

# INTERACTION BETWEEN A BEYOND-BAND DISCRETE SOLITON AND A DIRAC SOLITON IN BINARY WAVEGUIDE ARRAYS

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## Abstract

We investigate the intricate interactions between a beyond-band discrete solitons (BBDSs) and Dirac solitons in a binary waveguide array (BWA). Through comprehensive analysis, we demonstrate that the behavior of these soliton interactions remains largely unaffected by the presence of central peaks or dips at the center of each soliton, or the initial phase differences between them. Our research highlights the critical role of the BBDS intensity in determining the nature of its interaction with Dirac solitons. We reveal that low-intensity BBDS exhibit consistent attraction towards Dirac solitons across multiple encounters, while high-intensity BBDSs show an initial phase of attraction followed by a repulsion phase, causing a subsequent divergence. These findings not only enhance our understanding of soliton interactions on a fundamental level but also set the stage for the development of innovative optical communication and signal processing technologies leveraging the distinctive properties of BBDSs and Dirac solitons.

**Keywords:** *Beyond-band discrete soliton; Dirac soliton; binary waveguide array.*

## 1. Introduction

Waveguide arrays (WAs) are at the cutting edge of photonic research, embodying a unique interplay of discreteness, periodicity, and nonlinearity that has unveiled a multitude of fascinating photonic phenomena. These include, but are not limited to, the discrete diffraction [1-3] and the formation of discrete solitons [1, 4], phenomena that have not only enriched our understanding of light propagation in structured media but also bridged the gap between spatial and temporal nonlinear optical effects. The similarity of these effects to those observed in fiber optics [5, 6], such as diffractive resonant radiation [7], albeit manifested in the spatial domain, underscores the versatility and potential of waveguide arrays in exploring optical phenomena.

Beyond the fundamental insights, WAs offer a tangible platform for pioneering applications in photonic circuitry, enabling the development of all-optical routers and logic functions essential for the advancement of optical computing and information processing technologies. Waveguide arrays are also a great tool to investigate nonrelativistic quantum

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mechanics phenomena, including photonic Bloch oscillations [8-10], Zener tunneling [11, 12], and dynamic localization [13, 14], further amplifying their significance in both theoretical and applied physics.

The exploration of BWAs, a subclass of WAs characterized by the alternation of two types of waveguides, has opened new avenues for simulating relativistic quantum effects. Through BWAs, phenomena governed by the Dirac equation, such as Klein tunneling [15-17], Zitterbewegung [18, 19], Dirac solitons [20], and topological Jackiw-Rebbi states [21], have been earlier studied, providing a rich framework for understanding the complexities of relativistic quantum effects in a controlled, optical setting.

The study of discrete gap solitons in BWAs has revealed a variety of dynamic states including gap, out-gap solitons, breathers, and pulsons, expanding the understanding of light propagation in discrete systems. The pioneering work identified discrete gap solitons within a discrete diatomic lattice model [22], laying the groundwork for subsequent explorations into their nature and dynamics. A significant breakthrough was achieved when these discrete gap solitons were recognized as optical analogs of Dirac solitons, derived from the relativistic quantum one-dimensional Dirac equation with Kerr nonlinearity [23]. This revelation underscored the potential of BWAs to simulate complex relativistic quantum phenomena through optical means. Further research has deepened the understanding of Dirac solitons within BWAs, exploring their stability, interactions [24], and the formation of Dirac light bullets [20].

The concept of beyond-band discrete solitons (BBDSs) in BWAs has been recently introduced as a new class of localized structures with unique properties [25]. These BBDSs are localized structures that are not only out-gap but also beyond the band, as their detunings are outside the two bands of linear plane waves in a periodic BWA. This characteristic, along with their approximation through hyperbolic secant functions, distinguishes BBDSs from other soliton classes previously studied in BWAs [25]. The interaction between BBDSs has been systematically studied in [26].

This article aims to further the understandings on BBDSs by investigating the interactions between them and Dirac solitons. Given the importance of optical pulse and soliton interactions in both fundamental and applied nonlinear optics, our study seeks to provide comprehensive insights into these phenomena. The article is structured as follows: Section 2 revisits the foundational results on BBDSs necessary for our further analysis. In Section 3, we focus on the interaction between a weak BBDS and a Dirac soliton, while in Section 4, we examine the interaction between an intense BBDS and a Dirac soliton. We conclude in Section 5 with a synthesis of our findings and their important implications.

## 2. Equations governing beyond-band discrete solitons

The propagation of light beams within a discrete and periodic BWA, operating under the regime of Kerr nonlinearity, is governed by the subsequent dimensionless coupled-mode equations [16, 27]:

$$i \frac{da_n(z)}{dz} = -\kappa(a_{n+1} + a_{n-1}) + (-1)^n \sigma a_n - \gamma |a_n|^2 a_n \quad (1)$$

where  $a_n$  represents the amplitude of the electric field in the  $n$ -th waveguide, where  $n$  spans discretely over the set of  $[-(N-1)/2, \dots, -1, 0, 1, \dots, (N-1)/2]$ , with  $N$  denoting the total odd number of waveguides in the BWA. The variable  $z$  corresponds to the longitudinal coordinate along the axis of the BWAs,  $2\sigma$  and  $\kappa$  denote the propagation mismatch and the coupling coefficient between adjacent waveguides, respectively, while  $\gamma$  represents the nonlinear coefficient of the waveguides. For simplicity, it is assumed that all waveguides within the BWA possess an identical value for  $\gamma$ . It is pertinent to note that variables in the equations can be normalized such that the absolute values of both  $\gamma$  and  $\kappa$  equate to unity. Moreover, the same BWA can be employed to investigate scenarios wherein  $\sigma$  alternates its sign from 1 to -1 and vice versa, simply by shifting the position  $n$  of the waveguide system by any odd integer. This adjustment results in the reclassification of waveguides from even to odd and vice versa. Hence, if a waveguide was initially classified as even, following the inversion of  $\sigma$  to  $-\sigma$ , it will subsequently be regarded as odd, and conversely.

Utilizing the ansatz of a plane wave that propagates obliquely within BWAs entails:

$$a_n(Q) \sim \exp[i(Qn - \omega z)] \quad (2)$$

In the linear regime, the application of this approach yields the following dispersion relations [27]:

$$\omega_{\pm}(Q) = \pm \sqrt{\sigma^2 + 4\kappa^2 \cos^2 Q} \quad (3)$$

In the context,  $Q$  denotes the normalized wavenumber associated with the plane wave. This parameter further encapsulates the phase shift occurring between consecutive waveguides, a phenomenon attributable to the angled propagation of beams within BWAs. Now we can use the following ansatz to search for BBDSs in BWAs [25]:

$$a_n = b_n e^{i\delta z} \quad (4)$$

where  $b_n$  is a real coefficient, solely a function of  $n$ , and  $\delta$  represents the detuning of each BBDS. The detuning parameter for each BBDS corresponds to an eigenvalue, which must be determined based on the peak amplitude among other relevant parameters. Following

this, the ansatz (4) is substituted into Eq. (1), leading to the derivation of a corresponding system of algebraic equations [23]:

$$-\delta b_n + \kappa[b_{n+1} + b_{n-1}] - (-1)^n \sigma b_n + \gamma |b_n|^2 b_n = 0 \quad (5)$$

In the quasilinear regime, where the intensity of all components remains low, it is justifiable to neglect the nonlinear term present in Eqs. (1) and (5). Under such conditions of low intensity, an approximation can be employed to represent BBDSs in a simplified manner. Specifically, this simplification yields representations in the form of two hyperbolic functions, differentiated for even and odd components, as delineated in [25]:

$$\begin{bmatrix} b_{2n} \\ b_{2n-1} \end{bmatrix} = \begin{bmatrix} p \operatorname{sech}\left(\frac{2n}{n_0}\right) \\ q \operatorname{sech}\left(\frac{2n-1}{n_0}\right) \end{bmatrix} \quad (6)$$

where  $p$  and  $q$  are the peak amplitudes corresponding to the even and odd components, respectively. When the central amplitude  $b_0 = p$  of the BBDS is specified for the waveguide at  $n = 0$ , the two parameters  $\delta_l$  and  $q$  of the BBDS can be readily calculated as follows, according to [25]:

$$\delta_l = \pm \sqrt{\sigma^2 + 4\kappa^2} \quad (7)$$

$$q = \frac{b_0(\sigma \pm \sqrt{\sigma^2 + 4\kappa^2})}{2\kappa} \quad (8)$$

where  $\delta_l$  signifies the detuning parameter  $\delta$  in scenarios devoid of nonlinearity or within the low-intensity regime. As elucidated in [25], with Kerr nonlinearity, the sign of  $\gamma$  (positive or negative) dictates the adoption of the corresponding sign for the squares in Eqs. (7) and (8). Evident from Eq. (6), the even or odd components of BBDSs are characterized by a hyperbolic secant function, maintaining their sign consistency across the transverse direction of the BWAs as  $n$  varies. Additionally, as previously discussed, the detunings associated with these BBDSs extend beyond the two linear plane wave bands within BWAs. These characteristics distinctly differentiate BBDS from other discrete solitons observed in BWAs [25].

### 3. Interaction between a weak BBDS and a Dirac soliton

In this section, we investigate the interaction dynamics between a weak BBDS and a Dirac soliton. It is important to highlight that these solitons represent distinct categories within the spectrum of discrete solitons in BWAs. As delineated in the subsequent analysis, the characteristics of their interaction are invariant with respect to the initial

phase difference, denoted as  $\theta$ , between the two solitons. The initial condition for integraring Eq. (1) are as follows:

$$a_n(0) = S_{n+\Delta n_1}^{(1)} + rS_{n-\Delta n_2}^{(2)} e^{i\theta} \quad (9)$$

where  $S_{n+\Delta n_1}^{(1)}$  denotes the Dirac soliton with its center lying at the waveguide position  $n = -\Delta n_1$  and  $S_{n-\Delta n_2}^{(2)}$  denotes the BBDS with its center lying at the waveguide position  $n = \Delta n_2$ ,  $r$  is the scaling factor which is set to be unity in this work.

In Fig. 1, the dynamics of interaction between a weak BBDS and a Dirac soliton with variable peak amplitude and width is explored. Figure 1(a) illustrates the profiles of these solitons under the condition where the parameter  $\sigma = 1$  for the BWA, the phase difference  $\theta = 0$ , and the scaling factor  $r = 1$ . The Dirac soliton is positioned on the left, centered at  $n = -18$ , with a peak amplitude of  $a_{-18} = 0.4$  and a width parameter of  $n_0 = 5$ , as defined by the Dirac soliton solution detailed in Eq. (6) from [23]. Conversely, the weak BBDS is located on the right, with its center at  $n = 19$ , a peak amplitude of  $a_{19} = 0.2$ , and a detuning  $\delta \approx 2.3004$ . Consequently, the initial distance between the centers of the two solitons is  $D = 37$  waveguides. This separation distance is consistently maintained in Figs. 1(a), 1(b), and 1(c), but is increased to  $D = 41$  waveguides in Fig. 1(d), wherein the initial center of the Dirac soliton shifts to the waveguide at  $n = 23$ .

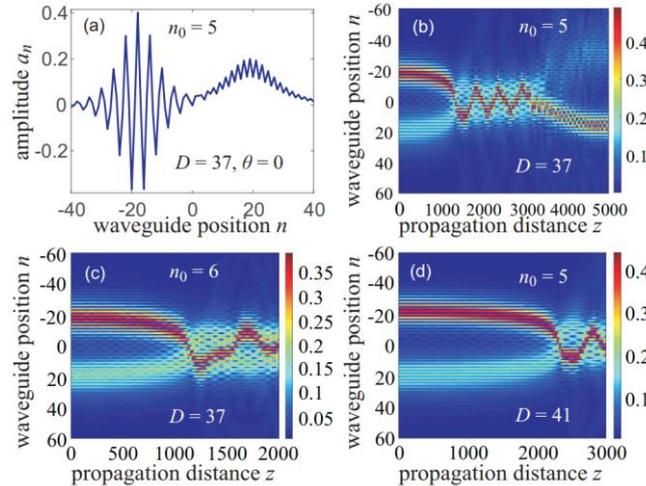


Fig. 1. The interaction between a weak BBDS and a Dirac soliton

(a) The configuration of two in-phase solitons: The Dirac soliton is positioned with its center at  $n = -18$  and exhibits a width parameter  $n_0 = 5$ , while the weak BBDS is located with its center at  $n = 19$  and has a peak amplitude of  $a_{19} = 0.2$  (b), and (c) Show the interaction dynamics between the BBDS and the Dirac soliton with an initial center-to-center distance of  $D = 37$  waveguides, for width parameters  $n_0 = 5$  and  $n_0 = 6$ , respectively. (d) Focuses on the interaction under the condition where  $n_0 = 5$  and the initial center-to-center distance is extended to  $D = 41$  waveguides. The parameters set for these interactions include  $\sigma = 1$ ,  $\gamma = 1$ ,  $\kappa = 1$ , and a phase difference of  $\theta = 0$ .

The interaction depicted between two solitons, as initially outlined in the profiles presented in Fig. 1(a), is elucidated further in Fig. 1(b). Initially, the solitons exhibit mutual attraction, progressively narrowing the distance between them until they intersect at an approximate distance of  $z \approx 1400$ . Subsequent to this intersection, the solitons engage in a series of oscillatory movements, characterized by brief periods of separation followed by convergence and recurrent crossings. These oscillatory interactions persist as the solitons maintain proximal trajectories. However, subsequent to their inaugural crossing, the BBDS incurs significant radiative losses towards the lattice's edges. This radiative dispersion is subsequently reflected by the lattice boundaries, delineated at  $n = -150$  and  $150$  within Fig. 1(b), contributing to a notably disordered background when  $z \geq 3500$ . When  $z \geq 3500$  the diminished BBDS ceases to retain its beam structure, attributable to the radiative loss and the background noise. Contrarily, the more robust Dirac solitons demonstrates resilience, persisting amidst the prevalent noise.

In Fig. 1(c), the parameter width  $n_0$  of the Dirac soliton is increased to 6, resulting in a diminution of its initial peak amplitude to approximately  $a_{-18} \approx 0.333$ . Conversely, Fig. 1(d) retains  $n_0 = 5$ , but extends the initial center-to-center separation between the two solitons to  $D = 41$  waveguides. Despite these modifications, the principal characteristics of the beam interactions depicted in Figs. 1(c) and 1(d) remain fundamentally akin to those observed in Fig. 1(b), with only quantitative variances evident among these depictions. For example, relative to Fig. 1(b), the initial point of intersection between the two beams in Fig. 1(c) is positioned closer to the input facet, indicative of a more pronounced initial overlap of the beams' extremities, whereas in Fig. 1(d), this crossing is situated further from the input facet, reflecting a lesser degree of initial overlap. These two solitons gradually close the gap between them at a distance  $z \geq 2200$ . It is observed that as the initial center-to-center distance between the solitons increases, their mutual attraction decreases. This behavior can be attributed to the increased width of the waveguide, which weakens the interactions between the solitons. As a result, a larger initial distance is required for the solitons to first intersect from the initial input facet.

#### **4. Interaction between an intense BBDS and a Dirac soliton**

In Fig. 2, the examination focuses on the interaction between an intense BBDS and a Dirac soliton, both of which exhibit variations in peak amplitude and width. Specifically, Fig. 2(a) delineates the profiles of these solitons under the conditions where the parameter  $\sigma = 1$  is applied to the BWA, with  $\theta = 0$ , and  $r = 0$ . The Dirac soliton, positioned on the left with its center at  $n = -18$  and peak amplitude at  $a_{-18} = 0.4$ , is

characterized by a width parameter  $n_0 = 5$ . Conversely, the BBDS is situated on the right, with its center at  $n = 18$ , featuring a central dip with an amplitude approximately  $a_{18} \approx 0.2363$  and a detuning  $\delta \approx 2.3004$ . The initial separation between the two solitons in Fig. 2 is consistently maintained at  $D = 36$  waveguides. Figure 2(b) illustrates the ensuing interaction, where the solitons initially exhibit mutual attraction, culminating in a collision at a distance  $z \approx 1750$ . Subsequent to this collision, a singular robust beam emerges, ascending, while a dispersion of weak and broad radiation is observed spreading.

In Figs. 2(c) and 2(d), the width parameter of the Dirac soliton  $n_0$  is augmented to values of 6 and 7, respectively. Consequently, the maximal amplitude of the Dirac soliton diminishes to 0.333 and 0.286, in each case. It is important to highlight that, apart from the alterations in  $n_0$ , all other parameters remain consistent with those delineated for Fig. 2(a). In Fig. 2(c), an initial attraction between two solitons is observed; they approach each other closely, reaching a proximity at a distance  $z \approx 800$ , yet without intersection, thereafter repelling and progressively distancing from one another. Contrastingly, Fig. 2(d) presents a divergent scenario wherein, although a weak attraction between the solitons is noted, it significantly differs in dynamics. Initially, the broad Dirac soliton and the BBDS appear to propagate parallel to the longitudinal axis. Subsequently, at  $z \approx 400$ , the BBDS is marginally pulled upwards towards the Dirac soliton, which, in turn, ascends further away from the BBDS. Following this interaction, the solitons rapidly diverge and proceed in opposite directions.

It is important to reiterate that the interaction between a Dirac soliton and a BBDS is, to a large extent, independent of the initial phase difference  $\theta$  that exists between them. This is totally different from the interaction between solitons of the same kinds, for instance, between two BBDSs [26] where their initial phase difference  $\theta$  plays a crucial role. Furthermore, our simulations, which are not presented herein, indicate that the nature of interactions between Dirac solitons and BBDS remains consistent, regardless of whether there is a dip or a peak at the center of each soliton. This stability is due to the complex dynamics of superposition and compensation when Dirac solitons interact with BBDS. These dynamics involve a more intricate exchange of energy and momentum than is typical in interactions between similar solitons. As a result, variations in soliton profiles, such as the presence of central peaks or dips, have a minimal impact on their interactions. It is noteworthy that this transition - from a peak to a dip at the soliton's center - can be achieved through a mere shift of the soliton's center by one waveguide within the same BWA.

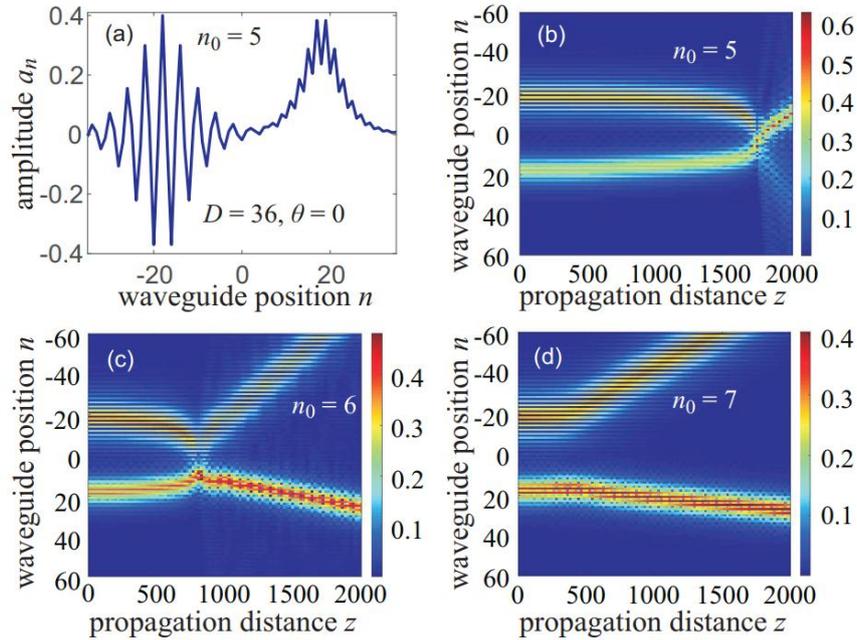


Fig. 2. An examination of the interaction between an intense BBDS and a Dirac soliton.

(a) The profiles of the two distinct solitons: A Dirac soliton positioned at  $n = -18$  with a parameter width,  $n_0 = 5$ , and an intense BBDS centered at  $n = 18$ ; (b, c, d) Illustrate the dynamics of interaction between these solitons for varying parameter widths  $n_0 = 5, 6,$  and  $7$  respectively. The interactions are characterized under the parameters:  $\sigma = 1, \gamma = 1, \kappa = 1,$  and a fixed center-to-center distance  $D = 36$ .

## 5. Conclusion

In conclusion, our study has advanced the understanding of the dynamics involved in the interaction between BBDS and Dirac solitons in a BWA. We have established that the qualitative behavior of these interactions is remarkably robust, showing little sensitivity to the characteristics of the solitons involved, such as the presence of a peak or a dip at their centers or their initial phase differences. Crucially, we have identified the intensity of the BBDS as a decisive factor in dictating the nature of the interaction with Dirac solitons. Specifically, a low-intensity BBDS consistently exhibits attraction to a Dirac soliton during repeated crossings, while a high-intensity BBDS initially attracts but subsequently repels the Dirac soliton, leading them to diverge over time. These findings not only provide new insights into the fundamental properties of soliton interactions but also pave the way for designing novel optical communication systems and signal processing technologies that leverage the unique characteristics of BBDS and Dirac solitons. Future research will be aimed at exploring the implications of these interactions in practical applications, particularly in enhancing the efficiency and stability of soliton-based transmission systems.

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## TƯƠNG TÁC GIỮA SOLITON RỜI RẠC BEYOND-BAND VÀ SOLITON DIRAC TRONG MẢNG ỚNG DẪN SÓNG NHỊ PHÂN

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**Tóm tắt:** Bài báo trình bày việc nghiên cứu sự tương tác giữa các soliton rời rạc beyond-band (BBDS) và soliton Dirac trong mảng ống dẫn sóng nhị phân BWA. Các kết quả tính toán mô phỏng chỉ ra rằng tương tác giữa hai loại soliton này phần lớn không bị ảnh hưởng bởi sự hiện diện của các đỉnh hoặc hố trung tâm và sự khác biệt pha ban đầu. Nghiên cứu cũng xác định vai trò quan trọng của cường độ BBDS trong ảnh hưởng đến sự tương tác của nó với soliton Dirac. Các BBDS cường độ thấp có xu hướng bị hút về phía soliton Dirac nhiều lần, trong khi BBDS cường độ cao chỉ bị hút về phía soliton Dirac ở giai đoạn ban đầu, sau đó chúng đẩy nhau, gây ra hiện tượng phân kỳ. Kết quả nghiên cứu này không chỉ cung cấp thêm thông tin quan trọng về sự tương tác của các soliton mà còn tạo tiền đề cho sự phát triển của các công nghệ truyền và xử lý tín hiệu quang học, dựa trên các tính chất đặc biệt của BBDS và soliton Dirac.

**Từ khóa:** *Soliton rời rạc beyond-band; soliton Dirac; mảng ống dẫn sóng nhị phân.*

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