CREATION OF BELL-LIKE STATE BY A NONLINEAR QUANTUM SCISSORS INTERACTIVE WITH TWO EXTERNAL COHERENT FIELDS

Doan Quoc Khoa
Faculty of Advanced Science and Technology, University of Science and Technology, The University of Danang, Vietnam

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**** Correspondence:***
dqkhoa@dut.udn.vn
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ABSTRACT
The study described in this paper focuses on a model featuring two nonlinear oscillators, which are stimulated by two external coherent fields, and are referred to as a nonlinear quantum scissors (NQS). To analyze the system's evolution, the NQS method for quantum state engineering was applied, leading to the discovery of a wave function that comprises a composite of n-photon states. Through the implementation of NQS, the system's optical states undergo a truncation process, resulting in the production of two-qubit states that can be achieved due to the oscillators' nonlinear characteristics and their interaction. As the system evolves, maximally entangled states referred to as Bell-like states are created.

Keywords: Kerr nonlinear coupler; Bell-like state; nonlinear quantum scissors.

1. Introduction

The application of quantum entanglement in quantum computing is of paramount importance [1]. It forms a fundamental component of numerous algorithms in quantum cryptography, quantum teleportation, and superdense coding. One way to produce entangled states is through the reciprocal interactions between the modes of optical fields, which results in the creation of entanglement in photon-number states [2]. Of all the methods available, this approach is the most promising due to the quick progress made in developing photon-number resolving detectors based on diverse mechanisms [3]. Hence, the generation of truncated states that are closely linked to quantum entanglement is of paramount importance in resolving key issues in the field of quantum information theory.

Currently, there is research on the NQS model for the two-mode state [4], which involves two quantum oscillators with Kerr-like nonlinearities. Various forms of coupling between the oscillators have been explored, including linear [5] and nonlinear [6] coupling. It has been demonstrated that the examined model can generate finite-dimensional quantum states, resulting in the production of truncated states of one, two, or three dimensions.

This paper deals with the study of a system comprising a nonlinear coupler and two external coherent fields [7]. The method described in [8] has been utilized to solve the system's evolution equation in the absence of damping. It has been acknowledged in [9] that the preparation of the system's initial Fock states is more challenging than that of the vacuum states only. However, despite imperfect initial state preparation, the truncated state can still have the desired form with a non-zero probability. Therefore, the initial conditions for the evolution equation can be selected arbitrarily.

The structure of this paper is as follows: Sec. 2 provides a comprehensive presentation of the model of Kerr-like nonlinear coupler. Sec. 3 discusses and describes the creation of maximally entangled states. The final Sec. 4 presents the conclusions of the study.

2. The Kerr-like nonlinear coupler model driven in two modes

The system under consideration is a Kerr-like nonlinear coupler with two nonlinear oscillators, denoted by χ_a and χ_b, and corresponding field modes *a* and *b*. The two oscillators are nonlinearly coupled to each other and are pumped by two external coherent fields. In the interaction picture, the Hamiltonian can be used to describe this system as

$$\hat{H} = \frac{\chi_a}{2}(\hat{a}^+)^2\hat{a}^2 + \frac{\chi_b}{2}\left(\hat{b}^+\right)^2\hat{b}^2 + \varepsilon(\hat{a}^+)^2\hat{b}^2 + \varepsilon^*\left(\hat{b}^+\right)^2\hat{a}^2 + \alpha\hat{a}^+ + \alpha^*\hat{a} + \beta\hat{b}^+ + \beta^*\hat{b} \quad (1)$$

where the operators $\hat{a}\left(\hat{b}\right)$ and $\hat{a}^+\left(\hat{b}^+\right)$, respectively, represent the annihilation and creation of bosonic particles for the *a* and *b* oscillation modes. The coupling between the two nonlinear oscillators is represented by the symbol ε, while the external coherent fields are coupled to the modes *a* and *b* with α and β, respectively.

The present model is limited to the case without damping. By utilizing the Schrödinger equation in the interaction picture, we derive equations of motion for the variable $c_{kl}(t)$

$$\begin{aligned} i\frac{d}{dt}c_{kl}(t) = &\left[\frac{1}{2}\chi_a k(k-1) + \frac{1}{2}\chi_b l(l-1)\right]c_{kl}(t) \\ &+ \varepsilon\sqrt{(l+2)(l+1)k(k-1)}c_{k-2,l+2}(t) \\ &+ \varepsilon^*\sqrt{(k+2)(k+1)l(l-1)}c_{k+2,l-2}(t) \\ &+ \alpha^*\sqrt{k+1}c_{k+1,l}(t) + \alpha\sqrt{k}c_{k-1,l}(t) \\ &+ \beta^*\sqrt{l+1}c_{k,l+1}(t) + \beta\sqrt{l}c_{k,l-1}(t). \end{aligned} \quad (2)$$

By using the formalism of NQS as in [4], the system's wave function can be represented solely using four states, namely $|2\rangle_a|0\rangle_b, |2\rangle_a|1\rangle_b, |1\rangle_a|2\rangle_b$ and $|0\rangle_a|2\rangle_b$, which have the following form

$$|\psi(t)\rangle_{cut} = c_{20}(t)|2\rangle_a|0\rangle_b + c_{21}(t)|2\rangle_a|1\rangle_b + c_{12}(t)|1\rangle_a|2\rangle_b + c_{02}(t)|0\rangle_a|2\rangle_b. \quad (3)$$

For the general case, set $\alpha = m\varepsilon$. Besides, assume that the coherent fields have the same real values, namely $\alpha = \beta$. By plugging in these values to equation (2), the complex probability amplitudes can be expressed as follows.

$$
\begin{aligned}
& i\frac{d}{dt}c_{20}(t) = 2\varepsilon c_{02}(t) + m\varepsilon^* c_{21}(t),\\
& i\frac{d}{dt}c_{21}(t) = m\varepsilon c_{20}(t),\\
& i\frac{d}{dt}c_{12}(t) = m\varepsilon c_{02}(t),\\
& i\frac{d}{dt}c_{02}(t) = 2\varepsilon^* c_{20}(t) + m\varepsilon^* c_{12}(t),
\end{aligned} \tag{4}
$$

Solving the equations (4) with the initial state is $|2\rangle_a |1\rangle_b$ leads to the solution of complex amplitudes that have simple form as follow

$$
\begin{aligned}
& c_{20}(t) = -\frac{i(\phi_1\phi_2)^{\frac{1}{2}}}{\phi_1+\phi_2}\left[\sin(\phi_1\varepsilon t) + \sin(\phi_2\varepsilon t)\right],\\
& c_{21}(t) = \frac{1}{\phi_1+\phi_2}\left[\phi_2 \cos(\phi_1\varepsilon t) + \phi_1 \cos(\phi_2\varepsilon t)\right],\\
& c_{12}(t) = -\frac{i}{\phi_1+\phi_2}\left[\phi_2 \sin(\phi_1\varepsilon t) - \phi_1 \sin(\phi_2\varepsilon t)\right],\\
& c_{02}(t) = \frac{(\phi_1\phi_2)^{\frac{1}{2}}}{\phi_1+\phi_2}\left[\cos(\phi_1\varepsilon t) - \cos(\phi_2\varepsilon t)\right],
\end{aligned} \tag{5}
$$

where $\varphi_1 = (m^2+1)^{\frac{1}{2}}+1$, and $\varphi_2 = (m^2+1)^{\frac{1}{2}}-1$.

3. Generating Bell-like states in the Kerr-like nonlinear coupler

Maximally entangled states, namely the output states $|\psi(t)\rangle_{cut}$ generated by our model, are described in this section. The time-evolution of the entanglement states of the system is depicted in terms of von Neumann entropy, which has been defined in [10], to obtain these states. From the expression (3) and full density matrix $\hat{\rho}_{ab} = |\psi\rangle_{cutcut}\langle\psi|$ as

$$
\begin{aligned}
\hat{\rho}_{ab} &= |\psi\rangle_{cutcut}\langle\psi|\\
&= |c_{02}|^2|0\rangle_a|2\rangle_{bb}\langle 2|_a\langle 0| + c_{02}c_{21}^*|0\rangle_a|2\rangle_{bb}\langle 1|_a\langle 2| + c_{02}c_{12}^*|0\rangle_a|2\rangle_{bb}\langle 2|_a\langle 1| + c_{02}c_{20}^*|0\rangle_a|2\rangle_{bb}\langle 0|_a\langle 2|\\
&+ c_{21}c_{02}^*|2\rangle_a|1\rangle_{bb}\langle 2|_a\langle 0| + |c_{21}|^2|2\rangle_a|1\rangle_{bb}\langle 1|_a\langle 2| + c_{21}c_{12}^*|2\rangle_a|1\rangle_{bb}\langle 2|_a\langle 1| + c_{21}c_{20}^*|2\rangle_a|1\rangle_{bb}\langle 0|_a\langle 2|\\
&+ c_{12}c_{02}^*|1\rangle_a|2\rangle_{bb}\langle 2|_a\langle 0| + c_{12}c_{21}^*|1\rangle_a|2\rangle_{bb}\langle 1|_a\langle 2| + |c_{12}|^2|1\rangle_a|2\rangle_{bb}\langle 2|_a\langle 1| + c_{12}c_{20}^*|1\rangle_a|2\rangle_{bb}\langle 0|_a\langle 2|\\
&+ c_{20}c_{02}|2\rangle_a|0\rangle_{bb}\langle 2|_a\langle 0| + c_{20}c_{21}^*|2\rangle_a|0\rangle_{bb}\langle 1|_a\langle 2| + c_{20}c_{12}|2\rangle_a|0\rangle_{bb}\langle 2|_a\langle 1| + |c_{20}|^2|2\rangle_a|0\rangle_{bb}\langle 0|_a\langle 2|,
\end{aligned} \tag{6}
$$

the partial trace of $\hat{\rho}_{ab}$ with respect to mode *b* can be expressed as follows:

$$\begin{aligned}\hat{\rho}_b &= Tr_a\rho_{ab} = {}_a\langle 0|\rho_{ab}|0\rangle_a + {}_a\langle 1|\rho_{ab}|1\rangle_a + {}_a\langle 2|\rho_{ab}|2\rangle_a \\ &= |c_{20}|^2|0\rangle_{bb}\langle 0| + c_{20}c_{21}^*|0\rangle_{bb}\langle 1| + c_{21}^*c_{20}^*|1\rangle_{bb}\langle 0| + |c_{21}|^2|1\rangle_{bb}\langle 1| + \left(|c_{02}|^2 + |c_{12}|^2\right)|2\rangle_{bb}\langle 2|.\end{aligned} \tag{7}$$

Then, the entangled entropy of our system is defined

$$E(t) = -Tr\rho_a \log_2 \rho_a = -Tr\rho_b \log_2 \rho_b = -\lambda_1 \log_2 \lambda_1 - \lambda_2 \log_2 \lambda_2 \tag{8}$$

where $\lambda_1 = |c_{02}|^2 + |c_{12}|^2$ and $\lambda_2 = |c_{20}|^2 + |c_{21}|^2$.

The dependence of entanglement entropy on the interaction constant ε is demonstrated in Figure 1, with an initial state of $|2\rangle_a|1\rangle_b$.

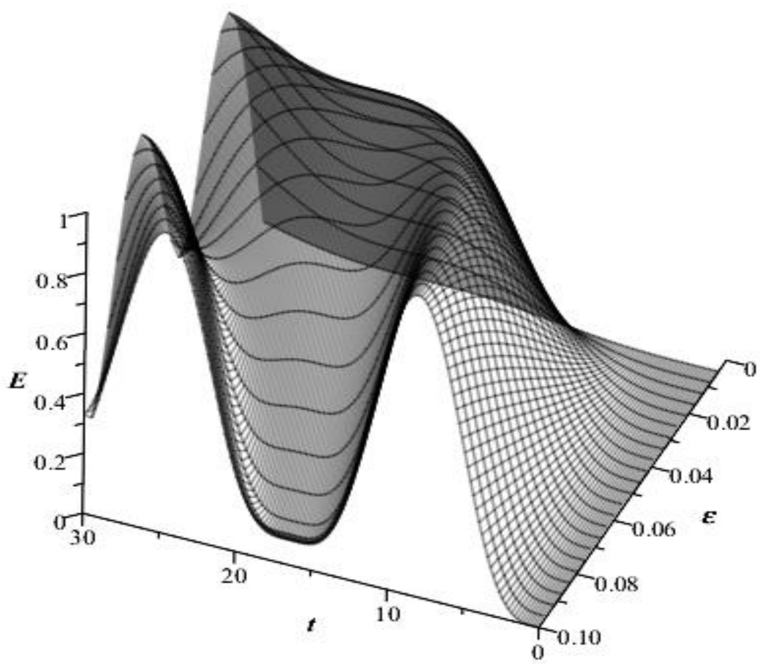

Figure 1: *The evolution of entanglement entropy depends on ε with the initial state is $|2\rangle_a|1\rangle_b$. The parameter m = 4 and time unit is scaled in 1/*

The Figure 1 demonstrates that the output state in the given figure exhibits entanglement, which indicates that the maximum values of entanglement entropy are determined by the interaction constant ε of the two modes. The peaks of entanglement values exhibit a periodic change. In order to find the value of the interaction constant that corresponds to the best maximally entanglement of output state with the periodic change of entanglement, we determined the reparation of entanglement values following moments of times and interaction values that is shown in figure 1. In this figure when the interaction constant ε = 0.04 we can obtain the best maximum values of entanglement. That is the reason for us to choose the value of interaction constant ε = α/4 to examine the time-evolution of entanglement of output state. In this case, we obtain the maximally entangled state that has the highest maximum values of entanglement. Furthermore, during the same time interval, the output state exhibits a greater number of instances with maximum entanglement values compared to the findings reported in [4].

Considered the output state probabilities in Bell-like states as

$$\begin{aligned}
&|B_1\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b + i|0\rangle_a|2\rangle_b\right), && |B_2\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b - i|0\rangle_a|2\rangle_b\right),\\
&|B_3\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_a|2\rangle_b + |2\rangle_a|1\rangle_b\right), && |B_4\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_a|2\rangle_b - |2\rangle_a|1\rangle_b\right),\\
&|B_5\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b + |1\rangle_a|2\rangle_b\right), && |B_6\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b - |1\rangle_a|2\rangle_b\right),\\
&|B_7\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b + i|1\rangle_a|2\rangle_b\right), && |B_8\rangle = \frac{1}{\sqrt{2}}\left(|2\rangle_a|0\rangle_b - i|1\rangle_a|2\rangle_b\right).
\end{aligned} \tag{9}$$

The probabilities corresponding to the output state in Bell-like states from $|B_1\rangle$ to $|B_8\rangle$ are plotted in figures from 2 to 5.

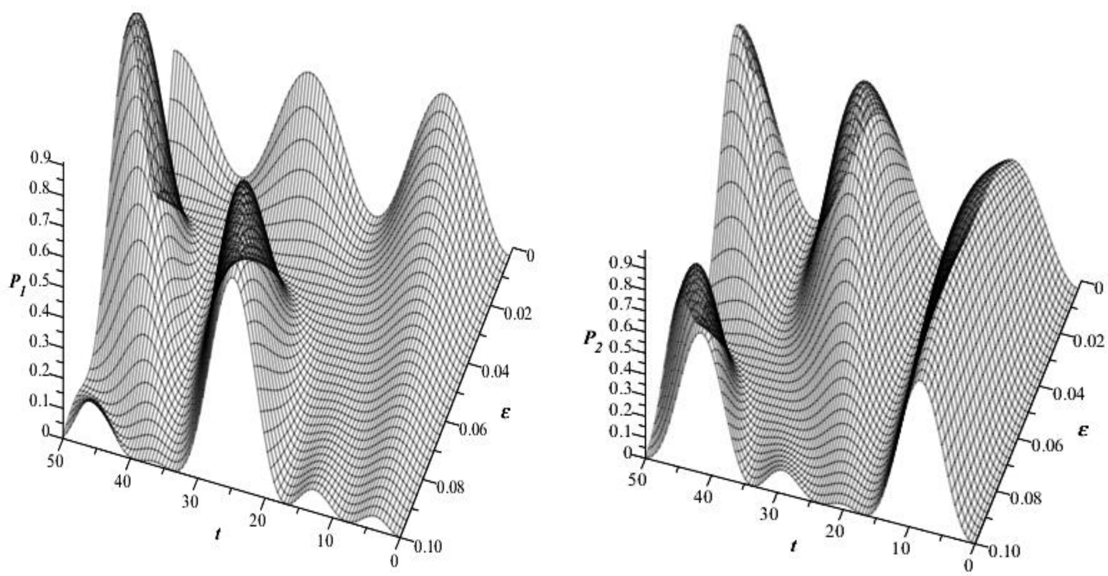

Figure 2: *The probabilities of the Bell-like states* $|B_1\rangle$ *and* $|B_2\rangle$ *corresponding to the initial state is* $|2\rangle_a|1\rangle_b$. *The parameters are the same in the previous figures*

Figure 2 shows that maximum values of the probabilities are close to the unit at some moments of time approximately for the case of the initial state is $|2\rangle_a|1\rangle_b$. These results are higher than the results that are shown in [10]. Then the states $|B_1\rangle$ and $|B_2\rangle$ can be created when entangled states come closing Bell-like states.

The probabilities corresponding to states $|B_3\rangle$, $|B_4\rangle$, $|B_5\rangle$, $|B_6\rangle$, $|B_7\rangle$ and $|B_8\rangle$ are calculated and the results are plotted in figures from 3 to 5. The time-evolution of probabilities corresponding to the output state in $|B_i\rangle$ (i = 3,...,8) are not higher than 0.8 and they have the same forms in pairs ($|B_3\rangle$, $|B_5\rangle$), ($|B_4\rangle$, $|B_6\rangle$) and ($|B_7\rangle$, $|B_8\rangle$) that are only different from the other by phase component. From this, we can conclude that these states cannot be created Bell-like states.

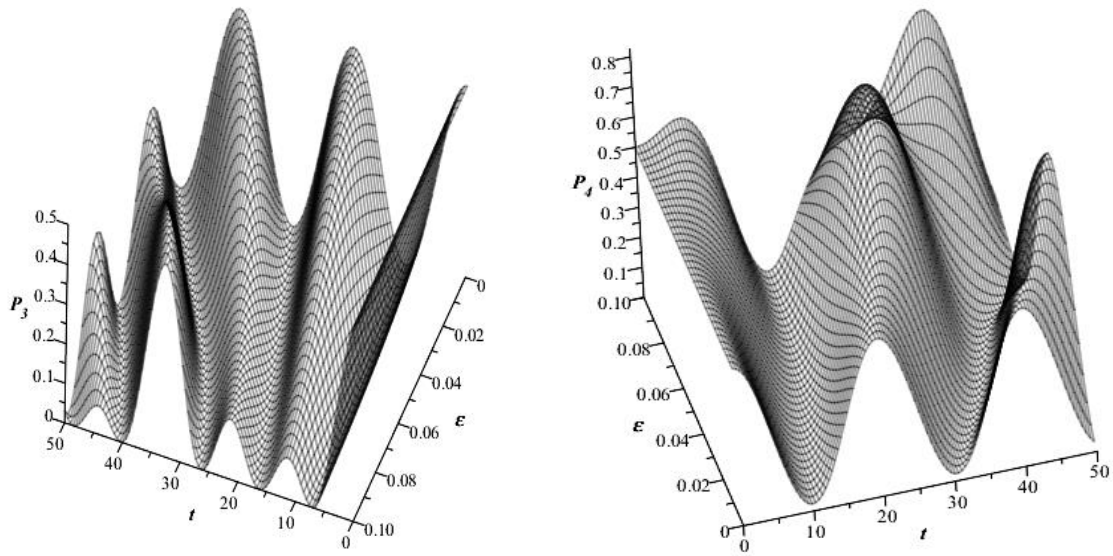

Figure 3: *The probabilities of the Bell-like states* $|B_3\rangle$ *and* $|B_4\rangle$ *corresponding to the initial state is* $|2\rangle_a|1\rangle_b$. *The parameters are the same in the previous figures*

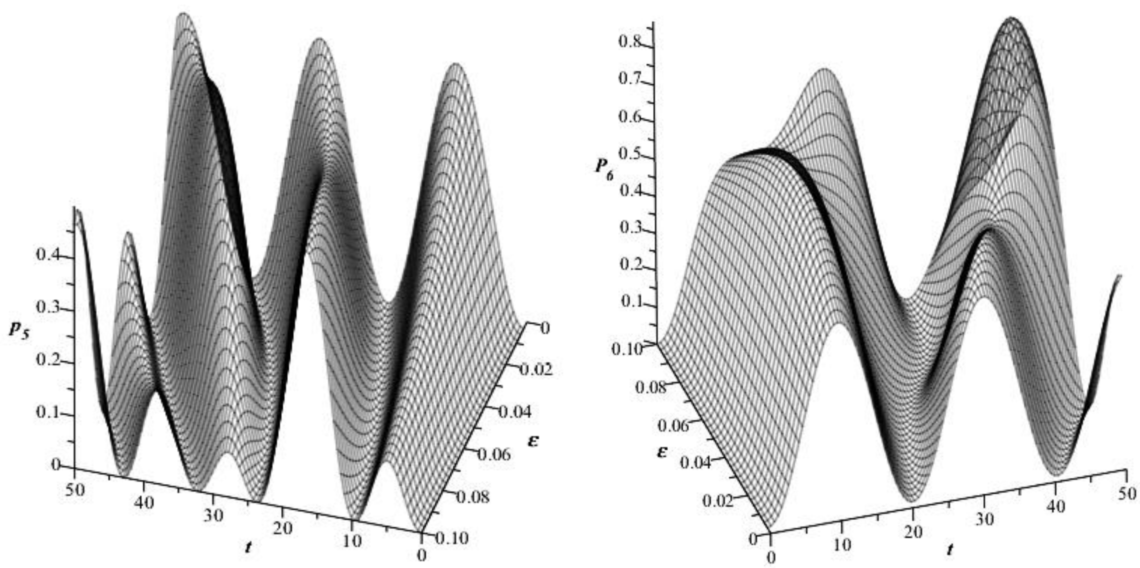

Figure 4: *The probabilities of the Bell-like states* $|B_5\rangle$ *and* $|B_6\rangle$ *corresponding to the initial state is* $|2\rangle_a|1\rangle_b$. *The parameters are the same in the previous figures*

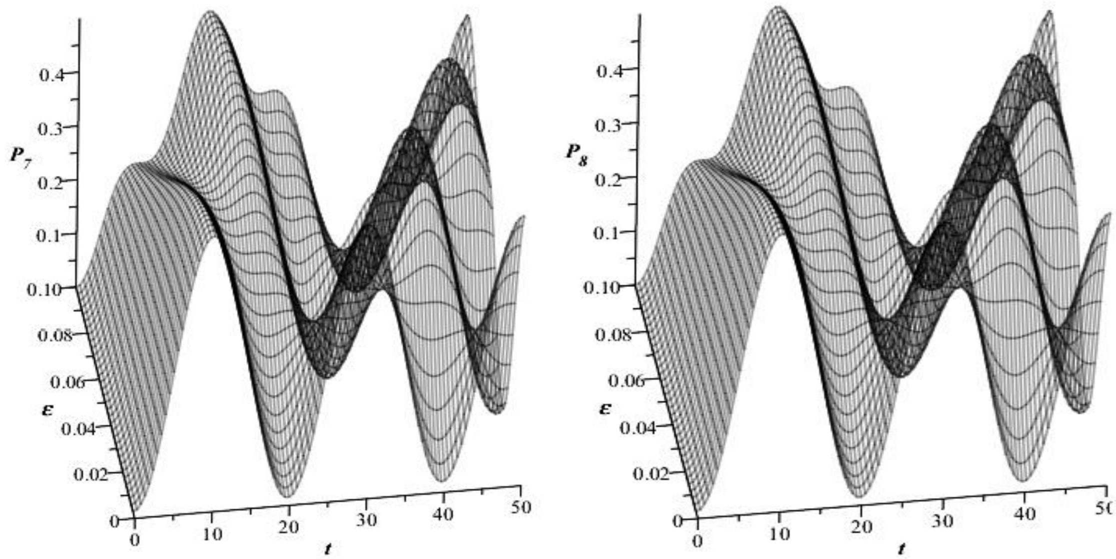

Figure 5: *The probabilities of the Bell-like states* $|B_7\rangle$ *and* $|B_8\rangle$ *corresponding to the initial state is* $|2\rangle_a|1\rangle_b$*. The parameters are the same in the previous figures*

4. Conclusions

This study investigates a Kerr-like nonlinear coupler model consisting of two nonlinear oscillators that are mutually coupled nonlinearly and stimulated by two external classical coherent fields in two modes. The system was found to evolve solely into four states, namely $|2\rangle_a|0\rangle_b, |2\rangle_a|1\rangle_b$, $|1\rangle_a|2\rangle_b$ and $|0\rangle_a|2\rangle_b$, when the NQS method was employed. It has been shown that the maximally values of entanglement entropy are approximately equal to unity in the big number of moments of time in comparison with the results in [10]. Thus, our model can create the Bell-like states with larger probabilities. Especially, we showed that when the interaction constant $\varepsilon = \alpha/4$, the entanglement has the best maximum values. Moreover, our results denote that the time-evolution of the system and entanglement are independent on phase difference between two external pumped fields. These results are different from the case of linear coupling between modes in [9].

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TÓM TẮT

SỰ SINH TRẠNG THÁI KIỂU BELL BỞI KÉO LƯỢNG TỬ PHI TUYẾN TƯƠNG TÁC VỚI HAI TRƯỜNG NGOÀI KẾT HỢP

Đoàn Quốc Khoa
Khoa Khoa học Công nghệ tiên tiến, Trường Đại học Bách khoa, Đại học Đà Nẵng, Việt Nam
Ngày nhận bài 27/3/2023, ngày nhận đăng 05/4/2023

Sự nghiên cứu được mô tả ở bài viết tập trung vào mô hình có hai dao động tử phi tuyến, được kích thích bởi hai trường ngoài kết hợp và được gọi là kéo lượng tử phi tuyến. Để phân tích sự tiến triển của hệ, phương pháp NQS cho kỹ thuật trạng thái lượng tử được áp dụng dẫn đến việc tìm được hàm sóng bao gồm một tổ hợp của các trạng thái *n*-photon. Thông qua việc triển khai NQS, các trạng thái quang học của hệ trải qua quá trình cắt dẫn đến sự tạo ra các trạng thái hai qubit có thể đạt được do các tính chất phi tuyến của các dao động tử và sự tương tác của chúng. Khi hệ tiến triển các trạng thái đan rối cực đại gọi là các trạng thái kiểu Bell được tạo ra.

Từ khóa: Bộ nối phi tuyến Kerr; trạng thái kiểu Bell; kéo lượng tử phi tuyến.