ELECTROMAGNETIC CORRECTION IN THE MUON DECAY

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Abstract

The influence of electromagnetic interaction in one photon approximation was applied to study the muon decay in electroweak interaction. The divergences that ariose in this process were infrared ones in the low energy region. The divergent treatment was carried out with two methods: the minimum mass method and the dimensional regularization method. As a result, an electromagnetic correction was calculated for the muon electroweak decay rate. The results that we received confirm that the Fermi theory of weak interaction is a non-renormalizable theory.

Keywords: Muon decay; weak interaction; infrared divergence; renormalization.

1. Introduction

The decay process of muon to electron according to the equation $\mu \rightarrow e + \tilde{v}_e + v_\mu$ has a lifetime of $\tau_\mu = 2.2 \times 10^{-6}$ sec [1]. At first sight, μ^- can decay electromagnetically into e^- by emitting photons. However, this process is observed with extremely small probability ($< 1.2 \times 10^{-11}$). Largely, the muon decay produce charged particles. Therefore these particles have to perform electromagnetic interactions (i.e., the interaction of charged particles (e and μ) with the electromagnetic field A_μ). The Hamiltonian of the electromagnetic interaction is $H_{int}^{em} = J_\mu A_\mu$, where $J_\mu(x)$ is the charge flow, $J_\mu(x) = ie\overline{\psi}_\mu(x)\gamma_\mu\psi_e(x) = ie\overline{\mu}(x)\gamma_\mu e(x)$, and A_μ is the electromagnetic potential. The electroweak decay occurs simultaneously due to two interactions: weak and electromagnetic ones. This is a very complicated problem and we have known that these interactions have been unified by a complete theory, which is called Glashow - Weinberg - Salam theory (the non-Abelian standard theory). Within the scope of this article, we only limit our research to the first-order approximation when expanding the decay rate according to the interaction constant G. The decay process has been calculated with the emission of "soft" real photons.

The scattering amplitude expression becomes invariant across the displacement

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because of the added contribution of bremsstrahlung radiation (photon radiation and absorption) and the decay rate expression will include new series of divergences. They are the infrared ones [2, 3] in the low-energy region. The infrared divergence is directly related to fields whose quanta have zero rest mass, like photons in quantum electrodynamics (QED) and gravitons in quantum gravity. Features for infrared singularities appear not only for the Green function, but also in matrix elements. These singularities must be determined by methods of quantum field theory [4].

Previous research results [3, 5, 6] have pointed out that the radiation or absorption of one photon has a greater probability than the radiation or absorption of two, or a large number of photons. Also, the infrared divergence separation in the expression of scattering amplitude is carried out simultaneously in two ways: the minimum mass method and the dimensional regularization method.

The article is organized as follows: The Feynman diagram for the muon decay and the matrix element corresponding to the one mentioned above introduced in section 2. Section 3 is devoted to the reduction of infrared divergence in total decay rate of muons. This divergence is treated by using the minimum mass method and the dimensional regularization method. The last section (section 4) shows discussions and comparison of the results obtained from the two methods above and directions for further research.

2. Muon electroweak decay rate in one photon approximation

The Feynman diagram corresponding to the muon electroweak decay in a first-order approximation can be shown in Fig. 1. Specifically, the decay process can be written as follows:

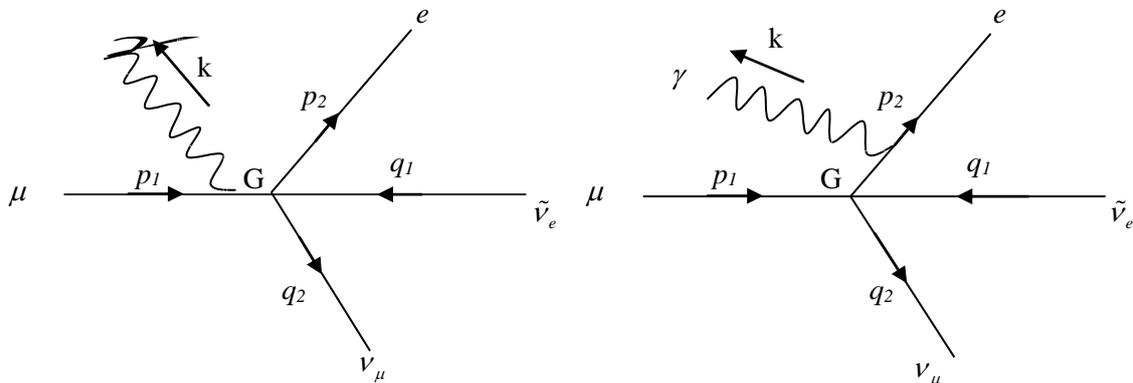$$\mu \rightarrow e + \tilde{\nu}_e + \nu_\mu \tag{1}$$

Fig. 1. Feynman diagram for muon scattering considering electromagnetic radiation.

The actual photons that are absorbed or radiated are related to the muon-electron flux. The scattering amplitude corresponding to the two process above, according to Feynman's rule, has the following form:

$$M_{fi,\gamma} = \frac{G}{\sqrt{2}}\left[\bar{u}_{\nu_\mu}(q_2)\gamma^\rho(1-\gamma_5)\frac{\hat{p}_1-\hat{k}+m_\mu}{(p_1-k)^2-m_\mu^2}(e\not{\epsilon})u_\mu(p_1)\right]\times\left[\bar{u}_e(p_2)\gamma_\rho(1-\gamma_5)v_{\bar{\nu}_e}(q_1)\right]$$

$$+\frac{G}{\sqrt{2}}\left[\bar{u}_e(p_2)\gamma_\rho(1-\gamma_5)v_{\bar{\nu}_e}(q_1)\right]\times\left[\bar{u}_e(p_2)\gamma_\rho(1-\gamma_5)\frac{\hat{p}_2+\hat{k}+m_\mu}{(p_2+k)^2-m_\mu^2}(e\not{\epsilon})v_{\bar{\nu}_e}(q_1)\right]$$

$$M_{fi,\gamma} = \frac{G}{\sqrt{2}}\left[\bar{u}_{\nu_\mu}(q_2)\gamma^\rho(1-\gamma_5)u_\mu(p_1)\right]\left[\bar{u}_e(p_2)\gamma_\rho(1-\gamma_5)v_{\bar{\nu}_e}(q_1)\right]$$

$$\times e\left(\frac{\hat{p}_1-\hat{k}+m_\mu}{(p_1-k)^2-m_\mu^2}\not{\epsilon}+\not{\epsilon}^*\frac{\hat{p}_2+\hat{k}+m_e}{(p_2+k)^2-m_e^2}\right), \tag{2}$$

where $\not{\epsilon}$ and $\not{\epsilon}^*$ are the polarization vectors of the photon, u and v denote the spinors for particles and anti-particles, respectively (momentum of each particle is given in parenthesis).

By using the equality $\not{\epsilon}^*(p_2+m_e)=\not{\epsilon}^*(m_e-p_2)+2p_2.\not{\epsilon}^*$, and considering the scattering on the mass shell, i.e., $p_1^2=m_\mu^2, p_2^2=m_e^2$, we will ignore the $\hat{k}$ value in the numerator of (2) and get:

$$M_{fi,\gamma} = M_{fi,0}\times e\left(\frac{p_2.\not{\epsilon}^*}{p_2.k}-\frac{p_1.\not{\epsilon}}{p_1.k}\right) = (M_{fi,0_\rho})J_\rho^{em} \tag{3}$$

$M_{fi,0}$ is defined as in document [1], and

$$J_\rho^{em} = e\left(\frac{p_{2\rho}}{p_2.k}-\frac{p_{1\rho}}{p_1.k}\right) \tag{4}$$

We see that the amplitude $M_{fi,\gamma}$ in (3) contains two factors: the first is the scattering amplitude without photon radiation, and the second is the current of the displacement J_ρ^{em}. The expression of J_ρ^{em} satisfies the continuity equation $kJ = 0$. It has a singularity at $k = 0$ that is the cause of the infrared divergence.

Squared by (2), summed by the spin states for final particles, spin - averaged for initial particles, and summed by the polarization states of the photon, we have:

$$\sum_{Spin,polarization}\left|M_{fi,\gamma}\right|^2 = \sum_{Spin}\left|M_{fi,0}\right|^2\times e^2\sum_{polarization}\left(\frac{p_2.\not{\epsilon}^*}{p_2.k}-\frac{p_1.\not{\epsilon}}{p_1.k}\right)^2 \tag{5}$$

Applying the decay rate formula [1] to process (1), we get:

$$\Gamma = \frac{G^2}{2p_1^0(2\pi)^5} \int \frac{d^3q_1}{2q_1^0} \frac{d^3q_2}{2q_2^0} \frac{d^3p_2}{2p_2^0} \frac{d^3k}{2k_0(2\pi)^3} \delta^{(4)}(q_1 + q_2 + p_2 - p_1 + k) \sum_{Spin,\alpha} \left| M_{fi,\gamma} \right|^2, \qquad (6)$$

where $\alpha = \pm 1$ is the polarization index of the photon.

Use (6) to deduce:

$$\Gamma = \Gamma_0 e^2 \int \frac{d^3k}{2k_0(2\pi)^3} \sum_{phancuc} \left\{ \frac{p_2.\not{\varepsilon}^*}{p_2.k} - \frac{p_1.\not{\varepsilon}}{p_1.k} \right\}^2, \qquad (7)$$

here Γ_0 is the muon decay rate without taking into account photon absorption or radiation. It is defined by the expression [1]:

$$\Gamma_0 = \frac{G^2 m_\mu^5}{192\pi^2} \qquad (8)$$

To sum all the polarization states of a photon, we do the following:

$$\sum_{\alpha=\pm 1} \left(\frac{p_2.\not{\varepsilon}_k^{\alpha*}}{p_2.k} - \frac{p_1.\varepsilon_k^\alpha}{p_1.k} \right) \left(\frac{p_2.\not{\varepsilon}_k^{\alpha*}}{p_2.k} - \frac{p_1.\varepsilon_k^\alpha}{p_1.k} \right) = J_x J_x + J_y J_y \qquad (9)$$

$$J^\mu = \frac{p_2^\mu}{p_2.k} - \frac{p_1^\mu}{p_1.k} \qquad (10)$$

However, from the condition of preserving current, $k_\mu J^\mu = 0$, and $J_0 = J_z$, we can write:

$$J_x J_x + J_y J_y = J_x J_x + J_y J_y + J_z J_z - J_0 J_0 = -J_\mu J^\mu \qquad (11)$$

From Eq. (7) - (10), we derive:

$$\Gamma = -\Gamma_0 e^2 \times \int_R \frac{d^3k}{2k_0(2\pi)^3} \left(\frac{p_{2\mu}}{p_2.k} - \frac{p_{1\mu}}{p_1.k} \right) \left(\frac{p_2^\mu}{p_2.k} - \frac{p_1^\mu}{p_1.k} \right) = \Gamma_0 e^2 I, \qquad (12)$$

where R is the integration region.

The divergence will be denoted as follows:

$$I = -\int_R \frac{d^3k}{2k_0(2\pi)^3} \left(\frac{p_{2\mu}}{p_2.k} - \frac{p_{1\mu}}{p_1.k} \right) \left(\frac{p_2^\mu}{p_2.k} - \frac{p_1^\mu}{p_1.k} \right) \qquad (13)$$

In the following, we will apply two methods to eliminate this divergence: the minimum mass method and the dimensional regularization method.

2.1. Treament for divergence using the minimum mass method

In QED, we often encounter infrared divergences (where the integrals will not converge in low energy regions). For the integrals to converge, we must attribute the photon additional mass $\lambda_{\min}$ in the propagator of photon, i.e., replacing $\dfrac{1}{k^2}$ with

$\dfrac{1}{k^2 + \lambda_{\min}^2}$ where $\lambda_{\min}^2 << m^2$, and m are the mass of leptons (electrons, or muons). This substitution is equivalent to slicing the integral at some lower limit when $k \approx \lambda_{\min}$. At the final result, we again let $\lambda_{\min} \to 0$.

To calculate the integral containing infrared divergence (13), we note that R is determined by the condition $0 \le |k| \le \varepsilon, E = \sqrt{p^2 + m^2}$. For brevity, we write λ in place of $\lambda_{\min}$ and $k_0 = \sqrt{k^2 + \lambda^2}, pk = \vec{p}\vec{k} - Ek_0$; ε is the limit so that integral (13) does not contain infrared divergence. If $p_{1,2}^2 + m^2 = 0$, and $d^3k = k^2 d|\vec{k}| d\Omega$, we have:

$$I = -\frac{1}{16\pi^3} \int_0^\varepsilon \frac{k^2 d|\vec{k}| d\Omega}{\sqrt{k^2 + \lambda^2}} \left\{ \frac{m^2}{(E_2 \sqrt{k^2 + \kappa^2} - \vec{p}_2\vec{k})^2} + \frac{m^2}{(E_1 \sqrt{k^2 + \lambda^2} - \vec{p}_1\vec{k})^2} + \right.$$
$$\left. + \frac{2\vec{p}_1\vec{p}_2}{(E_2 \sqrt{k^2 + \lambda^2} - \vec{p}_2\vec{k})(E_1 \sqrt{k^2 + \lambda^2} - \vec{p}_1\vec{k})} \right\}, \tag{14}$$

Here $d\Omega$ is the solid angular element containing the photon momentum k. Now we use the identity:

$$\frac{1}{(E_2 \sqrt{k^2 + \lambda^2} - \vec{p}_2\vec{k})(E_1 \sqrt{k^2 + \lambda^2} - \vec{p}_1\vec{k})} = \frac{1}{2} \int_{-1}^{1} \frac{dz}{(E_z \sqrt{k^2 + \lambda^2} - \vec{p}_z\vec{k})^2} \tag{15}$$

with

$$\vec{p}_z = \frac{1}{2}(1+z)\vec{p}_1 + \frac{1}{2}(1-z)\vec{p}_2; E_z = \frac{1}{2}(1+z)E_1 + \frac{1}{2}(1-z)E_2, \tag{16}$$

Since

$$\int \frac{d\Omega}{(E\sqrt{k^2 + \lambda^2} - \vec{p}\vec{k})^2} = \frac{4\pi}{k^2(E^2 - p^2) + \lambda^2 E^2} \tag{17}$$

We can show that:

$$\int_0^\varepsilon \frac{k^2 d\left|\vec{k}\right|}{\left[k^2(E^2-p^2)+\lambda^2 E^2\right]\sqrt{k^2+\lambda^2}} = \frac{1}{E^2-p^2}\ln\frac{2\varepsilon}{\lambda} - \frac{E}{2\left|\vec{p}\right|}\frac{1}{E^2-p^2}\ln\left|\frac{E+\left|\vec{p}\right|}{E-\left|\vec{p}\right|}\right| \tag{18}$$

Using Eq. (15) – (18) into Eq. (14), we have:

$$I = \frac{1}{4\pi^2}\left\{-\left[2+(\vec{p}_1\vec{p}_2-E_1E_2)\int_{-1}^1\frac{dz}{E_z^2-\vec{p}_z^2}\right]\ln\frac{2\varepsilon}{\lambda} + \frac{1}{2}\left[\frac{E_1}{\left|\vec{p}_1\right|}\ln\frac{E_1+\left|\vec{p}_1\right|}{E_1-\left|\vec{p}_1\right|} - \frac{E_2}{\left|\vec{p}_2\right|}\ln\frac{E_2+\left|\vec{p}_2\right|}{E_2-\left|\vec{p}_2\right|}\right]\right.$$
$$\left. + (\vec{p}_1\vec{p}_2-E_1E_2)\int_{-1}^1\frac{dz}{E_z^2-\vec{p}_z^2}\frac{E_z}{\left|\vec{p}_z\right|}\ln\frac{E_z+\left|\vec{p}_z\right|}{E_z-\left|\vec{p}_z\right|}\right\} \tag{19}$$

In the rest frame of muon $\vec{p}_1=0$, we set $E_2=mch2y, \vec{p}_2=msh2y,$ and note that:

$$E_z^2-\left|\vec{p}_z\right|^2 = ch^2y-z^2sh^2y, \quad \vec{p}_1\vec{p}_2-E_1E_2=-m^2ch2y$$

Therefore:

$$\int_{-1}^1\frac{dz}{E_z^2-\vec{p}_z^2} = \int_{-1}^{+1}\frac{dz}{ch^2y-z^2sh^2y} = \frac{1}{2chy}\int_{-1}^1\frac{(chy-zshy)+(chy+zshy)}{ch^2y-z^2sh^2y}dz$$

$$= \frac{2}{sh2y}\ln\left|\frac{chy+shy}{chy-shy}\right| = \frac{4y}{m^2sh2y}$$

Since $chy=\dfrac{e^y+e^{-y}}{2}, shy=\dfrac{e^y-e^{-y}}{2}$ and if $\left|\vec{p}_1\right|=x\to 0$ then

$$\frac{E_1}{\left|\vec{p}_1\right|}\ln\frac{E_1+\left|\vec{p}_1\right|}{E_1-\left|\vec{p}_1\right|} = \lim_{x\to 0}\frac{m}{x}\ln\frac{m+x}{m-x} = \lim_{x\to 0}\ln\left(\frac{m+x}{m-x}\right)^{\frac{m}{x}} = 2. \tag{20}$$

Similarly, we have the results:

$$\frac{E_2}{\left|\vec{p}_2\right|}\ln\frac{E_2+\left|\vec{p}_2\right|}{E_2-\left|\vec{p}_2\right|} = 4ycth2y; E_z=ch^2y-z^2sh^2y; \left|\vec{p}_z\right|=(1-z)shychy \tag{21}$$

$$J = \int_{-1}^1\frac{dz}{E^2-\vec{p}_z^2}\frac{E_z}{\left|\vec{p}_z\right|}\ln\frac{E_z+\left|\vec{p}_z\right|}{E_z-\left|\vec{p}_z\right|} = \frac{8y}{sh2y}h(2y) \tag{22}$$

where $h(y)=\dfrac{1}{y}\displaystyle\int_0^y xcthxdx$. Finally, we obtain the expression for I:

$$I = \frac{1}{4\pi^2}\left\{2(2\,ycth2y - 1)\left[\ln\frac{2\varepsilon}{\lambda} - \frac{1}{2}\right] + 4\,ycth2y(1 - h(2y))\right\} \tag{23}$$

For the sake of comparison with the calculation results of the dimensional regularization method in the following section, we set $2y = th^{-1}\beta$ and $cth2y = \frac{1}{\beta}$, in the rest frame of muon $\beta = |\vec{p}_1|/E_2$, where $E_2, \vec{p}_2$ are the energy and momentum of the electron, respectively.

From (23) we derive:

$$I = C_1(\beta)\{-\ln\lambda + \ln\varepsilon\} + C_2(\beta) \tag{24}$$

This expression diverges when $\lambda \to 0$.

Here $C_1(\beta)$ and $C_2(\beta)$ are determined according to the following formulas:

$$C_1(\beta) = \frac{1}{2\pi^2}\left[\frac{1}{\beta}th^{-1}\beta - 1\right] \tag{25}$$

$$C_2(\beta) = \frac{1}{4\pi^2}\left\{1 + \frac{th^{-1}\beta}{\beta}\left[1 - th^{-1}\beta\right] + 2\ln 2\left(\frac{1}{\beta}th^{-1}\beta - 1\right) + \frac{1}{\beta}L\left(\frac{2\beta}{1+\beta}\right)\right\} \tag{26}$$

$L(x) = \int_0^x \frac{dt}{t}\ln|1 - t|$ is the Spence function.

2.2. Treament for divergence using the dimensional regularization method

The method of treatment for divergence by the dimensional regularization method was first used in 1972 by G. 't Hoof and Veltman [7] to prove the renormalization of non-Abelian standard field theories. In the 4-dimensional manifold, to calculate integrals the imaginary impulses are replaced by integrals taken in the space of non-integer dimensions [8]. Then we have integral adjustment:

$$\frac{d^3k}{2k_0(2\pi)^3} \to \frac{d^{n-1}K}{2|K|(2\pi)^{n-1}},$$

where $K = (k_0, k)$ is the *n*- dimensional vector,

$$k_0 = |K| = \left(k_1^2 + k_2^2 + ... + k_{n-1}^2\right)^{\frac{1}{2}}.$$

In the rest frame of muon, the expression under the integral sign already includes the contribution of soft photons to the decay process.

The expression (13) was modified as follows:

$$I_n^{(\varepsilon)} = -I \int_\varepsilon \frac{d^{(n-1)}K}{(2\pi)^{n-1} 2|K|} \left(\frac{p_{2\mu}}{p_2.K} - \frac{p_{1\mu}}{p_1.K} \right) \left(\frac{p_2^\mu}{p_2.K} \right),$$

(27)

where n is the tuning parameter and ε is the notation to remind us that $|K|$ is in the region $0 \le K_0 = |K| \le \varepsilon$.

Muon is at rest in its frame of reference, therefore:

$$p_{1\mu} = p_1^\mu = 0; \, p_{2\mu} = p_2 \sin\theta,$$

(28)

here θ is the angle between p_2 and K.

Thus, equation (27) becomes:

$$I_n^{(\varepsilon)} = \int \frac{d^{n-1}K}{(2\pi)^{n-1} 2|K|} \left(\frac{p_2 \sin\theta}{|K|(E_2 - p_2 \cos\theta)} \right)^2 = \int \frac{d^{n-1}K}{(2\pi)^{n-1} 2|K|^3} \left(\frac{\frac{p_2}{E_2}\sin\theta}{1 - \frac{P_2}{E_2}\cos\theta} \right)^2$$

$$I_n^{(\varepsilon)} = \int \frac{d^{n-1}K}{(2\pi)^{n-1} 2|K|^3} \frac{\beta^2 \sin^2\theta}{(1 - \beta\cos\theta)^2},$$

(29)

where $\beta = |\vec{p}_2| / E_2$ and take the first axis along $\vec{p}_2$.

We can then integrate over the remaining angles and obtain:

$$I_n^{(\varepsilon)} = \frac{2}{(16\pi^2)^{\frac{1}{4}-n}} \frac{\beta^2}{\Gamma\left(\frac{1}{2}n-1\right)} \int_0^\varepsilon d|K||K|^{n-5} \int_{-1}^{+1} dx \frac{(1-x^2)^{\frac{1}{2}n-1}}{(1-\beta x)^2}$$

(30)

The integrals are performed in n-dimensional space and in the region where $|K|$ converges. By analytic expansion we see that the infrared divergence can appear as singularity of (30) at $n = 4$.

Notice that:

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$$\int_0^\varepsilon d\,|K|\,|K|^{n-5} = \frac{\varepsilon^{n-4}}{n-4}. \tag{31}$$

We substitute (31) into (30) and perform the expansion on the Laurent series at $n = 4$

$$I_n^{(\varepsilon)} = \frac{2}{\left(16\pi^2\right)^{\frac{1}{4}n}} \frac{\beta^2}{\Gamma\left(\frac{1}{2}n-1\right)} \frac{\varepsilon^{n-4}}{n-4} \int_{-1}^1 dx \frac{\left(1-x^2\right)^{\frac{1}{2}n-1}}{\left(1-\beta x\right)^2}, \tag{32}$$

$$I_n^{(\varepsilon)} = \frac{2\beta^2}{16\pi^2} \times \frac{1}{n-4} \times \frac{1}{\Gamma\left(\frac{1}{2}n-1\right)} \int_{-1}^{+1} dx \frac{\left(1-x^2\right)}{\left(1-\beta x\right)^2} \frac{\left(\varepsilon^2\right)^{(n-4)/2}\left(1-x^2\right)^{(n-4)/2}}{\left(16\pi^2\right)^{(n-4)/4}}. \tag{33}$$

Use the following expansions:

$$\Gamma\left(\frac{n}{2}-1\right) = 1 - \frac{n-4}{2}\gamma_E + O(n-4) \tag{34}$$

$$\frac{\left(\varepsilon^2\right)^{(n-4)/2}\left(1-x^2\right)^{(n-4)/2}}{\left(4\pi\right)^{(n-4)/2}} = \left(\frac{\varepsilon^2(1-x^2)}{4\pi}\right)^{(n-4)/2} = 1 + \frac{n-4}{2}\ln\frac{\varepsilon^2(1-x^2)}{4\pi} + O(n-4)$$

$$\approx 1 + \frac{n-4}{2}\left(2\ln\varepsilon + \ln\left(1-x^2\right) - 2\ln 2\sqrt{\pi}\right). \tag{35}$$

Substitute (34), (35) into (33), we receive:

$$I_n^{(\varepsilon)} = \frac{1}{8\pi^2}\beta^2\left[\frac{1}{n-4} + \ln(\varepsilon) + \frac{1}{2}\gamma - \ln\left(2\sqrt{\pi}\right)\right]\int_{-1}^1 dx \frac{\left(1-x^2\right)}{\left(1-\beta x\right)^2}$$

$$+ \frac{1}{16\pi^2}\int_{-1}^{+1} dx \frac{\left(1-x^2\right)}{\left(1-\beta x\right)^2}\ln\left(1-x^2\right) + O(n-4), \tag{36}$$

where

$$\int_{-1}^{+1} dx \frac{1-x^2}{\left(1-\beta x\right)^2} = \frac{4}{\beta^2}\left\{-\frac{1}{2\beta}\ln\frac{1-\beta}{1+\beta}-1\right\} = \frac{4}{\beta^2}\left\{\frac{1}{\beta}th^{-1}\beta-1\right\}. \tag{37}$$

Here, we use the formula:

$$th^{-1}\beta = -\frac{1}{2}\ln\left(\frac{1-\beta}{1+\beta}\right), \tag{38}$$

and

$$\int\limits_{-1}^{+1} dx \frac{1-x^2}{(1-\beta x)^2} \ln\left(1-x^2\right) = \frac{4}{\beta^2}\left[1 + \frac{th^{-1}\beta}{\beta}\left[1 - th^{-1}\beta\right] + 2\ln 2\left(\frac{1}{\beta}th^{-1}\beta - 1\right) + \frac{1}{\beta}L\left(\frac{2\beta}{1+\beta}\right)\right]. \quad (39)$$

The final evaluation of the above integrals gives us the result:

$$I_n^{(\varepsilon)} = C_1(\beta)\left[\frac{1}{n-4} + \ln(\varepsilon) + \frac{1}{2}\gamma_E - \ln\left(2\sqrt{\pi}\right)\right] + C_2(\beta) + O(n-4) \quad (40)$$

where $C_1(\beta)$ and $C_2(\beta)$ are determined by (25) and (26) respectively, γ_E is the Euler – Mascheroni constant.

3. Results and disscusion

We studied the electromagnetic correction in one real photon approximation for muon decay $\mu \to e + \tilde{\nu}_e + \nu_\mu + \gamma$, infrared divergence appeared in the matrix factor and decay rate expression. The infrared divergence here has been reduced by two methods: the minimum mass method and the dimensional regularization method. Comparing the results of the analytical expression for the decay rate of the process in the lowest order of two methods, we get the relationship as follow:

$$\ln \lambda = -\left(\frac{1}{n-4} + \frac{\gamma_E}{2} - \ln 2\sqrt{\pi}\right)$$

The infrared divergences here appear for physical processes with exchange particles of zero rest mass, for both virtual and real photons. To separate the infrared divergence emerging from radiative correction to the weak decay of muons, we need to compute the "soft" virtual photons, similarly to the infrared divergence in QED, which we have seen in a lot of theoretical literatures about divergence.

Another serious difficulty in the Fermi theory is that it is not a renormalizable theory [5, 6], which is very different from the case of quantum electrodynamics, being renormalizable. As we known, we have divergent integrals originating from higher – order diagrams even in QED as well. However, in QED we can remove those divergences by renormalizing the mass and charge into the observed physical values. On the other hand, in case of the Fermi interaction, if we try to follow the renormalization procedure analogously as in QED and go to the next higher order diagram, we encounter a more seriours divergence, which needs further renormalization constants. Then, to consider all higher order diagrams, we need an infinite set of

renormalization constants. That is, the theory cannot be renormalized [5, 6]. One of the intersting ideas to overcome those difficulties is the intermediate weak boson exchange model. That is, we will assume that the weak interaction is conducted through the exchange of a weakly interacting boson with a large mass (e.g. W particles in muon decay), which is similar to the electromagnetic interaction that does through the exchange of massless photons.

4. Conclusion

The content of the article extends the dimensional regularization method to eliminate divergence for weak interactions that are important in modern physics. This method can be used to eliminate ultraviolet and infrared divergences in quantum chromodynamics, in quantum gravitational field theory, etc., for specific physical processes as well as for unified quantum fields [8, 9]. These issues will be further studied by the group in the near future.

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BỔ CHÍNH ĐIỆN TỪ TRONG PHÂN RÃ MUON

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Tóm tắt: Ảnh hưởng của tương tác điện từ trong gần đúng một photon được áp dụng để xem xét sự phân rã muon trong tương tác điện yếu. Các phân kỳ phát sinh trong quá trình này là các phân kỳ hồng ngoại trong vùng năng lượng thấp. Việc khử các phân kỳ được thực hiện bằng hai phương pháp: phương pháp khối lượng tối thiểu và phương pháp chỉnh thứ nguyên. Kết quả là, một phần bổ chính tương tác điện từ được tính toán thêm cho tốc độ phân rã điện yếu của muon. Các kết quả một lần nữa xác nhận rằng lý thuyết Fermi về tương tác điện yếu là một lý thuyết không tái chuẩn hóa.

Từ khóa: Tương tác điện yếu; phân rã muon; phân kỳ hồng ngoại; tái chuẩn hóa.

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