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RESERVOIR OPTIMIZATION WITH DIFFERENTIAL EVOLUTION

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Abstract. Reservoir optimization, is one of recent problems, which has been researched by several methods such as Linear Programming (LP), Non-linear Programming (NLP), Genetic Algorithm (GA), and Dynamic Programming (DP). Differential Evolution (DE), a method in GA group, is recently applied in many fields, especially water management. This method is an improved variant of GA to converge and reach to the optimal solution faster than the traditional GA. It is also capable to apply for a wide range space, to a problem with complex, discontinuous, undifferential optimal function. Furthermore, this method does not require the gradient information of the space but easily find the global solution by a simple algorithm. In this paper, we introduce DE, compare to LP which was considered mathematically decades ago to prove DE's accuracy, then apply DE to Pleikrong, a reservoir in Vietnam, then discuss about the results.

Keywords: Differential evolution, reservoir, optimization.

1. INTRODUCTION

Water is one of few main components on the Earth that takes an important role in our life. For an agricultural country like Vietnam, water management has effects on many aspects such as electricity, water supplement, agriculture, and environment, ... Unfortunately, water is unequally distributed in space and time. To manage the water and distribute it for different purposes, reservoirs have been built and operated.

Most of reservoirs are built by blocking the rivers by dams. This detention causes many changes to the rivers and the downstream. That causes many changes to the hydro-dynamic statement of the river downstream. Reservoirs mainly serve multiple purposes such as hydro-power generation, flood control, downstream water supply, agricultural watering, ... The reservoirs objectives are mainly conflict, such as: water-deficient in dry season but dam-break threatening in flood season, therefore, it tends to store up the water in dry season and discharge the water in flood season. Also, each reservoir has different hydro-meteorological conditions, region water requirements and set of operation rules. How to optimize the objectives of a reservoir is a question that is still under consideration because of the large number of variables, the non-linearity of system dynamics, the stochastic of the inflows and many uncertainly parameters depended on the particular reservoir. That is the reason why making optimal operation decisions is challengeable [1].

An optimization problem needs to find a maximum or minimum of some object within given constraints. It can be described in the form

$$\max f(x) = f(x_1, x_2, \ldots, x_n), \quad x \in R^n \tag{1}$$

where $f(x)$ is an objective function.

Inequality and equality constraints are

$$g_j(x) \geq 0,\ h_k(x) = 0, \quad j = 1,2,\ldots,J;\ k = 1,2,\ldots,K. \tag{2}$$

In optimal reservoir problem, the objective functions are incorporate measurable such as efficiency (i.e., maximizing current and future discounted welfare), survivability (i.e., assuring future welfare exceeds minimum subsistence levels), and sustainability (i.e., maximizing cumulative improvement overtime) [2]. The constraints are the coefficients that belonged to the hydrology and operation structure of the reservoir such as limits on reservoir releases, storage in ranges of dead storage and power plant operation. Because the variables of the reservoir problem are nonlinear, non-differentiable, and non-continuous; the numbers of control and decision variables and constraints are large, therefore, this is a sophisticate problem. In the world, numbers of mathematical programming techniques have been researched and developed, such as LP, NLP, DP and GA, to solve this problem. Recently, there are also some applications of GA [3,4]; however, this is the first time DE is applied in reservoir optimization in Vietnam.

2. METHOD OF DIFFERENTIAL EVOLUTION

2.1. Outline of differential evolution

There are two types of reservoir optimization: long-term and short-term range optimization. The long-term optimization is for planning and the time step of the method can be yearly or monthly; while short-term type is for real management and the time step in this case mainly are daily or hourly [5]. DP often uses long time steps, while DE and LP are more flexible in time steps. We now focus on DE which is a method belonging to GAs group.

Genetic Algorithm is a method that was based on Darwin's theory of natural evolution. Holland (1975) first laid down the method Genetic Algorithm by using the idea of the principles of natural selection of biological organisms. In a genetic, a population of abstract representations of candidate solutions to an optimization problem are stochastically selected, recombined, mutated and then either eliminated or retained based on their relative fitness [6]. The approach has been successfully applied to a wide variety of problems [7]. That shows the efficiencies of Genetic Algorithm in optimization aspect.

To surpass the traditional methods, GAs must differ in some very fundamental ways. In [8], Goldberg identifies the following as the significant differences between GAs and more traditional optimization methods, such as: (1) work with a coding of the parameter set, not the parameter themselves; (2) search from a population of points, not a single point; (3) use objective function information, not derivatives or other auxiliary knowledge; (4) use probabilistic transition rules, not deterministic rules. Since then, GAs have been; developed into a powerful technique for identifying optimal solutions to complex problems.

However, traditional GAs use low mutation rates and fixed step sizes that cause trouble with problem having interdependent relationships among the decision variables [9].

In 1996, Price and Storn introduced a method named Differential Evolution to deal with nonlinear and non-differentiable continuous space minimal problem. Differential Evolution is a direct search in the family of GAs which reaches to a robustness in optimization and faster convergence to a given problem. It is also easy to use and requires only few control variables. It differs from other GAs in mutation and recombination phase. DE uses weighted differences between solution vectors to perturb the population, not a random quality as other GAs [10].

DE initial the population NP in N dimensions for the steps or generations distributed in the searching space

$$X_i = (x_{i1}, \ldots, x_{in})^T \quad i = 1, \ldots, NP \tag{3}$$

Mutation is a stepping-stone that creates new vector from the given ones. In the step G or generation G, a vector $V_i^{(G+1)} = \left(v_{i1}^{(G+1)}, v_{i2}^{(G+1)}, \ldots, v_{in}^{(G+1)}\right)^T$ is created from each individual vector $X_i(G)$ by adding a weighted differences between the given vectors [10].

There are some of mutation ways such as [9]

$$\begin{aligned}
V_i^{(G+1)} &= X_{r1}^{(G)} + F\left[X_{r2}^{(G)} - X_{r3}^{(G)}\right], \\
V_i^{(G+1)} &= X_{\text{best}}^{(G)} + F\left[X_{r1}^{(G)} - X_{r2}^{(G)}\right], \\
V_i^{(G+1)} &= X_{i}^{(G)} + F\left[X_{\text{best}}^{(G)} - X_{i}^{(G)}\right] + F\left[X_{r1}^{(G)} - X_{r2}^{(G)}\right], \\
V_i^{(G+1)} &= X_{\text{best}}^{(G)} + F\left[X_{r1}^{(G)} - X_{r2}^{(G)}\right] + F\left[X_{r3}^{(G)} - X_{r4}^{(G)}\right], \\
V_i^{(G+1)} &= X_{r1}^{(G)} + F\left[X_{r2}^{(G)} - X_{r3}^{(G)}\right] + F\left[X_{r4}^{(G)} - X_{r5}^{(G)}\right],
\end{aligned} \tag{4}$$

where $X_{\text{best}}^{(G)}$ is the best individual of G. $F > 0$ is a real parameter, called mutant constant, which controls the difference between two individuals, used to avoids the slow searching. $r1, r2, r3, r4, r5$ are random integers chosen from 1 to NP.

After mutant step, we will consider where choose new vectors for the next step or keep the old ones. This is called crossover. In this crossover, we randomly choose a number

$$rnbr(i) \in \{1, 2, \ldots, n\} \tag{5}$$

for each vector and create a new vector

$$U_i^{(G+1)} = \left(u_{i1}^{(G+1)}, u_{i2}^{(G+1)}, \ldots, u_{in}^{(G+1)}\right)^T \tag{6}$$

while

$$u_{ij}^{(G+1)} = \begin{cases} v_{ij}^{(G+1)} & \text{if}\,[randb(j) \le CR] \quad \text{or} \quad [j = rnbr(i)] \\ x_{ij}^{(G)} & \text{if}\,[randb(j) > CR] \quad \text{or} \quad [j \ne rnbr(i)] \end{cases} \tag{7}$$

where $j = 1, 2, \ldots, n$; $randb(j)$ is randomly chosen in $[0,1]$; CR is cross-over constant which is chosen in $[0,1]$.

Now, comparing new vectors to the old ones through the objective function, the vectors for the next step are chosen. Each vector that passes into the next generation is already compared and is a better one. The calculation steps will be repeated until the standard of the problem has reached [10].

$$u_{ij}^{(G+1)} = \begin{cases} v_{ij}^{(G+1)} & \text{if}\,[randb(j) \le CR] \quad \text{or} \quad [j = rnbr(i)] \\ x_{ij}^{(G)} & \text{if}\,[randb(j) > CR] \quad \text{or} \quad [j \ne rnbr(i)] \end{cases} \tag{8}$$

DE is wide-ranged used not only in reservoirs optimal problem [9, 11] but also in many fields and optimal problems [12–14].

In reservoir optimal problem, NP initial vector X_i could be the releases of NP days in calculating, the objective function could be the maximum electricity production of the plant in NP days and the constrains could be the limits on reservoir releases which based on the construction of the plant, minimum level for death storage and power plant operation, ...After calculating, we could find out the optimal operation to satisfy the objective function.

However, there is no mathematical proof of optimality by GA methods generally; and by DE methods particularly. DE is like all other heuristic programming methods that are based on rules-of-thumb, experience [2]. DE is said to be a method that "cannot guarantee termination to even local optimal solutions, but they are often capable of achieving global optimal solutions to problems where traditional algorithmic methods would fail to converge or get stuck in local optima" [2].

2.2. Application of differential evolution

We now use LP, a traditional optimal method that is well-proven to prove the accuracy and effectiveness of DE by applying in to the same object with the same conditions.

While reservoir optimal problems are mainly large-scale with long time horizon, LP is one of the most efficient methods are critically applied. LP requires all objective functions and constraints are linear or linearizable. An efficient method for moving LP is simplex one. If the solution of an optimization problem exists, then this simplex method can find it in the final steps [15].

2.2.1. *Comparing DE to LP*

We now apply both DE and LP into one object that is Pleirong plant in Vietnam.

Pleikrong hydropower plant was built in Kontum Province, Highland of Vietnam, at the upstream of Sesan river (Fig. 1). Pleikrong reservoir is the biggest one on the Sesan cascade and its purposes are: storing water for the whole Sesan cascade and producing electricity. The management of Pleikrong hydropower reservoir has a significant affect to other reservoirs in the cascade. In this paper, the objective function is to maximize the electricity production of Pleikrong plant.

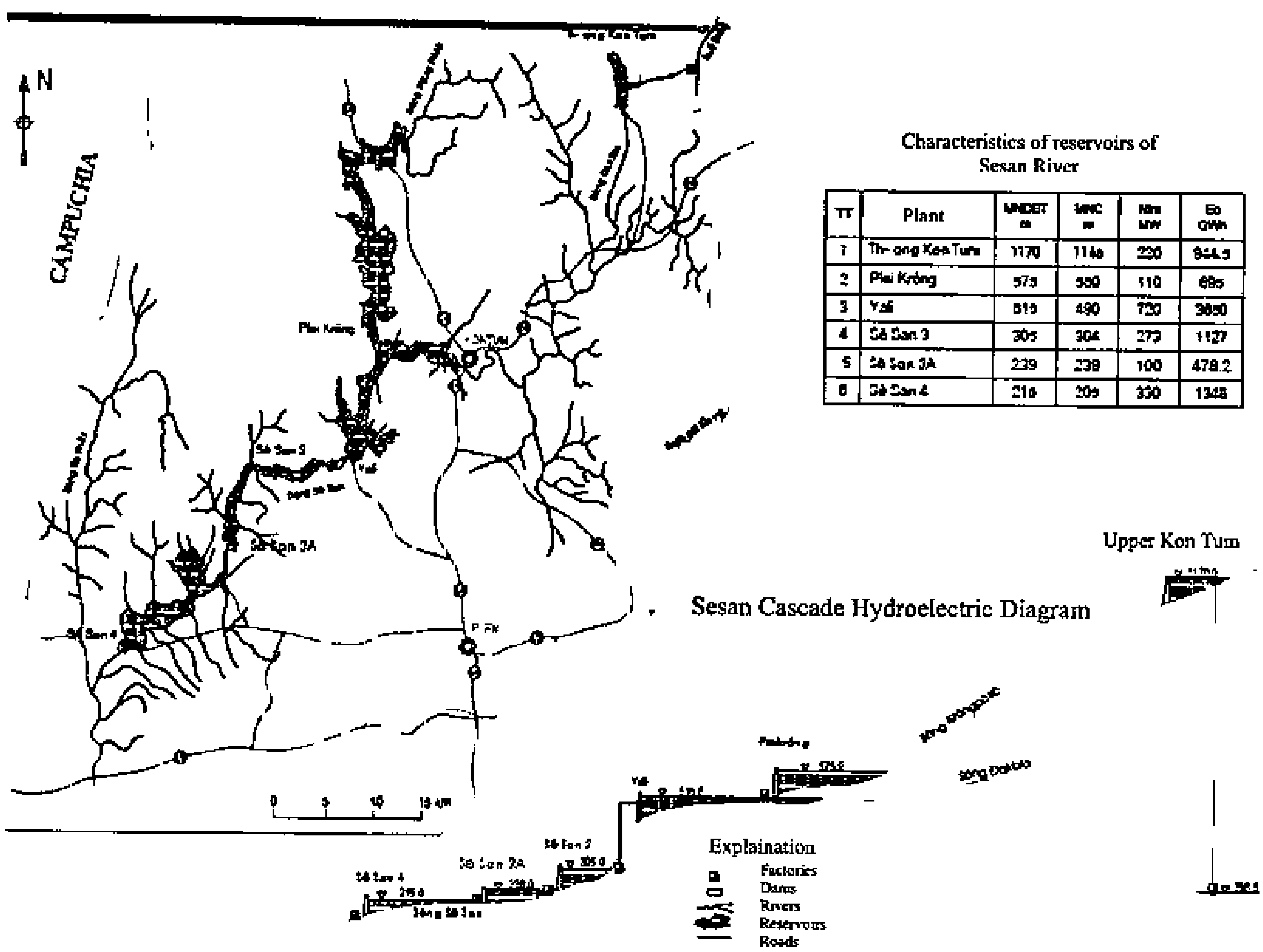

Fig. 1. The Sesan cascade

We consider an optimal problem in electricity production of Pleikrong plant in dry season, which starts at the beginning of December and ends in the next June. The reservoir management is followed the operation rules in the decision No. 1182 QD-TTg of the Government of Vietnam signed in July 17, 2014. In details: operation period in dry season is 10 day, from 1st December to next 15th February, Pleikrong reservoir needs to store the water as much as possible. From 16th Feb to 30th Jun, the main task of Pleikrong reservoir is producing electricity. To 30th Jun, the water level in the reservoir has to be at death level to prepare for the flood control in Sesan basin.

The object function of the optimal reservoir management problem that we consider is finding the maximum of electricity production from 11st February to 30th Jun (140 days). So, the problem is settled as below:

Finding the Q_i - average (in 10 days) release at time period $i, i = 1, 2, \ldots, 14$, to get the maximum of electricity production.

The electricity production of Pleikrong plant at time period i of 10 days is calculated by the following formula [16]

$$E_i = 9.8 * h_i * Q_i * k * 24 * 10/1000 \text{ (MWh)}, \tag{9}$$

where h_i - water height at time period i, Q_i - release at time period i, k - overall generation efficiency.

Then electricity production E is calculated as

$$E = E_1 + E_2 + \ldots + E_{14}. \tag{10}$$

In formula (9), water height h_i is nonlinearly dependent on the average release Q_i and the average inflow U_i at time period i. Those inflows in the year 2010 are shown in the Tab. 1. The data that is used in this paper was provided by the team of Institute of Mechanics of VAST in project of building the reservoir operation for Sesan cascade in dry season (under contract N 01/2011/QTVH - SESAN on June 02, 2011).

– *Using LP*

To apply LP to the problem of reservoir regulation, we need all the data, functions and constraints in linear forms. Therefore, the water height h_i in (2) is approximately transferred into constant (Tab. 1). The difference between the results received by this approximation and the real monthly data of the Pleikrong hydropower plant in 2010 is not much (Tab. 2).

Table 1. Monthly electricity production (MWh) in the 2010 year

No period	Time of period	Inflow (m^3/s)	Approx. height (m)
1	11/Feb - 20/Feb	47.73	52.00
2	21/Feb - 02/Mar	53.35	49.79
3	03/Mar - 12/Mar	23.79	47.58
4	13/Mar - 22/Mar	51.80	45.37
5	23/Mar - 01/Apr	31.98	43.16
6	02/Apr - 11/Apr	25.26	40.95
7	12/Apr - 21/Apr	44.56	38.74
8	22/Apr - 01/May	41.18	36.53
9	02/May - 11/May	14.54	34.31
10	12/May - 21/May	23.54	32.10
11	22/May - 31/May	0.09	30.00
12	01/Jun - 10/Jun	17.15	27.68
13	11/Jun - 20/Jun	44.76	25.47
14	21/Jun - 30/Jun	36.48	23.26

Using available LP software (for example: LP in MatLab; SIMPLEX PROCEDURE written by J. Morris, Naval Surface Weapons Center Dahlgren, Virginia, USA; Online

Table 2. Monthly electricity production (MWh) in the 2010 year

	Calculation data	Real data
March	31031	34883
April	43717	47932
May	29064	32354
June	8490	8027
Sum	**112303**	**123198**

software PHPSimplex, www.phpsimplex.com) [17], we get the following optimal releases as shown in Tab. 3.

Table 3. Optimal releases by LP

No period	Time of period	Optimal releases (m^3/s) by LP
1	11/Feb - 20/Feb	330.00
2	21/Feb - 02/Mar	330.00
3	03/Mar - 12/Mar	330.00
4	13/Mar - 22/Mar	224.14
5	23/Mar - 01/Apr	31.98
6	02/Apr - 11/Apr	25.26
7	12/Apr - 21/Apr	44.56
8	22/Apr - 01/May	41.18
9	02/May - 11/May	14.54
10	12/May - 21/May	23.54
11	22/May - 31/May	0.09
12	01/Jun - 10/Jun	17.15
13	11/Jun - 20/Jun	44.76
14	21/Jun - 30/Jun	36.48

With the maximum electrical production is 143107.20 MWh.

– *Using linear DE*

Recently, there is no DE software for reservoir regulations. We use the program DE_Fortran90, written by Dr. Feng-Sheng Wang Department of Chemical Engineering, National Chung Cheng University, Chia-Yi 621, Taiwan [18] and develop it for reservoir regulation (DE_Pleik01).

For verification of DE_Pleik01 we use the Eq. (9) with constants h_i in the Tab. 1, i.e. we consider the problem of optimal releases Q_i with linear objective function. DE_Pleik01 give the similar result in the Tab. 3. So, we can use DE_Pleik01 for optimal reservoir regulation.

Table 4. Optimal releases by DE for linear case

No period	Time of period	Optimal releases (m^3/s) by DE
1	11/Feb - 20/Feb	330.00
2	21/Feb - 02/Mar	330.00
3	03/Mar - 12/Mar	330.00
4	13/Mar - 22/Mar	224.13
5	23/Mar - 01/Apr	31.98
6	02/Apr - 11/Apr	25.26
7	12/Apr - 21/Apr	44.56
8	22/Apr - 01/May	41.18
9	02/May - 11/May	14.54
10	12/May - 21/May	23.54
11	22/May - 31/May	0.09
12	01/Jun - 10/Jun	17.15
13	11/Jun - 20/Jun	44.76
14	21/Jun - 30/Jun	36.48

With the maximum electric production is 143107.71 MWh.

The results in Tab. 2 and Tab. 3 show that DE is an accurate method and its results are effective comparing to a well-proven method like LP.

2.2.2. *Applying nonlinear DE to Pleikrong optimal regulation*

The relationship between water levels and storage volumes is not linear in the reality. As shown in (Fig. 2), the water height h_i is a nonlinear function of the release Q_i and the inflow U_i. We consider now the problem of optimal releases Q_i with nonlinear objective function.

Applying DE_Pleik01 with the nonlinear water height function h_i, we get the other result of optimal releases as shown in Tab. 5. With the maximum electric production in this case is 141109.71 MWh, which is much higher than the real production is about 134058.3 MWh.

Results for application of DE in linear (Tab. 3) and nonlinear (Tab. 5) cases are different. The biggest releases in Tab. 3 are in the beginning, but the biggest releases in Tab. 5 are in the end of the dry season. This difference is because the relation between water heights and the releases is nonlinear for real. We can use this nonlinear relation into nonlinear DE right away. However, this relation must be linearized to qualify the requirement of LP. That causes the differences between results of Tab. 3 and Tab. 5.

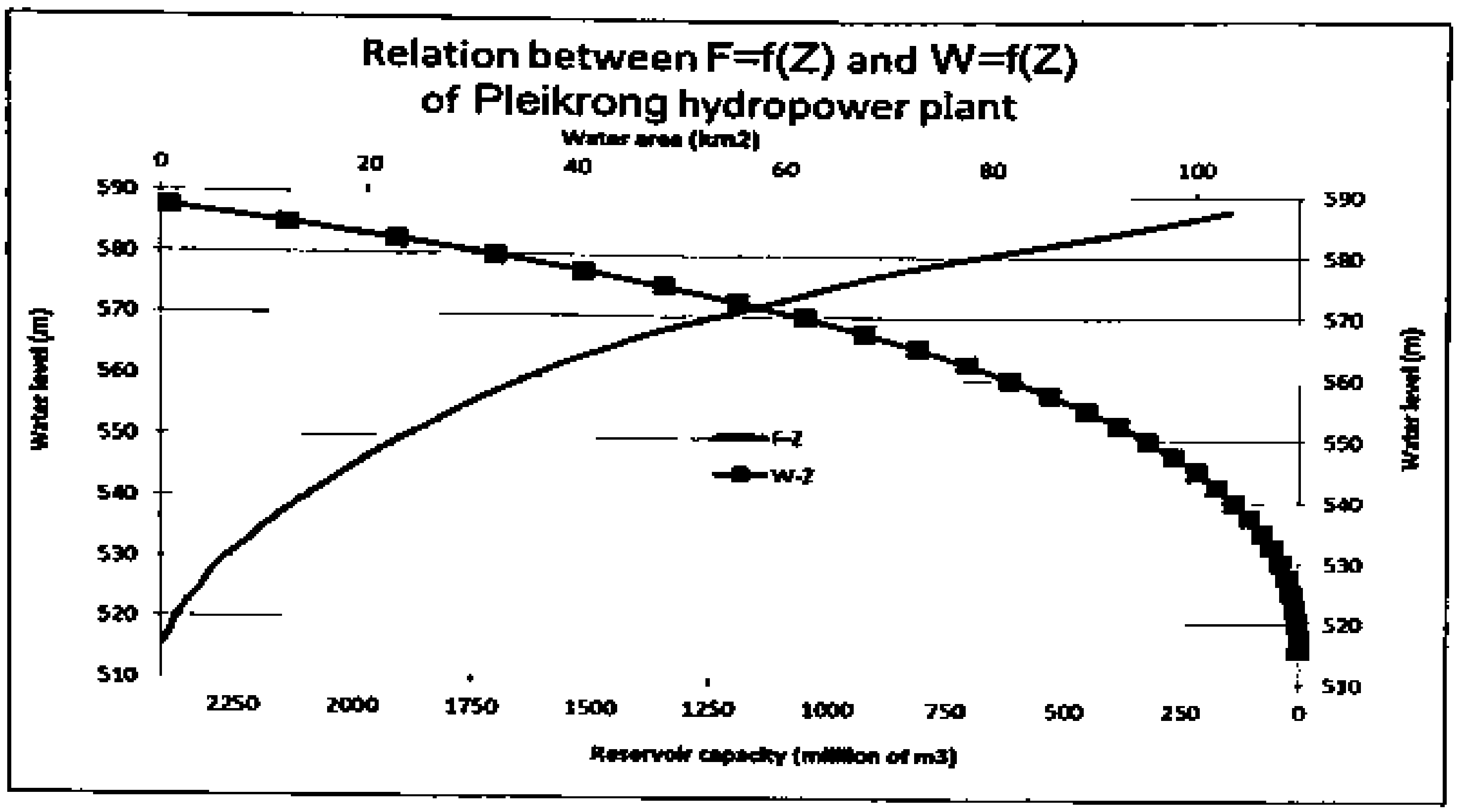

Fig. 2. Relationship between water levels Z and storage volumes W of the Pleikrong reservoir

Table 5. Optimal releases by DE for nonlinear case

No period	Time of period	Optimal releases (m^3/s) by DE
1	11/Feb - 20/Feb	0.0
2	21/Feb - 02/Mar	40.5
3	03/Mar - 12/Mar	23.8
4	13/Mar - 22/Mar	51.80
5	23/Mar - 01/Apr	31.98
6	02/Apr - 11/Apr	25.26
7	12/Apr - 21/Apr	44.56
8	22/Apr - 01/May	41.18
9	02/May - 11/May	14.54
10	12/May - 21/May	102.7
11	22/May - 31/May	193.5
12	01/Jun - 10/Jun	263.8
13	11/Jun - 20/Jun	330.0
14	21/Jun - 30/Jun	330.0

3. CONCLUSION

In many methods, DE is chosen in this paper to apply to a specific reservoir optimal problem because DE is a direct search method that can be linked directly with hydrologic and water quality simulation models without requiring simplifying assumption in calculation of derivatives [2]. DE is also a good tool that has been widely used and given good results. We use Linear Programming, which is efficient to solve large-scale problem, no need of the initial solutions, well-developed duality theory for sensitively analysis, convergence to global solutions and easily setup and use [2], as a measurement to prove the accuracy of Differential Evolution and show that DE is a promise method while it is still unproven mathematically. We are also using DE to other reservoirs in Sesan Cascade and for other years, and these results will be shown in another paper.

REFERENCES

[1] L. W. Mays and Y. K. Tung. *Hydrosystems engineering and management*. McGraw-Hill, Inc, (1992).

[2] J. W. Labadie. Optimal operation of multireservoir systems: state-of-the-art review. *Journal of Water Resources Planning and Management*, **130**, (2), (2004), pp. 93–111.

[3] L. Hung. *Multi-objective reservoirs optimization*. PhD thesis, Da Nang University, (2012).

[4] Q. T. Xuan. *Using multi-objective optimal methods to assess and optimize reservoir regulations - Case-study: Hoa Binh reservoir*. PhD thesis, Politecnico di Milano, (2011).

[5] A. W. Salami and B. F. Sule. Optimal water management modeling for hydropower system on river Niger in Nigeria. *Annals of Faculty of Engineering Hunedoara - International Journal of Engineering*, **3**, (2012), pp. 185–192.

[6] J. H. Holland. *Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence*. University of Michigan Press, (1975).

[7] M. Mitchell. *An introduction to genetic algorithms*. A Bradford Book The MIT Press, (1998).

[8] D. E. Goldberg. *Genetic algorithms and machine learning*, Vol. 3. Addison-Wesley, (1989).

[9] M. J. Reddy and D. N. Kumar. Multiobjective differential evolution with application to reservoir system optimization. *Journal of Computing in Civil Engineering*, **21**, (2), (2007), pp. 136–146.

[10] R. Storn and K. Price. Differential evolution - A simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, **11**, (4), (1997), pp. 341–359.

[11] M. J. Reddy and D. N. Kumar. Optimal reservoir operation using multi-objective evolutionary algorithm. *Water Resources Management*, **20**, (6), (2006), pp. 861–878.

[12] Y. Yare and G. K. Venayagamoorthy. A differential evolution approach to optimal generator maintenance scheduling of the Nigerian power system. In *Power and Energy Society General Meeting-Conversion and Delivery of Electrical Energy in the 21st Century, IEEE*, (2008), pp. 1–8.

[13] X. Yuan, Y. Zhang, L. Wang, and Y. Yuan. An enhanced differential evolution algorithm for daily optimal hydro generation scheduling. *Computers & Mathematics with Applications*, **55**, (11), (2008), pp. 2458–2468.

[14] D. Yadav, V. Sharma, and R. Naresh. Fixed head short term hydro thermal generation scheduling using differential evolution. *Asian Journal of Current Engineering and Maths*, **2**, (1), (2013), pp. 17–23.

[15] D. G. Luenberger and Y. Ye. *Linear and nonlinear programming*. Springer Science + Business Media, LLC, (2008).

[16] K. D. W. Nandalal and J. J. Bogardi. *Dynamic programming based operation of reservoirs: applicability and limits*. Cambridge University Press, (2007).

[17] http://www.phpsimplex.com.

[18] http://www1.icsi.berkeley.edu/storn/code.html.se.