SOME RESULTS ON THE CONCEPT OF $\{a_{m,n,i,j}\}$ – STOCHASTIC DOMINATION FOR DOUBLE ARRAYS OF RANDOM VARIABLES

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ABSTRACT

In this paper, we prove some results on the concept of $\{a_{m,n,i,j}\}$- stochastic domination for double arrays of random variables, where $\{k_m, m \geq 1\}$ and $\{l_n, n \geq 1\}$ are two sequences of positive integers and $\{a_{m,n,i,j}; 1 \leq i \leq k_m,\ 1 \leq j \leq l_n, m, n \geq 1\}$ is sequences of positive constants satisfying

$$\sup_{m\geq 1, n\geq 1} \sum_{i=1}^{k_m} \sum_{j=1}^{l_n} a_{m,n,i,j} = C_0, C_0 \in (0, \infty).$$

The main results establish double sum versions of Theorem 2.1 and Theorem 2.6 of Thành (2023).

Keywords: Stochastic domination; double array; slowly varying function; Cesàro stochastic domination.

1. Introduction and preliminaries

Firstly, we develop some results concerning a new concept of stochastic domination which leads to the concept of the Cesàro stochastic domination as a particular case.

A double array $\{X_{m,n}, m \geq 1,\ n \geq 1\}$ of random variables is said to be *stochastically dominated* by a random variable X if

$$\sup_{m\geq 1, n\geq 1} \mathbb{P}(|X_{m,n}| > x) \leq \mathbb{P}(|X| > x), \text{ for all } x \in \mathbb{R}. \quad (1.1)$$

The next concept was extended to the concept of the so-called Cesàro stochastic domination by Fazekas and Tómács [5] as follows. Let $\{k_m, m \geq 1\}$ and $\{l_n, n \geq 1\}$ be sequences of positive integers. A double array $\{X_{m,n}, m \geq 1, n \geq 1\}$ of random variables is said to be *tochastically dominated in the Cesàro sense* (or *weakly mean dominated*) by a random variable X if

$$\sup_{m\geq 1,n\geq 1}\frac{1}{k_m l_n}\sum_{i=1}^{k_m}\sum_{j=1}^{l_n}\mathbb{P}(|X_{i,j}|>x)\leq C\mathbb{P}(|X|>x),\ \text{ for all } x\in\mathbb{R}, \tag{1.2}$$

where $C>0$ is a constant.

Following Thành [12], we introduce a new concept of $\{a_{m,n,i,j}\}$-stochastically dominated for a double array of random variables as follows.

Let $\{a_{m,n,i,j}; 1\leq i\leq k_m, 1\leq j\leq l_n, m,n\geq 1\}$ be sequences of positive constants satisfying

$$\sup_{m\geq 1,n\geq 1}\sum_{i=1}^{k_m}\sum_{j=1}^{l_n}a_{m,n,i,j}=C_0, C_0\in(0,\infty). \tag{1.3}$$

A double array $\{X_{m,n}, m\geq 1, n\geq 1\}$ of random variables is said to be $\{a_{m,n,i,j}\}$-*stochastically dominated* by a random variable X if

$$\sup_{m\geq 1,n\geq 1}\sum_{i=1}^{k_m}\sum_{j=1}^{l_n}a_{m,n,i,j}\mathbb{P}(|X_{i,j}|>x)\leq C_0\mathbb{P}(|X|>x),\ \text{ for all } x\in\mathbb{R}. \tag{1.4}$$

In view of the definition in (1.2), one may be tempted to give an apparently weaker definition of $\{X_{m,n}, m\geq 1, n\geq 1\}$ being $\{a_{m,n,i,j}\}$-stochastically dominated by a random variable Y, namely that

$$\sup_{m\geq 1,n\geq 1}\sum_{i=1}^{k_m}\sum_{j=1}^{l_n}a_{m,n,i,j}\mathbb{P}(|X_{i,j}|>x)\leq C\mathbb{P}(|Y|>x),\ \text{ for all } x\in\mathbb{R}, \tag{1.5}$$

for some finite constant $C>0$.

Concerning definition of the Cesàro stochastic domination, we can simply choose $C=1$ in (1.2). If $a_{m,n,i,j}=1/k_m l_n, 1\leq i\leq k_m, 1\leq j\leq l_n, m,n\geq 1$, then it is obvious that $C_0=1$, and the concept of $\{a_{m,n,i,j}\}$-stochastic domination reduces to the concept of stochastic domination in the Cesàro sense.

The laws of large numbers with the norming sequences are of the form $f(\cdot)L(\cdot)$ where $L(\cdot)$ is a slowly varying function were studied by many authors. We refer to Anh et al. [[1]], Anh and Hien [[2]], Gut[[7],[8]], Matsumoto and Nakata [[9]] and others. The notion of slowly varying function can be found in [[11], Chapter 1]. A real-valued function $L(\cdot)$ is said to be *slowly varying* (at infinity) if it is a positive and measurable function on $[A,\infty)$ for some $A\geq 0$, and for each $\lambda>0$,

$$\lim_{x\to\infty}\frac{L(\lambda x)}{L(x)}=1.$$

Let $L(\cdot)$ be a slowly varying function. Then by Bingham et al.[[3], Theorem 1.5.13], there exists a slowly varying function $\tilde{L}(\cdot)$, unique up to asymptotic equivalence, satisfying

$$\lim_{x\to\infty} L(x)\tilde{L}\left(xL(x)\right) = 1 \text{ and } \lim_{x\to\infty} \tilde{L}(x)L\left(x\tilde{L}(x)\right) = 1. \tag{1.6}$$

The function $\widetilde{L}$ is called the de Bruijn conjugate of L, and $\left(L, \widetilde{L}\right)$ is called a (slowly varying) conjugate pair (see, e.g., [[3], p.29]).

The following lemma shows that we can approximate a slowly varying function $L(\cdot)$ by a differentiable slowly varying function $L_1(\cdot)$. We can see Anh et al. [[1], Lemma 2.2], Galambos and Seneta [[6], p.111], and other references.

Lemma 1.1. *For any slowly varying function $L(\cdot)$ defined on $[A,\infty)$ for some $A \geq 0$, there exists a differentiable slowly varying function $L_1(\cdot)$ defined on $[B,\infty)$ for some $B \geq A$ such that*

$$\lim_{x\to\infty} \frac{L(x)}{L_1(x)} = 1 \text{ and } \lim_{x\to\infty} \frac{xL_1'(x)}{L_1(x)} = 0.$$

Conversely, if $L(\cdot)$ is a positive differentiable function satisfying

$$\lim_{x\to\infty} \frac{xL'(x)}{L(x)} = 0, \tag{1.7}$$

then $L(\cdot)$ is a slowly varying function.

The following lemma is the simple result on the expectation of a nonnegative random variable, can see Rosalsky and Thành [10] for a proof.

Lemma 1.2. *Let $h : [0,\infty) \to [0,\infty)$ be a measurable function with $h(0) = 0$ which is bounded on $[0, A]$ and differentiable on $[A,\infty)$ for some $A \geq 0$. If ξ is a nonnegative random variable, then*

$$\mathbb{E}(h(\xi)) = \mathbb{E}(h(\xi)\mathbf{1}(\xi \leq A)) + h(A) + \int_A^\infty h'(x)\mathbb{P}(\xi > x)dx.$$

Throughout this paper, $\mathbf{1}(A)$ denotes the indicator function for set A. For $x \geq 0$, let $\log x$ denote the logarithm base 2 of $\max\{2, x\}$. For $x \geq 0$ and for a fixed positive integer ν, let

$$\log_\nu(x) := (\log x)(\log\log x)\dots(\log\cdots\log x), \tag{1.8}$$

and

$$\log_\nu^{(2)}(x) := (\log x)(\log\log x)\dots(\log\cdots\log x)^2, \tag{1.9}$$

where in both (1.8) and (1.9), there are ν factors. For example, $\log_2(x) = (\log x)(\log\log x)$, $\log_3^{(2)}(x) = (\log x)(\log\log x)(\log\log\log x)^2$, and so on.

2 Main results

The following theorem is a simple result and its proof is similar to that of Theorem 2.1 of Thành [12].

Theorem 2.1. *Let* $\{k_m, m \geq 1\}$ *and* $\{l_n, n \geq 1\}$ *be sequences of positive integers,* $\{X_{m,n}, m \geq 1, n \geq 1\}$ *be a double array of random variables,* $\{a_{m,n,i,j}; 1 \leq i \leq k_m, 1 \leq j \leq l_n, m, n \geq 1\}$ *be sequences of positive constants satisfying* (1.3) *and let*

$$F(x) = 1 - \frac{1}{C_0} \sup_{m\geq 1, n\geq 1} \sum_{i=1}^{k_m} \sum_{j=1}^{l_n} a_{m,n,i,j} \mathbb{P}(|X_{i,j}| > x),\ x \in \mathbb{R}.$$

Then $F(\cdot)$ *is the distribution function of a random variable* X *if and only if* $\lim_{x\to\infty} F(x) = 1$. *In such a case,* $\{X_{m,n}, m \geq 1, n \geq 1\}$ *is* $\{a_{m,n,i,j}\}$*-stochastically dominated by* X.

Proof. It is clear that $F(\cdot)$ is nondecreasing, and

$$\lim_{x\to -\infty} F(x) = 1 - \frac{1}{C_0} \sup_{m\geq 1, n\geq 1} \sum_{i=1}^{k_m} \sum_{j=1}^{l_n} a_{m,n,i,j} = 0.$$

Let $\varepsilon > 0$ be arbitrary. For $a \in \mathbb{R}$, let $m_0 \geq 1, n_0 \geq 1$ be such that

$$\frac{1}{C_0} \sum_{i=1}^{k_{m_0}} \sum_{j=1}^{l_{n_0}} a_{m_0,n_0,i,j} \mathbb{P}(|X_{i,j}| > a) > \frac{1}{C_0} \sup_{m\geq 1, n\geq 1} \sum_{i=1}^{k_m} \sum_{j=1}^{l_n} a_{m,n,i,j} \mathbb{P}(|X_{i,j}| > a) - \varepsilon/2,$$

or equivalently,

$$1 - \frac{1}{C_0} \sum_{i=1}^{k_{m_0}} \sum_{j=1}^{l_{n_0}} a_{m_0,n_0,i,j} \mathbb{P}(|X_{i,j}| > a) < F(a) + \varepsilon/2. \tag{2.1}$$

Since the function

$$x \mapsto \frac{1}{C_0} \sum_{i=1}^{k_{m_0}} \sum_{j=1}^{l_{n_0}} a_{m_0,n_0,i,j} \mathbb{P}(|X_{i,j}| > x),\ x \in \mathbb{R},$$

is nonincreasing and right continuous, there exists $\delta > 0$ such that

$$-\varepsilon/2 < \frac{1}{C_0} \sum_{i=1}^{k_{m_0}} \sum_{j=1}^{l_{n_0}} a_{m_0,n_0,i,j} \mathbb{P}(|X_{i,j}| > x) - \frac{1}{C_0} \sum_{i=1}^{k_{m_0}} \sum_{j=1}^{l_{n_0}} a_{m_0,n_0,i,j} \mathbb{P}(|X_{i,j}| > a) \leq 0.$$

for all x such that $0 \le x - a < \delta$

Therefore, for x satisfying $0 \le x - a < \delta$, we have

$$\begin{aligned} F(x) - \varepsilon &= 1 - \frac{1}{C_0} \sup_{m\ge1,n\ge1} \sum_{i=1}^{k_m}\sum_{j=1}^{l_n} a_{m,n,i,j}\mathbb{P}(|X_{i,j}| > x) - \varepsilon \\ &\le 1 - \frac{1}{C_0} \sum_{i=1}^{k_{m_0}}\sum_{j=1}^{l_{n_0}} a_{m_0,n_0,i,j}\mathbb{P}(|X_{i,j}| > x) - \varepsilon \\ &< 1 - \frac{1}{C_0} \sum_{i=1}^{k_{m_0}}\sum_{j=1}^{l_{n_0}} a_{m_0,n_0,i,j}\mathbb{P}(|X_{i,j}| > a) - \varepsilon/2 \\ &< F(a) \text{ (by (2.1))} \end{aligned}$$

and so $|F(x) - F(a)| < \varepsilon$. Thus $\lim_{x\to a^+} F(x) = F(a)$. Since $a \in \mathbb{R}$ is arbitrary, this implies that F is right continuous on $\mathbb{R}$. Since $F(\cdot)$ is nondecreasing, right continuous and $\lim_{x\to-\infty} F(x) = 0$, it is the distribution function of a random variable X if and only if $\lim_{x\to\infty} F(x) = 1$. By definition of $F(\cdot)$, $\{X_{m,n}, m \ge 1, n \ge 1\}$ is $\{a_{m,n,i,j}\}$-stochastically dominated by X.

□

The next theorem shows that bounded moment conditions on a double array of random variables $\{X_{m,n}, m \ge 1, n \ge 1\}$ with respect to weights $\{a_{m,n,i,j}; 1 \le i \le k_m, 1 \le j \le l_n, m, n \ge 1\}$ can accomplish $\{a_{m,n,i,j}\}$-stochastic domination. This result extends the Theorem 2.6 of Thành [12].

Theorem 2.2. *Let $\{k_m, m \ge 1\}$ and $\{l_n, n \ge 1\}$ be sequences of positive integers, $\{X_{m,n}, m \ge 1, n \ge 1\}$ be a double array of random variables, $\{a_{m,n,i,j}; 1 \le i \le k_m, 1 \le j \le l_n, m, n \ge 1\}$ be sequences of positive constants satisfying (1.3). Let $p > 0$ and let ν be a fixed positive integer. Let $L(\cdot)$ be a slowly varying function. If*

$$\sup_{m\ge1,n\ge1} \sum_{i=1}^{k_m}\sum_{j=1}^{l_n} a_{m,n,i,j}\mathbb{E}\left(|X_{i,j}|^p L(|X_{i,j}|)\log_\nu^{(2)}(|X_{i,j}|)\right) < \infty, \tag{2.2}$$

then there exists a random variable X with distribution function

$$F(x) = 1 - \frac{1}{C_0} \sup_{m\ge1,n\ge1} \sum_{i=1}^{k_m}\sum_{j=1}^{l_n} a_{m,n,i,j}\mathbb{P}(|X_{i,j}| > x),\ x \in \mathbb{R} \tag{2.3}$$

such that $\{X_{m,n}, m \ge 1, n \ge 1\}$ is $\{a_{m,n,i,j}\}$-stochastically dominated by X, and

$$\mathbb{E}\left(|X|^p L(|X|)\right) < \infty. \tag{2.4}$$

Proof. Set

$$g(x) = x^p L(x)\log_\nu^{(2)}(x), \text{ and } h(x) = x^p L(x),\ x \geq 0.$$

Since $\lim_{x\to\infty} g(x) = \infty$ and $g(\cdot)$ is strictly increasing on $[0,\infty)$ as we have assumed before, we have from Markov's inequality and (2.2) that

$$0 \leq \lim_{x\to\infty} \sup_{m\geq 1,n\geq 1} \sum_{i=1}^{k_m}\sum_{j=1}^{l_n} a_{m,n,i,j}\mathbb{P}(|X_{i,j}| > x) \leq \lim_{x\to\infty}\frac{1}{g(x)} \sup_{m\geq 1,n\geq 1} \sum_{i=1}^{k_m}\sum_{j=1}^{l_n} a_{m,n,i,j}\mathbb{E}(g(|X_{i,j}|)) = 0.$$

By Theorem 2.1, the array $\{X_{m,n}, m \geq 1, n \geq 1\}$ is $\{a_{m,n,i,j}\}$-stochastically dominated by a random variable X with distribution function $F(\cdot)$ given in (2.3).

Next, we prove (2.4). We firstly consider the case where the slowly varying function $L(\cdot)$ is differentiable on an infinite interval far enough from 0, and

$$\lim_{x\to\infty}\frac{xL'(x)}{L(x)} = 0. \tag{2.5}$$

By (2.5), there exists $B \geq 0$ such that

$$\left|\frac{xL'(x)}{L(x)}\right| \leq \frac{p}{2},\ x > B.$$

It follows that

$$h'(x) = px^{p-1}L(x) + x^p L'(x) = x^{p-1}L(x)\left(p + \frac{xL'(x)}{L(x)}\right) \leq \frac{3px^{p-1}L(x)}{2},\ x \geq B. \tag{2.6}$$

Therefore, there exists a constant C_1 such that

$$\begin{aligned}
\mathbb{E}(h(X)) &= \mathbb{E}(h(X)\mathbf{1}(X \leq B)) + h(B) + \int_B^\infty h'(x)\mathbb{P}(X > x)\mathrm{d}x \\
&\leq C_1 + \frac{3p}{2}\int_B^\infty x^{p-1}L(x)\mathbb{P}(X > x)\mathrm{d}x \\
&= C_1 + \frac{3p}{2C_0}\int_B^\infty x^{p-1}L(x) \sup_{m\geq 1,n\geq 1}\sum_{i=1}^{k_m}\sum_{j=1}^{l_n} a_{m,n,i,j}\mathbb{P}(|X_{i,j}| > x)\mathrm{d}x \\
&\leq C_1 + \frac{3p}{2C_0}\int_B^\infty \frac{1}{x\log_\nu^{(2)}(x)} \sup_{m\geq 1,n\geq 1}\sum_{i=1}^{k_m}\sum_{j=1}^{l_n} a_{m,n,i,j}\mathbb{E}\left(g(|X_{i,j}|)\right)\mathrm{d}x \\
&= C_1 + \frac{3p}{2C_0}\sup_{m\geq 1,n\geq 1}\sum_{i=1}^{k_m}\sum_{j=1}^{l_n} a_{m,n,i,j}\mathbb{E}\left(g(|X_{i,j}|)\right)\int_B^\infty \frac{\mathrm{d}x}{x\log_\nu^{(2)}(x)} < \infty,
\end{aligned}$$

where we have applied Lemma 1.2 in the first equality, (2.6) in the first inequality, Markov's inequality in the second inequality, and (2.2) in the last inequality. Thus we obtain (2.4) in this case.

By Lemma 1.1, there exists a slowly varying function $L_1(\cdot)$ which is differentiable on $[B_1, \infty)$ for some B_1 large enough, and satisfies

$$\lim_{x\to\infty} \frac{L_1(x)}{L(x)} = 1 \tag{2.7}$$

and

$$\lim_{x\to\infty} \frac{xL_1'(x)}{L_1(x)} = 0.$$

For $i \geq 1, j \geq 1$, we have from (2.7) that for all B_2 large enough

$$\begin{aligned} &\mathbb{E}\left(|X_{i,j}|^p L_1(|X_{i,j}|)\log_\nu^{(2)}(|X_{i,j}|)\right) \\ &\quad= \mathbb{E}\left(|X_{i,j}|^p L_1(|X_{i,j}|)\log_\nu^{(2)}(|X_{i,j}|)\mathbf{1}(|X_{i,j}| \leq B_2)\right) \\ &\qquad+ \mathbb{E}\left(|X_{i,j}|^p L_1(|X_{i,j}|)\log_\nu^{(2)}(|X_{i,j}|)\mathbf{1}(|X_{i,j}| > B_2)\right) \\ &\quad\leq C_2 + 2\mathbb{E}\left(|X_{i,j}|^p L(|X_{i,j}|)\log_\nu^{(2)}(|X_{i,j}|)\mathbf{1}(|X_{i,j}| > B_2)\right), \end{aligned} \tag{2.8}$$

where C_2 is a finite constant. Combining (2.2) and (2.8) yields

$$\sup_{m\geq 1, n\geq 1} \sum_{i=1}^{k_m}\sum_{j=1}^{l_n} a_{m,n,i,j}\mathbb{E}\left(|X_{i,j}|^p L_1(|X_{i,j}|)\log_\nu^{(2)}(|X_{i,j}|)\right) < \infty. \tag{2.9}$$

Proceeding exactly the same manner as the first case with $L(\cdot)$ is replaced by $L_1(\cdot)$, we obtain (2.4). The proof of the theorem is completed. □ □

Remark 2.3. A weaker version of Theorem 2.2 for stochastic domination (without the appearance of the slowly varying function $L(\cdot)$) was proved by Rosalsky and Thành (see Theorem 2.5 (ii) and (iii) in Rosalsky and Thành [10]). Typical examples of slowly varying functions $L(\cdot)$ for (2.2) are $L(x) \equiv 1$ and $L(x) \equiv L_1(x)(\log_\nu^{(2)}(x))^{-1}$, where $L_1(\cdot)$ is another slowly varying function. Theorem 2.2 is proved by employing an idea from Galambos and Seneta [6].

The following corollary follows immediately from Theorem 2.2. We note that the result in Corollary 2.4 was stated for the d-dimensional array of random variables with $\nu = 2$, which was shown by Dat et al. in [4]. The new result we present shows that $\{X_{m,n}, m \geq 1, n \geq 1\}$ be a double array of random variables and ν be a fixed positive integer.

Corollary 2.4. *Let $\{k_m, m \geq 1\}$ and $\{l_n, n \geq 1\}$ be sequences of positive integers, let $\{X_{m,n}, m \geq 1, n \geq 1\}$ be a double array of random variables and let $L(\cdot)$ be a slowly* *varying function. Let $p > 0$ and let ν be a fixed positive integer. If*

$$\sup_{m\ge1,n\ge1} \frac{1}{k_m l_n} \sum_{i=1}^{k_m} \sum_{j=1}^{l_n} \mathbb{E}\left(|X_{i,j}|^p L(|X_{i,j}|) \log_\nu^{(2)}(|X_{i,j}|)\right) < \infty,$$

then there exists a random variable X with distribution function

$$F(x) = 1 - \sup_{m\ge1,n\ge1} \frac{1}{k_m l_n} \sum_{i=1}^{k_m} \sum_{j=1}^{l_n} \mathbb{P}(|X_{i,j}| > x),\ x \in \mathbb{R}$$

such that $\{X_{m,n}, m \ge 1, n \ge 1\}$ is stochastically dominated in the Cesàro sense by X, and

$$\mathbb{E}\left(|X|^p L(|X|)\right) < \infty.$$

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TÓM TẮT

MỘT SỐ KẾT QUẢ ĐỐI VỚI MẢNG HAI CHIỀU CÁC BIẾN NGẪU NHIÊN $\{a_{m,n,i,j}\}$-BỊ CHẶN NGẪU NHIÊN

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Ngày nhận bài 06/11/2024, ngày nhận đăng 12/02/2025

Trong bài báo này, chúng tôi chứng minh một số kết quả đối với mảng hai chiều các biến ngẫu nhiên $\{a_{m,n,i,j}\}$-bị chặn ngẫu nhiên, trong đó $\{k_m, m \geq 1\}$ và $\{l_n, n \geq 1\}$ là dãy các số nguyên dương, $\{a_{m,n,i,j}; 1 \leq i \leq k_m, 1 \leq j \leq l_n, m, n \geq 1\}$ là các dãy các hằng số dương thỏa mãn điều kiện

$$\sup_{m\geq 1, n\geq 1} \sum_{i=1}^{k_m} \sum_{j=1}^{l_n} a_{m,n,i,j} = C_0, C_0 \in (0, \infty).$$

Kết quả chính của bài báo là phát triển Định lý 2.1 và Định lý 2.6 của Thành (2023) cho trường hợp mảng hai chiều các biến ngẫu nhiên.

Từ khóa: Bị chặn ngẫu nhiên; mảng hai chiều; hàm biến đổi chậm; bị chặn ngẫu nhiên theo nghĩa Cesàro.