

PARAMETRIC RESONANCE OF ACOUSTIC AND OPTICAL PHONONS IN A DOPED SEMICONDUCTOR SUPERLATTICE IN THE PRESENCE OF A LASER FIELD

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Abstract: In this paper, we analytically investigated the possibility of parametric resonance of acoustic and optical phonons. We obtained a general dispersion equation for parametric amplification and transformation of phonons. The dispersions of the resonant acoustic phonon modes and the threshold amplitude of the field for acoustic phonon parametric amplification are obtained. The parametric amplification for acoustic phonons in a doped semiconductor superlattice can occur under the condition that the amplitude of the external electromagnetic field is higher than the threshold amplitude.

Keyword: Phonon; acoustic phonon; optical phonon; semiconductor superlattice.

1. INTRODUCTION

Resonance effects in general and parametric resonance are important processes in physics research. For many beneficial resonance processes, we try to strengthen but there are also processes that people limit to eliminate. The study of parametric resonance and parameter enhancement in low-dimensional systems is expected to provide many important bases for many applications in modern physics and engineering, especially in low-dimensional materials engineering, microelectronics technology, information technology... [1], [2].

As we know, in the presence of electromagnetic waves, the electron gas environment becomes non-stop, when the parametric resonance condition is satisfied, the parameter interaction or the same type of stimulus will appear (phonon-phonon) or between phonon-plasmon types, meaning that the process of converting energy from one type of stimulus to another stimulus appears.

Resonance of phonon parameters and optical phonons in conventional semiconductor semiconductors in the presence of electrons has been studied in recent years [3-5]. This phenomenon can be understood as follows: when the presence of electromagnetic waves (laser field) with frequency Ω will appear frequency density electronic waves $\omega_{\bar{q}} \pm \ell\Omega$ ($\ell=1,2,\dots$). If a certain frequency of these electron density waves coincides with the optical phonon frequency $\nu_{\bar{q}}$, the optical phonons are increased. These optical phonons then generate electron density waves with frequency $\nu_{\bar{q}} \pm \ell\Omega$ ($\ell=1,2,\dots$), and when a frequency of the electron density wave coincides with the frequency of some acoustic phonon $\omega_{\bar{q}}$, it increases the acoustic phonon.

In this paper, we study the parametric resonance between acoustic phonons and optical phonons in doped semiconductor super-lattice (DSSL) [6] in the case of degenerate electron gas, from that we find the dispersion resonance frequency and the amplitude condition of the laser field for this resonance.

II. THE GENERAL DISPERSION EQUATION

The research model is a DSSL created from two identical but doped semiconductors. The DSSL is created by the periodic spatial distribution of charges. An example of such a DSSL is created by the recirculation arrangement of thin-type GaAs semiconductor layers (GaAs:Si) and GaAs p-type (GaAs:Be) separated by non-doped classes (called is n-i-p-i crystal).

Suppose the electromagnetic wave propagates along the axis of the Oz axis of the super-network penetrating into the sample, the vector of electric field strength of the electromagnetic wave takes the form $\vec{E} = \vec{E}_0 \sin(\Omega t)$, \vec{E}_0 direction perpendicular to the axis Oz. Assuming the DSSL does not confine the phonon, the electron energy is quantized and each state of the electron is characterized by the quantization index n and the wave vector \vec{k}_\perp directed perpendicularly to the axis Oz. Hamiltonian of the electron-phonon system when a laser field is present, in the secondary quantum representation, assuming two phonons are present but not dispersed (block phonons) of the form [7]:

$$\hat{H}(t) = \hat{H}_0(t) + \hat{H}_{e-ac} + \hat{H}_{e-op} \quad (1)$$

where $\hat{H}_0(t)$ is the Hamiltonian of the electron-phonon system that does not interact with the laser field.

$$\hat{H}_0(t) = \sum_n \varepsilon_n \left(\vec{k}_\perp - \frac{e}{\hbar c} \vec{A}(t) \right) a_n^+ a_n + \sum_{\vec{q}} \hbar \omega_{\vec{q}} b_{\vec{q}}^+ b_{\vec{q}} + \sum_{\vec{q}} \hbar \nu_{\vec{q}} c_{\vec{q}}^+ c_{\vec{q}} \quad (2)$$

where $|n\rangle = |\vec{k}_\perp, n\rangle$ is the electron state corresponding to the wave vector \vec{k}_\perp ; a_n^+, a_n are the creation and annihilation operators of electron in the $|n\rangle$ state; $b_{\vec{q}}^+, b_{\vec{q}}$ ($c_{\vec{q}}^+, c_{\vec{q}}$) are the creation and annihilation operators of acoustic phonon (optical phonon) with wave vector \vec{q} ; $\omega_{\vec{q}}$ ($\nu_{\vec{q}}$) is the frequency of acoustic phonon (optical phonon) that due to the electron-phonon interaction; c is the speed of light in a vacuum; ε_n is the energy spectrum of electrons in the DSSL, in the form [6]:

$$\varepsilon_n(\vec{k}_\perp) = \hbar \omega_p \left(n + \frac{1}{2} \right) + \frac{\hbar^2 \vec{k}_\perp^2}{2m_e} = \varepsilon_n + \frac{\hbar^2 \vec{k}_\perp^2}{2m_e} \quad (3)$$

where $n = 0, 1, 2, \dots$ and $\omega_p = \left(\frac{4\pi e^2 n_D}{\kappa_0 m_e} \right)$ is the plasma frequency caused by donor impurities with a concentration of doped n_D , κ_0 is static dielectric constant, e is electron charge, m_e is the effective mass of the electron; $\vec{A}(t)$ is the vector potential associated with the electric field intensity vector of the laser field:

$$\vec{A} = \vec{A}_0 \cos(\Omega t), A_0 = \frac{cE_0}{\Omega} \quad (4)$$

The Hamiltonian interaction between electron-phonon $\hat{H}_{e-ac}, \hat{H}_{e-op}$ given by:

$$\hat{H}_{e-ac} = \sum_{\vec{q}} \sum_{n,n'} G_{n,n'}(\vec{q}) a_n^+ a_n (b_{-\vec{q}}^+ + b_{\vec{q}}) \quad (5)$$

$$\hat{H}_{e-op} = \sum_{\vec{q}} \sum_{n,n'} D_{n,n'}(\vec{q}) a_n^+ a_n (c_{-\vec{q}}^+ + c_{\vec{q}}) \quad (6)$$

With the assumption of block phonons, the electron-phonon interaction coefficients $G_{n,n'}(\vec{q})$, $D_{n,n'}(\vec{q})$ have form:

$$G_{n,n'}(\vec{q}) = G_{\vec{q}} M_{n,n'}(q_z), D_{n,n'}(\vec{q}) = D_{\vec{q}} M_{n,n'}(q_z) \quad (7)$$

$$\text{with: } |G_{\vec{q}}|^2 = \frac{\hbar q \xi^2}{2\rho v_a V}, |D_{\vec{q}}|^2 = \frac{\hbar e^2 v_{\vec{q}}}{2Vq^2} \left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0} \right) \quad (8)$$

where V, ρ, v_a, ξ are the volume, the density, the acoustic velocity and the deformation potential constant, respectively; χ_0, χ_∞ are the static and the high-frequency dielectric constants, respectively; and $M_{n,n'}(q_z)$ is electron form factor in DSSL, it has form [7]:

$$M_{n,n'} = \sum_{j=1}^{N_d} \int_0^d \Phi_n(z-jd) \Phi_{n'}(z-jd) \exp(iq_z d) dz \quad (9)$$

$\Phi_n(z)$ is the eigenfunction of the electron for an individual potential well, N_d is the number of periods of the DSSL, d is the periods of the DSSL.

In order to establish a set of quantum transport equations for acoustic and optical phonons, we use the general quantum distribution functions for the phonons [6]. $\langle \Psi \rangle_t = Tr \langle \hat{W} \hat{\Psi} \rangle_t$, where \hat{W} is the density matrix operator; $\langle \Psi \rangle_t$ denotes a statistical average at the moment t .

We look for quantum dynamic equations for $\langle b_{\vec{q}} \rangle_t$:

$$\frac{\partial}{\partial t} \langle b_{\vec{q}} \rangle_t = \frac{i}{\hbar} \langle [\hat{H}(t), b_{\vec{q}}] \rangle \quad (10)$$

Performing algebraic calculations, we get quantum dynamic equations:

$$\begin{aligned} \frac{\partial}{\partial t} \langle b_{\vec{q}} \rangle_t &= \frac{i}{\hbar^2} \sum_{n,n',\vec{k}_\perp} \sum_{s,s'=-\infty}^{+\infty} J_s(\lambda) J_{s'}(\lambda) \exp(i(s-s')\Omega t) (f_n(\vec{k}_\perp) - f_{n'}(\vec{k}_\perp - \vec{q})) \times \\ &\times \int_{-\infty}^t \left\{ |G_{n,n'}(\vec{q})|^2 (\langle b_{\vec{q}} \rangle_{t'} + \langle b_{-\vec{q}}^+ \rangle_{t'}) + G_{n,n'}(-\vec{q}) D_{n,n'}(\vec{q}) (\langle c_{\vec{q}} \rangle_{t'} + \langle c_{-\vec{q}}^+ \rangle_{t'}) \right\} \times \\ &\times \exp\left(\frac{i}{\hbar} \left[\varepsilon_n(\vec{k}_\perp - \vec{q}) - \varepsilon_{n'}(\vec{k}_\perp) + s\hbar\Omega \right] (t' - t) dt' \right) \end{aligned} \quad (11)$$

Here $J_s(\lambda)$ is Bessel function, $f_n(\vec{k}_\perp)$ is the distribution function of the electron in $|n\rangle$ state.

From Eq. (11) we can obtain an equation for the Fourier transformation $B_{\vec{q}}(\omega)$ and $\langle b_{\vec{q}} \rangle_t$:

$$\begin{aligned}
 (\omega - \omega_{\vec{q}})B_{\vec{q}}(\omega) &= \frac{2}{\hbar^2} \sum_{n,n'} \sum_{\ell=-\infty}^{+\infty} |G_{n,n'}(\vec{q})|^2 \frac{\omega_{\vec{q}}}{\omega - \ell\Omega + \omega_{\vec{q}}} B_{\vec{q}}(\omega - \ell\Omega) P_{\ell}(\vec{q}, \omega) + \\
 &+ \frac{2}{\hbar^2} \sum_{n,n'} \sum_{\ell=-\infty}^{+\infty} G_{n,n'}(-\vec{q}) D_{n,n'}(\vec{q}) \frac{\nu_{\vec{q}}}{\omega - \ell\Omega + \nu_{\vec{q}}} C_{\vec{q}}(\omega - \ell\Omega) P_{\ell}(\vec{q}, \omega)
 \end{aligned} \tag{12}$$

Similarly for $C_{\vec{q}}(\omega)$ we also have:

$$\begin{aligned}
 (\omega - \omega_{\vec{q}})C_{\vec{q}}(\omega) &= \frac{2}{\hbar^2} \sum_{n,n'} \sum_{\ell=-\infty}^{+\infty} |D_{n,n'}(\vec{q})|^2 \frac{\nu_{\vec{q}}}{\omega - \ell\Omega + \nu_{\vec{q}}} C_{\vec{q}}(\omega - \ell\Omega) P_{\ell}(\vec{q}, \omega) + \\
 &+ \frac{2}{\hbar^2} \sum_{n,n'} \sum_{\ell=-\infty}^{+\infty} G_{n,n'}(\vec{q}) D_{n,n'}(-\vec{q}) \frac{\omega_{\vec{q}}}{\omega - \ell\Omega + \omega_{\vec{q}}} B_{\vec{q}}(\omega - \ell\Omega) P_{\ell}(\vec{q}, \omega)
 \end{aligned} \tag{13}$$

where
$$P_{\ell}(\omega, \vec{q}) = \sum_s J_s(\lambda) J_{s+\ell}(\lambda) \Gamma_{\vec{q}}(\omega + s\Omega);$$

$$\Gamma_{\vec{q}}(\omega + s\Omega) = \sum_{\vec{k}_{\perp}} \frac{\hbar [f_{n'}(\vec{k}_{\perp}) - f_{n'}(\vec{k}_{\perp} - \vec{q})]}{\varepsilon_{n'}(\vec{k}_{\perp}) - \varepsilon_{n'}(\vec{k}_{\perp} - \vec{q}) - \hbar(\omega + s\Omega) - i\delta}$$

Here, we pay attention to the hypothesis of the thermal segment of interaction by multiplying the factor $e^{i\delta}$ ($\delta \rightarrow +0$).

We see equations (12) and (13) describe the interaction between two phonons and the other. So if we only consider the interaction between two different types of phonons in the first term of the right side of the two equations we only get $\ell = 0$.

Rewriting the equation and transformation we obtain the general dispersion equation for parametric resonance between acoustic phonon and optical phonon:

$$\begin{aligned}
 \left[\omega^2 - \omega_{\vec{q}}^2 - \frac{2}{\hbar^2} \sum_{n,n'} |G_{n,n'}(\vec{q})|^2 \omega_{\vec{q}} P_0(\vec{q}, \omega) \right] \left[\omega^2 - \nu_{\vec{q}}^2 - \frac{2}{\hbar^2} \sum_{n,n'} |D_{n,n'}(\vec{q})|^2 \nu_{\vec{q}} P_0(\vec{q}, \omega) \right] = \\
 = \frac{4}{\hbar^2} \sum_{n,n'} \sum_{\ell=-\infty}^{\ell=+\infty} |G_{n,n'}(\vec{q})|^2 |D_{n,n'}(\vec{q})|^2 \omega_{\vec{q}} \nu_{\vec{q}} P_{\ell}(\vec{q}, \omega) P_{\ell}(\vec{q}, \omega - \ell\Omega)
 \end{aligned} \tag{14}$$

The general dispersion equation (14) for parametric resonance of the two types of phonons we just found plays a decisive role in the phonon re-normalization study by interacting with electrons in the presence of the electromagnetic field. From this equation, we can determine the incremental conditions and transform the parameters of these stimuli to another.

The dispersion equation (14) is general and can be used for degenerate and non-degenerate electronic gases, for both receivers one or more photons.

III. CONDITIONS FOR INCREASING PHONON PARAMETRIC RESONANCE IN CASE OF DEGENERATIVE ELECTRONIC GAS

When the condition of resonate parameter between acoustic phonon and optical phonon is done, ie $|\omega_{\vec{q}} - N\Omega| = \nu_{\vec{q}}$ (N is a specified integer) the sum is followed ℓ , on the right side of equation (14) there is only one term left $\ell = N$.

To pretend that the dispersion equation (14) is very complex, here we consider only the first parameter resonance case $\omega_{\bar{q}} \pm v_{\bar{q}} = \Omega$ and $|G_{n,n'}(\bar{q})|^2 |D_{n,n'}(\bar{q})|^2 \ll 1$, in the case there are:

$$\omega^2 - \omega_{\bar{q}}^2 - \frac{2}{\hbar^2} \sum_{n,n'} |G_{n,n'}(\bar{q})|^2 \omega_{\bar{q}} P_0(\bar{q}, \omega) = 0 \quad (15)$$

$$\omega^2 - v_{\bar{q}}^2 - \frac{2}{\hbar^2} \sum_{n,n'} |D_{n,n'}(\bar{q})|^2 v_{\bar{q}} P_0(\bar{q}, \omega) = 0 \quad (16)$$

Spectrum of acoustic phonon and optical phonon is written in the form $\omega_{ac}(\bar{q}) = \omega_a + i\gamma_a$; $\omega_{op}(\bar{q}) = \omega_o + i\gamma_o$.

For acoustic phonon, we have:

$$\begin{cases} \omega_a \approx \omega_{\bar{q}} + \frac{1}{\hbar^2} \sum_{n,n'} |G_{n,n'}(\bar{q})|^2 \text{Re} P_0(\bar{q}, \omega_{\bar{q}}) \\ \gamma_a = -\frac{1}{\hbar^2} \sum_{n,n'} |G_{n,n'}(\bar{q})|^2 \text{Im} P_0(\bar{q}, \omega_{\bar{q}}) \end{cases} \quad (17)$$

For optical phonon, we have:

$$\begin{cases} v_o \approx \omega_{\bar{q}} + \frac{1}{\hbar^2} \sum_{n,n'} |D_{n,n'}(\bar{q})|^2 \text{Re} P_0(\bar{q}, v_{\bar{q}}) \\ \gamma_o = -\frac{1}{\hbar^2} \sum_{n,n'} |D_{n,n'}(\bar{q})|^2 \text{Im} P_0(\bar{q}, v_{\bar{q}}) \end{cases} \quad (18)$$

If $\omega_{\bar{q}} \equiv v_{\bar{q}}$ and the wave vector overlapping acoustic waves and optical waves merged together, this time is the greatest resonance, assuming at (ω_0, q_0) . We examine dependence on ω and q near the intersection (ω_0, q_0) according to the electronic interaction constant and phonon is[7]:

$$\omega_{\pm}^{\pm} = \omega_a + \frac{1}{2} \left[(v_a \pm v_o) \Delta(q) - i(\gamma_a + \gamma_o) \pm \sqrt{[(v_a \mp v_o) \Delta(q) - i(\gamma_a - \gamma_o)]^2 \pm \Lambda^2} \right] \quad (19)$$

Where $v_a(v_o)$ is the group velocity of acoustic phonon (optical phonon); ω_a is acoustic phonon frequency re-normalized due to electron-phonon interaction; $\Delta(q) = q - q_0$ is the intersection distance of the dispersion line; $\Delta(q) \ll q$ and $\Lambda = \frac{2}{\hbar^2} \sum_{n,n'} |G_{n,n'}(\bar{q})| |D_{n,n'}(\bar{q})| P_N(\bar{q}, \omega_{\bar{q}})$.

In the expression (19): the signs (\pm) in the subscript of ω_{\pm}^{\pm} correspond to the signs (\pm) in front of the root and the signs (\pm) in the superscript of ω_{\pm}^{\pm} correspond to the other sign pairs. These marks are chosen from the selection of resonance conditions. From equation (19), the expression ω_{+}^{-} corresponds to the increase in acoustic phonon parameters when selecting resonance conditions $v_{\bar{q}} + \omega_{\bar{q}} = N\Omega$. We find an increase in

the number of acoustic phonons for the case $\Delta(q) = 0$. The condition for an increase in acoustic phonon parameters is imaginary parts of ω_{\pm}^{\pm} must be positive, ie:

$$\text{Im } \omega_{+}^{-} = \frac{1}{2} \left(-(\gamma_a + \gamma_0) + \sqrt{(\gamma_a - \gamma_0)^2 + \Lambda^2} \right) > 0 \quad (20)$$

In case of $N = 1$, we get:

$$\Lambda = \frac{2}{\hbar^2} \sum_{n,n'} |G_{n,n'}(\vec{q})| |D_{n,n'}(\vec{q})| P_N(\vec{q}, \omega_{\vec{q}}) = \frac{2}{\hbar^2} \sum_{n,n'} |G_{n,n'}(\vec{q})| |D_{n,n'}(\vec{q})| \frac{\lambda}{2} [\Gamma_{\vec{q}}(\omega_{\vec{q}}) - \Gamma_{\vec{q}}(\omega_{\vec{q}} - \Omega)] \quad (21)$$

Replace expression (21) into (20) we get:

$$\lambda^2 > 4 \frac{\text{Im } \Gamma_{\vec{q}}(\omega_{\vec{q}}) \Gamma_{\vec{q}}(v_{\vec{q}})}{(\text{Re } \Gamma_{\vec{q}}(\omega_{\vec{q}}) - \text{Re } \Gamma_{\vec{q}}(\omega_{\vec{q}} - \Omega))^2} \quad (22)$$

When thermal energy $k_B T$ is much smaller than Fermi energy, electronic gas degrades. The gas distribution function now takes the form:

$$f_n(\vec{k}_{\perp}) = \theta(\varepsilon_F - \varepsilon_n(\vec{k}_{\perp})) = \begin{cases} 1 & \text{when } \varepsilon_F > \varepsilon_n(\vec{k}_{\perp}) \\ 0 & \text{when } \varepsilon_F < \varepsilon_n(\vec{k}_{\perp}) \end{cases} \quad (23)$$

Perform calculations instead (23) into (22), note $\lambda = \frac{eqE_0}{m_e \Omega^2}$ we get the condition

of the electromagnetic field amplitude to have an increase in acoustic phonon:

$$E_0 > \frac{2m_e^3 \Omega}{e \hbar^3 q^3} \frac{\varepsilon_{nn'}(\omega_{\vec{q}}) (\varepsilon_{nn'}(\omega_{\vec{q}}) + \hbar \Omega)}{\sqrt{2m_e (\varepsilon_F - \varepsilon_{n'})} - \sqrt{2m_e (\varepsilon_F - \varepsilon_n - \alpha q^2)}} \times \\ \times \left[\left(\frac{2\hbar^2 q^2}{m_e} (\varepsilon_F - \varepsilon_{n'}) - \varepsilon_{nn'}(v_{\vec{q}}) \right) - \left(\frac{2\hbar^2 q^2}{m_e} (\varepsilon_F - \varepsilon_n + \hbar v_{\vec{q}}) - \varepsilon_{nn'}^2(v_{\vec{q}}) \right)^{1/2} \right]^{1/2} \times \quad (24) \\ \times \left[\left(\frac{2\hbar^2 q^2}{m_e} (\varepsilon_F - \varepsilon_{n'}) - \varepsilon_{nn'}(\omega_{\vec{q}}) \right) - \left(\frac{2\hbar^2 q^2}{m_e} (\varepsilon_F - \varepsilon_n + \hbar \omega_{\vec{q}}) - \varepsilon_{nn'}^2(\omega_{\vec{q}}) \right)^{1/2} \right]^{1/2} = E_{th}$$

$$\text{here } \varepsilon_{nn'}(\omega) = \varepsilon_{n'} - \varepsilon_n - \hbar \omega - \alpha q^2; \alpha = \frac{\hbar^2}{2m_e} .$$

IV. CONCLUSIONS

In this paper, we analytically investigated the possibility of parametric resonance of acoustic and optical phonons in DSSL. We obtained a general dispersion equation for parametric amplification and transformation of phonons. However, an analytical solution to the equation could only be obtained within some limitations. Using these limitations for simplicity, we obtained dispersions of the resonant acoustic phonon modes and the threshold amplitude of the field for acoustic phonon parametric amplification. The parametric amplification for acoustic phonons in a doped superlattice could occur under

the condition that the amplitude of the external electromagnetic field is higher than the threshold amplitude. Analytical expressions show that the threshold amplitude depends on the field, the material and the physical conditions.

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TÓM TẮT

CỘNG HƯỞNG THAM SỐ CỦA PHONON ÂM VÀ PHONON QUANG TRONG SIÊU MẠNG PHA TẠP KHI CÓ MẶT TRƯỜNG LASER

Trong bài báo này, chúng tôi đã thiết lập phương trình động lượng tử cho quá trình cộng hưởng tham số giữa phonon âm và phonon quang trong siêu mạng bán dẫn pha tạp dưới tác dụng của trường laser và điều kiện gia tăng tham số phonon âm trong siêu mạng bán dẫn pha tạp cho trường hợp khí điện tử suy biến. Sự gia tăng tham số cho các phonon âm trong siêu mạng bán dẫn pha tạp có thể xảy ra khi biên độ trường điện từ ngoài lớn hơn biên độ ngưỡng.

Từ khóa: Phonon; phonon âm; phonon quang; siêu mạng bán dẫn.