulations are implemented on the Matlab environment for the plant (11) using the controller (12). The results of the identification of nonlinear components and external disturbance $\mathbf{f}_\Sigma = \mathbf{f}(\mathbf{x}) + \mathbf{d}(t) = [f_{\Sigma 1}, f_{\Sigma 2}]^T$ using algorithms (24), (37), (38) are shown in Fig. 3.

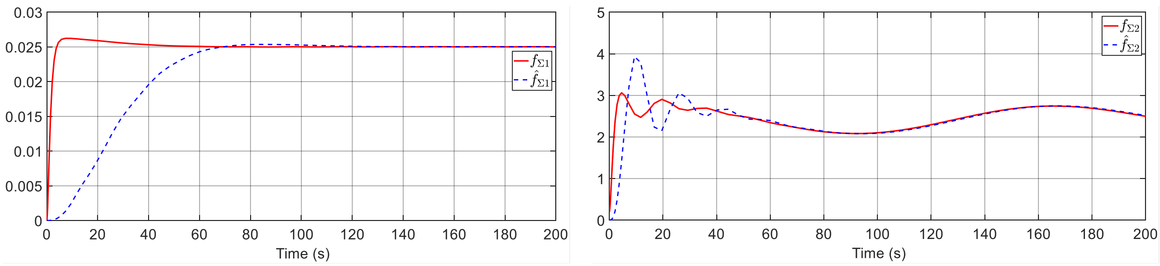

Fig 3. *The identification vectors* $\hat{\mathbf{f}}_{\Sigma}$.

The simulation results in Fig. 3 show that the algorithm to identify the components of change in plant kinematics has worked properly.

The results of the control law $\mathbf{u}_{ad}$ (40) to compensate for the nonlinear component and the external disturbance are expressed through the error $\mathbf{e_c} = [e_{c1}, e_{c2}]^T$ between the actual plant (11) and the linear kinematic (41). The simulation results are shown in Fig. 4.

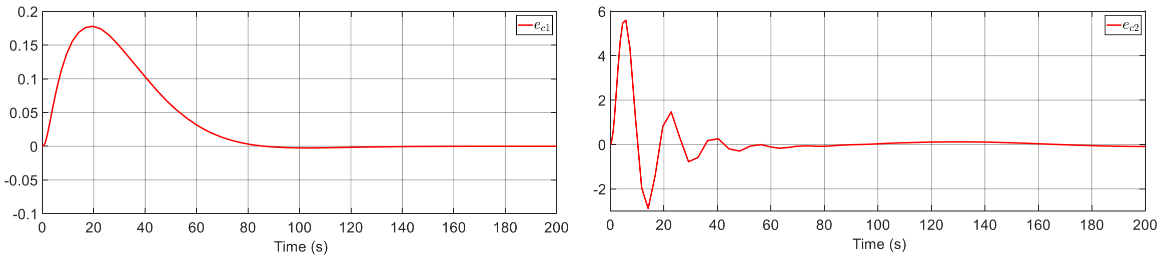

Fig 4. *The error between (11) and linear model (41)*

The simulation results in Fig. 4 show that the control law $\mathbf{u}_{ad}$ (40) has compensated for the nonlinear components and external disturbance in the plant kinematics with the offset error asymptotically zero.

The results of tracking the state vectors of the system with the desired signal vector $\mathbf{x}_d = [1.0 \quad 12.15]^T$ using the sliding mode control law (54) are shown in Fig. 5.

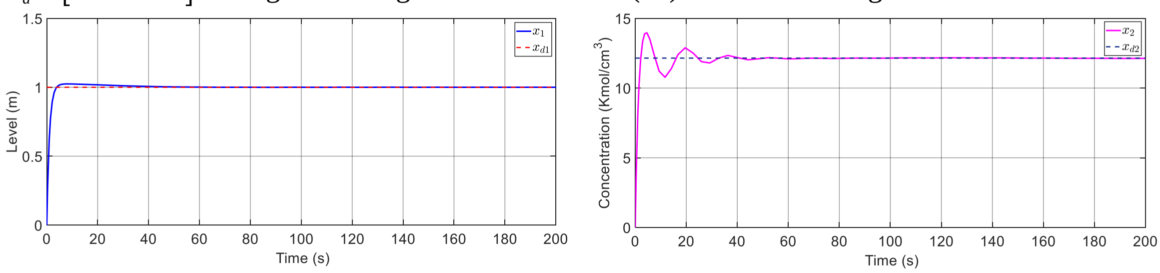

Fig 5. *Responses of the system for the desired signals* $\mathbf{x}_d = [1.0 \quad 12.15]^T$.

From Fig. 5, it is shown that the system's response has to track to the desired signal $h_0 = 1.0$ m and $c_{30} = 12.15$ kmol/m^3. Thus, with the controller (12), the continuous mixing technology control system has created a desired concentration and volume solution with guaranteed control quality. The simulation results have proved the correctness and effectiveness of the article's proposed control law.

5. Conclusion

The article has synthesized the adaptive sliding mode control law for continuous mixing technology. The law for identifying vectors of nonlinear functions and external disturbance has been built. From the identification results, building a compensation structure for their influence on the system, the compensation error depends on the approximate error of the RBF neural network. On the other hand, the RBF neural network can approximate with arbitrarily small precision so that this error can be ignored. The control law of continuous mixing technology is built on the principle of sliding mode control. When the adaptive algorithm converges, the uncertainty elements are compensated so that the chattering effect in the sliding mode control law is reduced to a minimum, which overcomes the limitation of the classical sliding mode control method. The control system proposed by this paper is adaptable, resistant to interference, and has good control quality. The simulation results once again proved the correctness and effectiveness of the proposed method.

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