

# Control strategy of optimal power flow for microgrid islanded operation based on dynamic programming

## Xây dựng chiến lược điều khiển tối ưu công suất phát cho hệ thống điện lai ở chế độ ốc đảo sử dụng phương pháp quy hoạch động

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### Tóm tắt

Năng lượng tái tạo đã trở thành một giải pháp quan trọng để thay thế thế hệ thông thường cung cấp năng lượng cho hệ thống điện microgrid (MG), tuy nhiên, sự dao động và gián đoạn do tính ngẫu nhiên của nguồn và phụ tải ảnh hưởng đến luồng công suất trong hệ thống. Bài viết này trình bày chiến lược điều khiển tối ưu dòng công suất (OPL) của MG hoạt động ở chế độ ốc đảo bằng cách phát triển mô hình toán học. MG bao gồm máy phát diesel, hệ thống lưu trữ năng lượng (ESS), máy phát điện sức gió (WTG) và hệ thống pin năng lượng mặt trời (PV). Đối với đối tượng nghiên cứu của bài báo, việc lập kế hoạch vận hành được thực hiện dựa trên giả thiết công suất tải, công suất WTG và công suất PV phải được dự báo ngắn hạn, để từ đó lập lịch cho nguồn phát (DEG) và ESS. Mục tiêu của bài toán điều khiển là tìm công suất đầu ra DEG và công suất định mức của biến tần cho ESS bằng cách giảm thiểu tổng chi phí năng lượng và sử dụng lập trình động dẫn đến phương trình Hamilton-Jacobi-Bellman. Kết quả thí nghiệm được trình bày và cho thấy hiệu quả của điều khiển tối ưu đáp ứng đúng theo yêu cầu.

### Keywords

Microgrid, hierarchical control, optimal control, photovoltaic, wind power, diesel power, optimal power flow, Hamilton-Jacobi-Bellman equation, distributed generation

### Abstract<sup>1</sup>

Renewable energy has become a significant solution to replace conventional generation for supplying electrical energy of nowadays microgrid (MG), however, fluctuation and intermittency resulted from uncertainty source and load demand impact on power flow of system. This paper presents the control strategy for optimal power flow (OPL) of islanded MG by developing the mathematical model. The MG consists of diesel engine generator, energy storage system (ESS), wind turbine generator (WTG), and photovoltaic generator (PV). In the control system considered, operation planning is realized based on profiles such that the demand load, WTG power, and PV power must be forecasted in short-period, meanwhile the dispatch source (i.e., DEG) and ESS need scheduling. The objective of the control problem is to find the DEG output power and the rated power of inverter for ESS by minimizing the total cost of energy and using dynamic programming that leads to the Hamilton-Jacobi-Bellman equation. Experimental results are presented, and show the effectiveness of optimal control such that the generation allows demand profile.

### Symbols and Abbreviations

Symbol	Unit	Description
DEG		Diesel Engine Generator
WTG		Wind Turbine Generator
PV		Photovoltaic
$X_e$		State variable
$P_L$	kW	Demand load
$U, W$	kW	Control variable
$P_{DEG}$	kW	DEG output power
$P_{WTG}$	kW	WTG output power
$P_{PV}$	kW	PV output power
$P_{max/min}^{DEG}$	kW	Upper/lower bounds of $P_{DEG}$
$P_{max/min}^{WTG}$	kW	Upper/lower bounds of $P_{WTG}$
$P_{max/min}^{PV}$	kW	Upper/lower bounds of $P_{PV}$
$P_{inv}$	kW	Rated power of inverter
$G(.)$	\$/t	Running cost function
$J(t, \varepsilon, U, W)$	\$	Overall Cost of system
$v(t, X_e)$	\$	Value function

### 1. Introduction

Due to disadvantage of fossil fuel sources for environment all over the world, the installation of renewable energy sources power plants is necessary. On the other hand, the conventional electric power systems are rapid changes to alleviate environmental conditions, bring smartgrids and solutions, and respond to the customer demands. Thus, we focus on the dealing with changes in loads, and uncertainties of wind

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power, photovoltaic (PV) that separate the system into islanded microgrid. The concept of microgrid was proposed by the Consortium for Electric Reliability Technology Solutions (CERTS) so as enhance the stability and flexibility of power system. Microgrid is a small grid in which distributed generations (DGs) and electric loads are placed together and controlled efficiently in an integrated manner [1]. It provides the electrical energy to islands, the installation of DG with small-scale power generation technologies has rapidly increased in many countries at a reduced cost and a higher efficiency. Microgrid can operate in grid-connected, and islanded operating modes [2], [3], [4], [5]. The principal roles of the microgrid control structure are [2], [3], [6], [7], [8], [9] as voltage, frequency, and angle regulation for operating modes; proper load sharing and distributed energy resources; microgrid synchronization with the main grid (utility grid); power flow control between the microgrid and the main grid, or within microgrid; and optimizing the microgrid operating cost.

Concerning the problem of optimal power flow also *energy management* that's the tertiary control level in the hierarchical control structure including *the primary, secondary, tertiary, and emergency control* [2], [10], basically we must consider the uncontrollable nature of wind, solar power as well as load change raises uncertainty for power system operation on the one part, the application and integration of DGs into the system is complex on the other part. Among these aspects are, the investigation of impact of distributed generation on the power fluctuations from penetration of wind, photovoltaic power is presented in [11], [12], and [13]. According to [14], [15], [16], the authors have considered the hybrid power system whose the energy storage/thermal unit has a high potential for providing regulation power to meet the reverse requirements.

One of important works as in [17], which examines the optimal problem as the investigation of optimal power flow by adopting the interval optimization in which the wind power is defined in range of values; the interval power flow problem for DC power flow can be formulated as a non-convex and nonlinear programming. In consequence, to ensure system reliability, the forecasting uncertainty must be considered into generation scheduling, and interval power flow provides promising approach to achieve the boundary information of system statuses under uncertainties. More importantly, the contribution in [17] is a landmark for class of optimal power flow problem.

For the load flow problem which is typically formulated as a set of non-linear equations as functions of bus voltages has taken some advantage. Such stochastic demand has been developed and presented in [18]–[20], those authors used stochastic, fuzzy, and probability programming techniques to model the uncertainties. Although the works in [18]–[20] have been

specified to load flow problem with some algorithms, the aforementioned methods are typically dependent on predefined probability distribution function or membership function of uncertainty. Furthermore, it is hard to identify accurate probability distribution functions due to data availability and stochastic nature of the uncertainty.

The research in all these directions [11]–[22] for the case of power flow was based on non-convex, and non-linear programming and the DG units have known locations and running all the time. To add to the complexity of the problem, in the real life systems, the operation of these DG units, undergoes different scenarios according to the strategies of the electricity producers and the needs of the consumers. Therefore, some uncertainties are introduced in the operation of such units and thus, stochastic modeling of systems involving DG units becomes of great interest. The sources of the uncertainties in the operation of the DG systems at a certain hour of the day include. These uncertainties affect the modeling and evaluation of the system capacity, power losses.

In this work, we continue to develop the optimal control problem that we did in [21], [22] and [23], however, the MG adds energy storage system instead of consisting of diesel engine generator, wind turbine generator (WTG), and photovoltaic generator (PV). This is because the ESS plays important role in relation renewable energy system e.g., wind integration include load shifting which uses off-peak storage for on-peak dispatch at the system level; regulation (voltage and frequency support); power quality [24],[25]. We formulate the optimal control problem for the power flow under uncertainties by minimizing the cost of electricity production. Therefore utilizes the energy balance equation in real time to formulate the optimal power flow problem as optimal control problem of a linear system by using dynamic programming. It will be expedient to derive an algorithm similar to the Bellman principle where the optimality conditions satisfy Hamilton-Jacobi-Bellman (HJB) equations, and the value function is convex. In addition, the HJB equation leads optimal feedback control for power flow problem, and is to make the resulting system relatively insensitive to fluctuations that can deal with uncertainties of power system considered.

## 2. Description of the microgrid system

For microgrid power system having distributed generations (DGs) such as that photovoltaic (PV), wind turbine generator (WTG), diesel engine generator (DEG), and energy storage system (ESS). Figure 1 shows the typical structure of MG. The MG has two connection modes (AC bus and DC bus), and it can operate in grid-connected mode or islanded mode. However, in this paper, we consider the MG in only islanded mode as study objective.

In the Figure 1, let  $P_{PV}(t)$ ,  $P_{WTG}(t)$ , and  $P_{DEG}(t)$  be the non-dispatchable output powers of PV, WTG, and dispatchable output power of DEG respectively at time  $t$ . In consequence, let  $P_L(t)$  and  $\Delta P_L(t)$  be the demand load and the loss of power in the MG at time  $t$ .

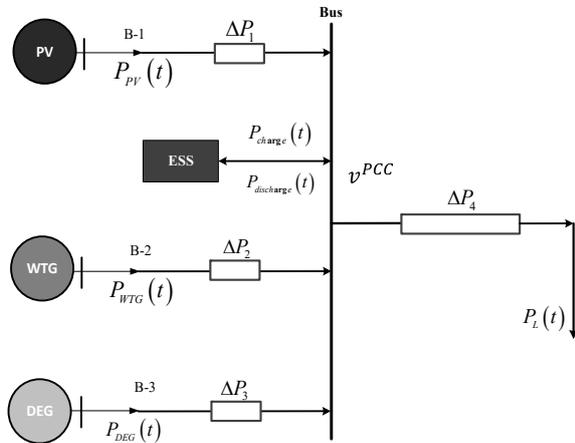


Fig. 1 Simplified schematic of the studied system

In what following, we describe the modeling of the considered microgrid. First of all, the balance of power satisfies such as:

$$P_{DEG}(t) + P_{WTG}(t) + P_{PV}(t) + P_{ESS}(t) = \hat{P}_L(t) \quad (1)$$

where  $P_{ESS}(t) = P_{discharge}(t) - P_{charge}(t)$  refers to the the power of inverter at time  $t$ ;  $P_{discharge}(t)$  and  $P_{charge}(t)$  are discharge and charge power at time  $t$ ; the total demand load and loss of power is  $\hat{P}_L(t) = P_L(t) + \Delta P_L(t)$ .

In reality, the produced power by PV and WTG depends on the environmental conditions, and the demand depends on the power consumption habits. Hence, the fluctuations in loads, PV, and WTG output power are adjusted by control in the DEG output power and ESS. Therefore, the expression of the power balance in equation (1) for randomness in such DG associated with demand can be described as follows:

$$P_{DEG}(t) + P_{ESS}(t) - (\hat{P}_L(t) - P_{WTG}(t) - P_{PV}(t)) = 0 \quad (2)$$

The power generation of DEG as well as ESS must evolve the fluctuations in loads and DGs in equation (2) called the *optimal power flow* (OPF) problem. In literature, the OPF problem is significant to consider in level 3 (*tertiary control*) [26], [2], [3] and [10]. And OPF has been investigated based on linear and non-linear programming such [14], [27], [28], [16], [29], and [17]. In contrary, we will formulate the considered problem as an optimal control one in order to find out the optimal policies for operation of microgrid in the next section.

### 3. Mathematical formulation

In this section, we consider the studied system described in Section 2. The control problem is considered in finite time (i.e.,  $T=24h$ ) in order to find the DEGs output power and discharge/charge of inverter. Clearly, to formulate a model, we shall assume that the power generation in the real time and the total power generation of DGs described as in equation (2).

#### 3.1 Selection of intermediate variables

Next, let's consider  $\varepsilon(t)$  be the difference of power balance as *intermediate variable*:

$$\varepsilon(t) = P_{DEG}(t) + P_{ESS}(t) - (\hat{P}_L(t) - P_{WTG}(t) - P_{PV}(t)) \quad (3)$$

By using the control variables  $U(t) = P_{DEG}(t)$  and  $W(t) = P_{ESS}(t)$ , the equation (3) becomes

$$\varepsilon(t) = U(t) + W(t) - (\hat{P}_L(t) - P_{WTG}(t) - P_{PV}(t)) \quad (4)$$

where,  $U(t) \in \mathcal{A}_U(t) \in \mathfrak{R}^+ = [0, +\infty)$  and  $W(t) \in \mathcal{A}_W(t) \in \mathfrak{R} = (-\infty, +\infty)$ .  $\mathcal{A}_U(t)$  and  $\mathcal{A}_W(t)$  are the set of *admissible controls*.

To explain the intermediate variable  $\varepsilon(t)$ , we make sure that the time periods are  $\forall t \in [0, T]$  with  $T = 24h$ .

- If  $\varepsilon(t) > 0$  the system is in power unbalance positive sequence – the generation is more than the load demand.
- If  $\varepsilon(t) < 0$  the system is in power unbalance negative sequence - the generation is less than the load demand.

The intermediate variable  $\varepsilon(t)$  is *adopted* for mathematical modeling in order to find the control strategy of DEG output power and ESS power so that the value of  $\varepsilon(t)$  keeps being equal to zero ( $\varepsilon(t) = 0$ ) at any time  $t$ . For  $\varepsilon(t)$  varies at any time  $t$  whose the value is  $-M_\varepsilon \leq \varepsilon(t) \leq M_\varepsilon$  ( $M_\varepsilon$  is *finite positive*), it might be velocity of  $X_\varepsilon(t)$  called *state variable*. We use dynamic programming to formulate the model by making use of state variable as follows:

$$\frac{dX_\varepsilon(t)}{dt} = \varepsilon(t), \forall t \in [0, T] \quad (5)$$

Let  $f_\varepsilon(t, X_\varepsilon; U, W), \forall t \in [0, T]$  be the state function, the equation (5) can be replaced by:

$$\frac{dX_\varepsilon(t)}{dt} = f_\varepsilon(t, X_\varepsilon; U, W), \forall t \in [0, T] \quad (6)$$

Since the  $\varepsilon(t) \in [-M_\varepsilon, M_\varepsilon]$ , the value of  $X_\varepsilon(t)$  can be:

$$X_\varepsilon(t) \in [-|T-t|M_\varepsilon, |T-t|M_\varepsilon], \forall t \in [0, T] \quad (7)$$

or  $X_\varepsilon(t) \in \Re$ .

### 3.1 Establishment of objective function

Suppose  $J(t, \varepsilon; U, W)$  is the cost function for the system [5], given that we start it at time  $t$ . That is

$$J(t, \varepsilon; U, W) = \Pi_\varepsilon(t) + \Pi_{DEG}(t) \quad (8)$$

In which, the first part  $\Pi_\varepsilon(t)$  is defined

$$\Pi_\varepsilon(t) = \int_t^T \frac{1}{T} c_\varepsilon \left[ \int_{\tau-s}^\tau (\varepsilon_p(\tau) + \varepsilon_n(\tau)) d\tau \right] ds \quad (9)$$

where,

$\varepsilon_n(t) = |\varepsilon(t)|$ , if  $\varepsilon(t) < 0$ ;  $\varepsilon_p(t) = \varepsilon(t)$ , if  $\varepsilon(t) > 0$ ,  $c_\varepsilon$  is punishment cost of  $\varepsilon(t)$  [\$/kWh]. The second part  $\Pi_{DEG}(t)$  is defined:

$$\int_t^T c_{DEG} U(s) ds \quad (10)$$

where,  $c_{DEG}$  is operating cost of diesel engine generator [\$/kWh]. Our main objective is to obtain *admissible control*  $\mathcal{A}_U(t)$  and  $\mathcal{A}_W(t)$  that optimizes the cost function (11) as follows:

$$\min_{\substack{U(t) \in \mathcal{A}_U(t) \\ W(t) \in \mathcal{A}_W(t)}} J(t, \varepsilon; U, W), \quad t \in [0, T] \quad (11)$$

### 3.2 Constraints

#### (1) Power balance

$$P_{DEG}(t) + P_{WTG}(t) + P_{PV}(t) + P_{ESS}(t) = \widehat{P}_L(t) \quad (12)$$

#### (2) Energy balance

$$E_{DEG} + E_{WTG} + E_{PV} = E_L, \quad \forall t \in [0, T] \quad (13)$$

where,  $E_L$  is the cumulative load demand,  $E_{WTG}$  and  $E_{PV}$  are the cumulative wind power and PV power, and  $E_{DEG}$  is defined:

$$E_{DEG} = \int_0^T P_{DEG}(s) ds \quad (14)$$

#### (3) Outputs constraint conditions for $\forall t \in [0, T]$

$$\sum_{k=1}^{N_w} P_{WTG,k}^{\min} \leq P_{WTG}(t) \leq \sum_{k=1}^{N_w} P_{WTG,k}^{\max} \quad (15)$$

$$P_{PV}^{\min} \leq P_{PV}(t) \leq P_{PV}^{\max} \quad (16)$$

$$\sum_{k=1}^{N_D} P_{DEG,k}^{\min}(t) \leq P_{DEG}(t) \leq \sum_{k=1}^{N_D} P_{DEG,k}^{\max}(t) \quad (17)$$

$$-P_{inv} \leq P_{ESS}(t) \leq P_{inv} \quad (18)$$

where,  $N_D$  is the number of DEG,  $N_w$  is the number of WTG.

#### (4) Capacity of energy storage system

$$E_0 - \int_0^T \left[ \frac{P_{ESS}^{discharge}(t)}{\eta_{inv}} + \eta_{inv} P_{ESS}^{charge}(t) \right] dt \leq E_{ESS} \quad (19)$$

where,  $\eta$  is the efficiency of inverter for charging/discharging process,  $E_0$  is initial state of ESS,  $E_{ESS}$  is the maximum state of charge (SOC).

#### (5) Operating reverse constraint

$$\sum_i P_{DEG,i}^{\max} u_{DEG,i}(t) - P_L(t) \geq R \quad (20)$$

where,  $u_{DEG,i}$  is the binary variable indicating the state of DEG unit,  $R$  is the required operating reserve.

#### (6) Minimum required number of diesel generator online (MRN)

$$\sum_i u_{DEG,i}(t) \geq MRN \quad (21)$$

### 3.3 Bellman Optimality Principle

From the equation (8), clearly we can rewrite:

$$\begin{aligned} J(t, X_\varepsilon) &= \int_t^T \frac{1}{T} c_\varepsilon (X_\varepsilon^p(s) + X_\varepsilon^n(s)) ds + \int_t^T c_{DEG} U(s) ds \\ &= \int_t^T G(s, X_\varepsilon; U, W) ds \end{aligned} \quad (22)$$

where:

$$X_\varepsilon^p(s) = \int_0^s \varepsilon_p(\tau) d\tau; X_\varepsilon^n(s) = \int_0^s \varepsilon_n(\tau) d\tau$$

$$G(t, X_\varepsilon; U, W) = \frac{1}{T} c_\varepsilon (X_\varepsilon^p(t) + X_\varepsilon^n(t)) + c_{DEG} U(t)$$

Now, let's define the value function whose value is minimum value of the objective function  $J(\cdot)$  of the system,  $v(t, X_\varepsilon): \Re^+ \times \Re^+ \rightarrow \Re^+$ . Given that we start it at  $t$

$$v(t, X_\varepsilon) = \min_{\substack{U(t) \in \mathcal{A}_U(t) \\ W(t) \in \mathcal{A}_W(t)}} J(t, \varepsilon; U, W), \quad t \in [0, T] \quad (23)$$

Taking  $t < t + \Delta t < T$ , we have that

$$\begin{aligned} v(t, X_\varepsilon) &= \min_{\substack{U(t) \in \mathcal{A}_U(t) \\ W(t) \in \mathcal{A}_W(t)}} \left\{ \int_t^{t+\Delta t} G(s, \cdot) ds + \int_{t+\Delta t}^T G(s, \cdot) ds \right\} \\ &= \min_{\substack{U(t) \in \mathcal{A}_U(t) \\ W(t) \in \mathcal{A}_W(t)}} \left\{ \int_t^{t+\Delta t} G(\cdot) ds \right\} + v(t + \Delta t, X_\varepsilon(t + \Delta t)) \end{aligned} \quad (24)$$

The Bellman principle of optimality [30], (see also [31] and [32] for more details), states that if  $U^*(t, \cdot)$  and  $W^*(t, \cdot)$  are optimal control policies exercised over the time interval  $[t, T]$  for a given start state at time  $t$ , then if we operated this policy up to time  $t + \Delta t$  then the remaining component of the policy will be optimal for the control problem over  $[t + \Delta t, T]$  with start state being the current state at time  $t + \Delta t$ . If we assume the existence of such

an optimal policy, then, as  $\Delta t \rightarrow 0^+$ , we are lead to the partial differential equation (Hamilton-Jacobi-Bellman equation) for  $v(t, X)$ .

$$0 = \min_{\substack{U(t) \in \mathcal{A}_U(t) \\ W(t) \in \mathcal{A}_W(t)}} \left\{ G(t, X_\varepsilon, U, W) + v_t(t, X_\varepsilon) + v_{X_\varepsilon}(t, X_\varepsilon) f_\varepsilon(t, X_\varepsilon; U, W) \right\} \quad (25)$$

where the terms  $v_t(t, X_\varepsilon)$ , and  $v_{X_\varepsilon}(t, X_\varepsilon)$  denote the partial derivatives of  $v(t, X_\varepsilon)$  of value function respect to time  $t$  and state variable  $X$ , respectively.

In principle, once a minimizing solution  $U^*(t, X_\varepsilon)$  and  $W^*(t, X_\varepsilon)$  are known, it may be used as closed loop feedback: that is, the control policies are taken as these functions of the current state parameters. The optimal policies are therefore given by

$$U^*(t) + W^*(t) \in \arg \min_{\substack{U(t) \in \mathcal{A}_U(t) \\ W(t) \in \mathcal{A}_W(t)}} \left\{ G(t, \cdot) + v_{X_\varepsilon}(t, \cdot) f_\varepsilon(t, \cdot) \right\} \quad (26)$$

### 3.4 Optimal trajectory

The production target is specified for the difference of power balance  $\varepsilon(t)$  having to make by time  $T$ , the

production period. The cumulative electricity production is the total energy produced by time  $t$ . The cumulative electricity production must equal to total demand at time  $T$ ; that is one of the objectives is to ensure that

$$\int_{t_0}^t [U(s) + W(s) + P_{WTG}(s) + P_{PV}(s)] ds = \int_{t_0}^t [P_L(s) + \Delta P_L(s)] ds \quad (27)$$

By definition, the difference of energy compared to energy balance in equation (12) whose value is described as follows:

$$X_\varepsilon(t) = X_{\varepsilon,0} + \int_{t_0}^t [U(\cdot) + W(\cdot) - (\hat{P}_L(\cdot) - (P_{WTG}(\cdot) + P_{PV}(\cdot)))] ds \quad (28)$$

where  $X_{\varepsilon,0} = X_\varepsilon(t_0)$ . Our objective is to meet electricity production as closely as possible at every time  $t$  for period  $T$ , i.e., to keep  $X_\varepsilon(t)$  close to zero.

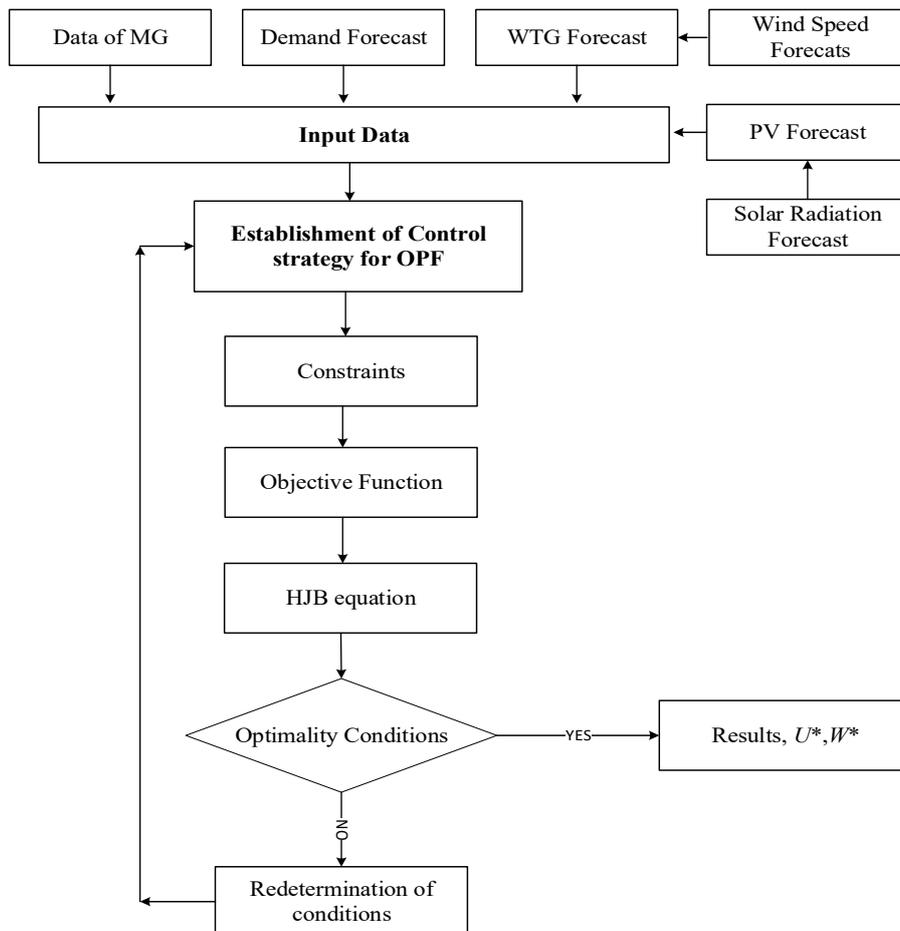


Fig. 2 Flowchart for control strategy of OPF

### 3.5 Strategy of Power management

The purpose of the mathematical model is the supply the power to the MG with the load demand, WTG power and PV power forecasts. So the flowchart for the considered optimal control problem presents in the Fig 2 in which there is two step as follows:

- *Step 1: Determination of input data*  
That consists of the demand forecast, WTG power forecast and PV power forecast.
- *Step 2: Establishment of control strategy for OPF*

We must describe the established energy constraints, determine the objective function, and establish the HJB equation with the optimality conditions in order to test. During testing, if the optimality conditions are corrected then stop the program, if not return the establishment of control strategy for OPF.

### 4. Numerical approach

Facing the difficulty to solve the HJB equation by analyzing, in this section, we develop the numerical method for solving the optimality conditions represented in equation (25). This method is based on Kushner's approach [33].

By adopting the algorithm in [34], the HJB equation (25) which includes the derivative of value function of  $v(t, X_\epsilon)$  can be solved. Let  $\Delta X_\epsilon > 0$  and  $\Delta t > 0$  denote the length of the finite difference interval of the variable  $X_\epsilon$  and  $t$  respectively. The first-orders partial derivative of the value functions  $v_t(\cdot)$  and  $v_{X_\epsilon}(\cdot)$  in equation (25) are calculated by the following expressions:

$$v_t(t, X_\epsilon) = \frac{v(t + \Delta t, X_\epsilon) - v(t, X_\epsilon)}{\Delta t} \quad (29)$$

$$v_{X_\epsilon}(t, \cdot) = \begin{cases} \frac{v(t, X_\epsilon + \Delta X_\epsilon) - v(t, \cdot)}{\Delta X_\epsilon} & \text{if } f(\cdot) \geq 0 \\ \frac{v(t, \cdot) - v(t, X_\epsilon - \Delta X_\epsilon)}{\Delta X} & \text{otherwise} \end{cases} \quad (30)$$

Using  $\Delta X_\epsilon$  and  $\Delta t$ , and after manipulations, the HJB equations can be rewritten as follows:

$$0 = \min_{\substack{U^\Delta(\cdot) \\ W^\Delta(\cdot)}} \left\{ G(\cdot) + \frac{v_\Delta(t + \Delta t, \cdot) - v_\Delta(\cdot)}{\Delta t} + f^i(t, \cdot) \frac{(\mp) v_\Delta(\cdot)(\cdot, X_\epsilon \pm \Delta X_\epsilon) \mp v_\Delta(\cdot)}{\Delta X_\epsilon} \right\} \quad (31)$$

The next theorem shows that  $v_\Delta(t, X_\epsilon)$  is an approximation to  $v(t, X_\epsilon)$  for small step size  $\Delta X_\epsilon$ .

### Theorem 3.1

Let  $v_\Delta(t, X_\epsilon)$  denote a solution to HJB equation (25). Assume that there are constants  $C_g$  and  $K_g$  such that

$$0 \leq v_\Delta(t, X_\epsilon) \leq C_g (1 + |X_\epsilon|^{K_g}) \quad (32)$$

then

$$\lim_{\Delta \rightarrow 0} v_\Delta(t, X_\epsilon) = v(t, X_\epsilon) \quad (33)$$

### Proof

The proof of this theorem is adopted from the one in [35] for the case of deterministic control problem.

## 5. Numerical example

In this example, we make use of the forecast of load demand, total of PV power, and total of wind power generations (see more in [36]–[39], [40] for the forecast method).

### 5.1. Input data

The load demand, WTG power and PV power are presented in Figures 3-5. The constraints of system parameters in equations (12-21) for MG presents in the Table 1.

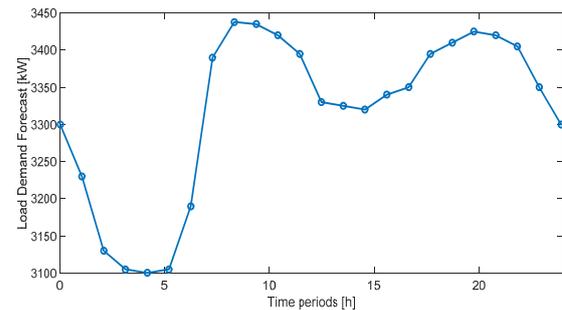


Fig. 3 Demand forecast [kW]

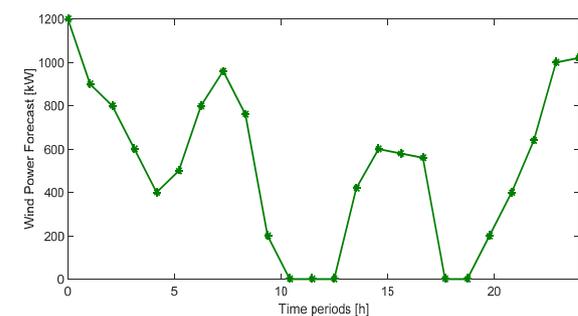


Fig. 4 Wind power forecast [kW]

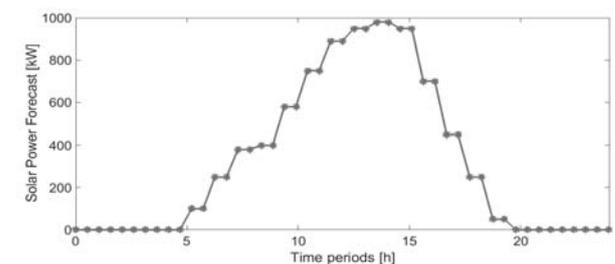


Fig. 5 PV power forecast [kW]

**Tab. 1** Parameters of optimal power flow model

No	Description	Unit	Lower	Upper
1	Time interval	h	0	24
2	Total of WTG power	kW	0	2000
3	Total of PV power	kW	0	3000
4	Load demand and loss power	kW	500	4000
5	DEG1 power	kW	600	5000
6	DEG2 power	kW	0	7560
7	Power of inverter for ESS	kW	0	400
8	Operating cost of DEG, $c_{DEG}$	\$	0,15	-
9	Penalty cost, $c_\varepsilon$	\$	0,20	-
10	Initial state of ESS, $E_0$	kWh	1000	-
11	Capacity of ESS	kWh	0	5000

**5.2. Optimal control problem**

The study is to aim the optimization of objective function presented in equation (23). As the results, the optimality conditions satisfy the HBJ equation (25) where:

$$f_\varepsilon(t, X_\varepsilon, U, W) = U_{DEG}(t) + W(t) - (P_L(t) + \Delta P_L(t) - (P_{WTG}(t) + P_{PV}(t))) \quad (34)$$

The purpose is to find the output power of DEG as well as the ESS state of charge by using the Kushner approach in above.

**5.3. Result and Discussion**

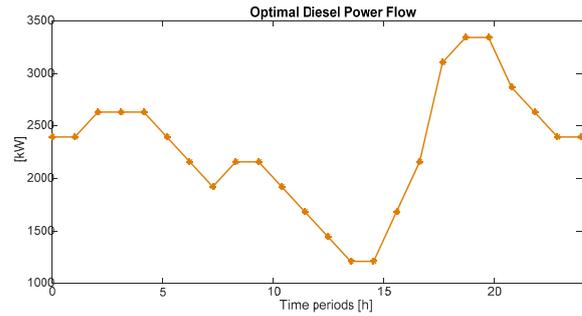
The optimal control problem is solved by coding in the Matlab the HJB equation (31). It takes about 20 minutes to perform this program on Intel® 2.70 Core™ i7-7500 (8GB RAM). The evaluation of error between the cumulative load demand and the generation is described :

$$Err(t) = \frac{|E_{gen}(t) - E_{load}(t)|}{E_{load}(t)} 100\% \quad (35)$$

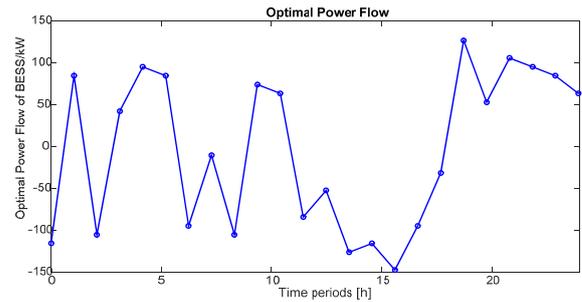
where:  $E_{load}(t)$  is the cumulative load demand,  $E_{gen}(t)$  is the cumulative generation.

The results are illustrated in figures 6-10. Figure 6 represents the optimal production of DEG output power versus time  $t$  in the interval  $[0,24h]$ . Meanwhile the figure 7 represents the power flow of ESS (charging and discharging). For the chosen parameters, the DEG follows demand profile (demand forecast). At the peak hour, DEG generates at their nominal power according to the demand. Moreover, the demand is adequately covered (i.e.,

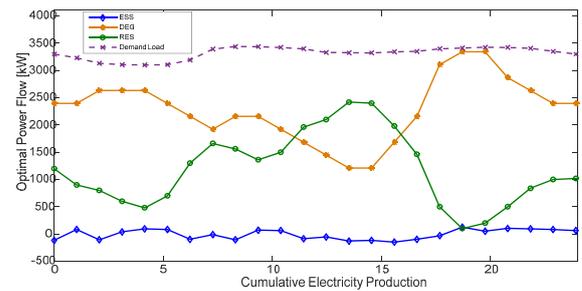
$$P_L(t) + \Delta P_L = P_{DEG}(t) + P_{WTG}(t) + P_{PV}(t) + P_{ESS}(t) \text{ for all } t).$$



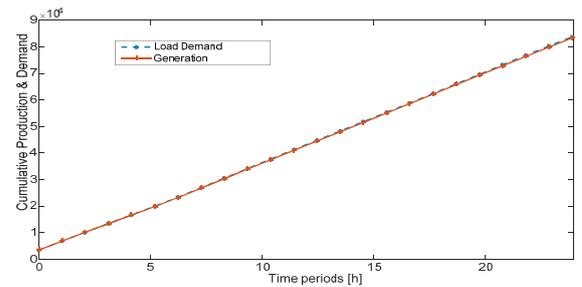
**Fig. 6** Optimal output power of DEG [kW]



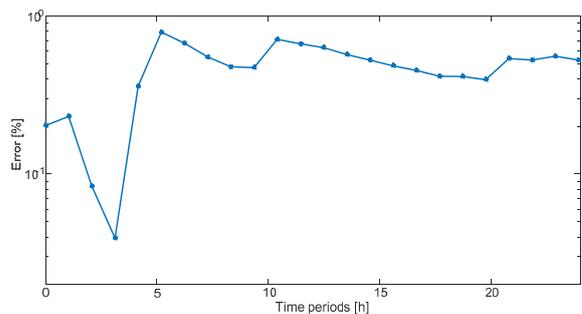
**Fig. 7** Optimal power flow of ESS [kW]



**Fig. 8** Optimal output powers [kW]



**Fig. 9** Cumulative production vs load demand [kWh]



**Fig. 10** Error  $E_{gen}$  and  $E_{load}$  demand [%]

Figure 9 represents the cumulative electricity production of MG and the load demand versus time  $t$ . This figure shows that, the effectiveness of optimal control gives the birth to optimal power flow of MG, thus the characteristic is linear instead, showing that the optimal control approach can be implemented with appropriate results. Figure 10 is illustrated the Error between cumulative generation and load demand.

## 6. Conclusions

This paper has studied the problem of optimal output power flow of MG. We proposed an approach based on dynamic programming (Bellman's principle) applied to the dispatch of DGs in a microgrid have been formulated instead of using the Mixed Integer Linear Programming so that leads to optimal control problem. The main objective of the study is to find optimal output power of DEG and ESS with uncertain power injection to implement a distributed tertiary control level in hierarchical control strategy of the microgrid, which allows the DEG and ESS to operate at an economic optimum. As a result, the new model is as a control problem whose optimality conditions have been established as Hamilton-Jacobi-Bellman equation. Moreover, the proposed model makes considered system joint between the optimal power flow and optimal control problem.

We also applied our proposed model to a real life system of an islanded MG at small-scale with demand, PV power, and WTG power forecast, and the DEG output power is control variable. The results of test system have demonstrated the effectiveness of the proposed method.

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