# An Application of Active Disturbance Rejection Control for a 1 DOF-Flexible Link Manipulator

# Ứng dụng điều khiển loại bỏ nhiễu chủ động cho tay máy robot linh hoạt một bậc tự do

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## Tóm tắt
Tay máy linh hoạt (FLM) hiện nay được sử dụng trong nhiều lĩnh vực. Tuy nhiên, việc điều khiển tay máy linh hoạt khá phức tạp do tính chất linh hoạt của hệ thống và thiếu thông tin về độ lệch của tay máy. Bộ điều khiển loại bỏ nhiễu chủ động (ADRC) đang được quan tâm nghiên cứu để thay thế cho bộ điều khiển PID truyền thống. Nhờ các ưu điểm như dễ chỉnh định, cho đáp ứng nhanh và tính bền vững khi tham số quá trình thay đổi, ADRC có tiềm năng trong ứng dụng điều khiển tay máy linh hoạt. Bài báo này đề cập việc thiết kế bộ điều khiển ADRC cho bài toán điều khiển vị trí cho tay máy linh hoạt một bậc tự do. Mô hình tay máy một bậc tự do được xây dựng thông qua phương trình Euler-Lagrange, tạo tiền đề cho việc tính toán các tham số của bộ điều khiển ADRC. Các kết quả mô phỏng cho thấy đáp ứng tốt của hệ kín.

## Keywords
Flexible link manipulator, active disturbance rejection, extended state observer, Euler-Lagrange

## Abstract [1]
Flexible Link Manipulators (FLM) have been used in many fields. However, the control of FLM is complicated due to the nature of flexibility in the system and unavailable information of link deflections. Active Disturbance Rejection Control (ADRC) has recently been interested in as an alternative to traditional PID controller. Due to its simple tuning, fast response, and robustness against process parameter variations, ADRC has a great potential to tackle FLM control problem. In the paper, an ADRC approach is applied to position control problem of 1DOF-FLM. The system model is derived based on Euler-Lagrange equations formulating a foundation for calculating ADRC parameters. Simulations results show good performance of the closed-loop system.

## Abbreviation

| | |
|---|---|
| FLM | Flexible Link Manipulator |
| PID | Proportional Integral Derivative |
| ADRC | Active Disturbance Rejection Control |
| ESO | Extended State Observer |

## 1. Introduction
Currently the demand for industrial robots is increasing in complex production processes with purpose of improving the productivity, the working conditions, the quality and the competitiveness of the products. The modern industrial robots also have strong adaptability, intelligence with simple and flexible control structures. Therefore, robots are applied in many other fields such as aerospace, construction, medical, etc. Due to multidisciplinary demand, the robot must be flexible, operate with high precision and consume less power. In order to build power and high efficient robot manipulators and to increase the operation speed, the focus is switched towards development of light weight manipulators. The Flexible Link Manipulator (FLM) is one of a number of types of manipulators that meet the design trend. Flexible link manipulators are assumed to have rigid joints and flexible links. The links are considered to have low stiffness because of lightweight materials.

The control of FLM is complicated due to the nature of flexibility in the system, depends on the dynamics and inertia of the object. The requirement for the controller is to make the manipulator to track the desired position in expected time with minimum vibration of the flexible structure. The oscillations of link only decrease when model has high stiffness, or the movement of the system is very slow. Therefore, it is necessary to use a suitable control method for controlling the system. Recently, there are several control methods for FLM system such as PID control [1], feedback linearization approach [2], sliding mode control [3], and adaptive control [4]. Most of the approaches in the literature require measurements of all state variables and the robustness is often highly model dependent.

In recent years, Active Disturbance Rejection Control (ADRC) is interested in to replace the traditional PID controller. This concept was originally proposed by J. Han [5, 6] but only become transparent

[1] *Ngày nhận bài*: 09/12/2017; *Ngày nhận bản sửa lần 1*: 23/01/2018; *Ngày nhận bản sửa lần 2*: 03/04/2018; *Ngày chấp nhận*: 27/04/2018; *Phản biện*: Lê Tiến Dũng, Dương Hoài Nghĩa, Nguyễn Phùng Quang

to application engineers since a new parameter tuning method is proposed in [7]. This control method shows several advantages for disturbance rejection and for process with inaccurate parameters. ADRC is a powerful control method where system models are expanded with a new state variable, including all unknown kinetic and disturbance, that commonly happens in system formulation. The new state is estimated by using the Extended State Observer (ESO). An application of ADRC for rigid coupling motion control system can be found in [8]. In addition, [9] and [10] deal with flexible system control problem with the assistance of ESO to provide estimation of unmeasured state. However, the control design process is complicated for engineering applications since the requirement of many differential actions. In [11, 12 and 13], the authors referred to decoupling control for multivariable system using ADRC. The ADRC tracking performances is also studied on three-axis didactic radar antenna control system through the frequency domain analysis [14]. These studies show the advantages and potential of ADRC approach in system control.

Thanks to simple control structure and good performance, an application of ADRC for 1DOF-FLM position control is considered in the paper. This paper is structured as follow. The 1DOF-FLM is presented in section 2. In section 3, we present the general idea of second-order ADRC as well as the parameters tuning procedure of the ADRC. Subsequently, the application of ADRC for 1DOF-FLM and some simulation results are given in section 4, followed by several concluding remarks.

## 2. System description

The system under consideration is a homogeneous material flexible link, mass evenly distributed, coupled to a motor shaft as shown in Fig. 1. The link is driven by a motor shaft to reach the desired position. Due to flexible nature, the flexible link at a given deflection results in an end-point displacement D as shown in Fig. 2 [3].

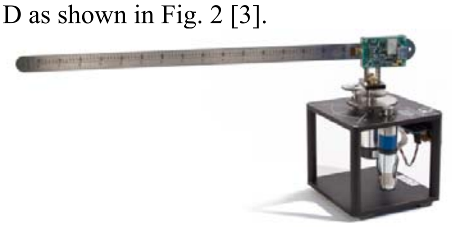

**Fig. 1** *An experimental set up of flexible link*

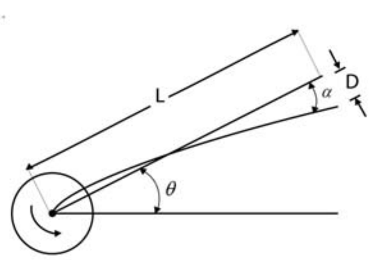


**Fig. 2** *Schematic diagram of flexible link*

Tip deflection angle $\alpha$ is assumed to be small. Hence, the following hold:

$$\alpha = \frac{D}{L} \tag{1}$$

Link stiffness $K_{stiff}$ provides restoring force. In controlling the tip of the link, it is sufficient to use a simplified model that will adequately describe the motion of the end-point. The equation of rotatory spring is:

$$J_{link}\ddot{\alpha} = -K_{stiff}\alpha \tag{2}$$

Otherwise:

$$\ddot{\alpha} = -\omega_c^2\alpha \tag{3}$$

where $\omega_c$ is damped natural frequency. Combining equations (2) and (3) results in:

$$K_{stiff} = \omega_c^2 J_{link} \tag{4}$$

The link is modeled as a link having stiffness $K_{stiff}$ length of the link $L$, mass of the link $M$ and the rotating about its endpoint with moment of inertia given as:

$$J_{link} = \frac{ML^2}{3} \tag{5}$$

The system dynamics is obtained using the Euler - Lagrange formulation. The potential energy in the system is:

$$V = P.E_{spring} = \frac{1}{2}K_{stiff}\alpha^2 \tag{6}$$

The kinetic energies in the system arise from the moving hub and flexible link and is calculated as:

$$\begin{aligned} T &= T_1 + T_2 = K.E_{Hub} + K.E_{Link} \\ &= \frac{1}{2}J_{eq}\dot{\theta}^2 + \frac{1}{2}J_{link}(\dot{\theta}+\dot{\alpha})^2 \end{aligned} \tag{7}$$

Forming the Lagrangian as bellow:

$$\begin{aligned} L &= T - V \\ &= \frac{1}{2}J_{eq}\dot{\theta}^2 + \frac{1}{2}J_{link}(\dot{\theta}+\dot{\alpha})^2 - \frac{1}{2}K_{stiff}\alpha^2 \end{aligned} \tag{8}$$

The two generalized co-ordinates are $\alpha$ and $\theta$. The Euler-Lagrange equations are:

$$\frac{\delta}{\delta t}\left(\frac{\delta L}{\delta\dot{\theta}}\right) - \frac{\delta L}{\delta\theta} = T_l - B_{eq}\dot{\theta} \tag{9}$$

$$\frac{\delta}{\delta t}\left(\frac{\delta L}{\delta\dot{\alpha}}\right) - \frac{\delta L}{\delta\alpha} = 0 \tag{10}$$

Solving equations (9) and (10) gives:

$$J_{eq}\ddot{\theta} + J_{link}(\dot{\theta}+\dot{\alpha}) = T_l - B_{eq}\dot{\theta} \tag{11}$$

$$K_{stiff}\alpha + J_{link}(\ddot{\theta}+\ddot{\alpha}) = 0 \tag{12}$$

Rearranging (11) and (12) yields:

$$\ddot{\theta} = \frac{T_l}{J_{eq}} - \frac{B_{eq}}{J_{eq}}\dot{\theta} + \frac{K_{stiff}\alpha}{J_{eq}} \tag{13}$$

$$\ddot{\alpha} = -K_{stiff}\left(\frac{1}{J_{link}} + \frac{1}{J_{eq}}\right)\alpha + \frac{B_{eq}}{J_{eq}}\dot{\theta} - \frac{T_l}{J_{eq}} \tag{14}$$

With the actuator model as in [3], we arrive at the following equations:

$$\ddot{\theta} = -p_1\dot{\theta} + p_2\alpha + p_3V_m \quad (15)$$

$$\ddot{\alpha} = p_1\dot{\theta} - p_4\alpha - p_3V_m \quad (16)$$

where:

$$p_1 = \frac{\eta_m\eta_g K_g^2 K_m K_t + B_{eq}R_m}{J_{eq}R_m},\ p_2 = \frac{K_{stiff}}{J_{eq}}$$

$$p_3 = \frac{\eta_m\eta_g K_g K_t}{R_m J_{eq}},\ p_4 = K_{stiff}\left(\frac{1}{J_{eq}} + \frac{1}{J_{link}}\right)$$

The parameters of the FLM are given as [15] and [16]:

| Symbol | Meaning | Value (Unit) |
|---|---|---|
| L | Length of link | 0.381 (m) |
| M | Mass of link | 0.065 (kg) |
| $\omega_c$ | Damped natural frequency | 6π (Hz) |
| $J_{link}$ | Moment of inertia of link | 0.0031452(kg.m$^2$) |
| $K_{stiff}$ | Stiffness of link | 1.1175 (N.m/rad) |
| $\alpha$ | Tip deflection | (rad) |
| $\theta$ | Position of link angle | (rad) |
| $V_m$ | DC input voltage | (V) |
| $I_m$ | DC input current | (A) |
| $L_m$ | Motor armature inductance | (H) |
| $R_m$ | Motor armature resistance | 2.6 (Ω) |
| $E_m$ | Equivalent back emf | (V) |
| $\theta_m$ | Motor shaft position | (rad) |
| $K_m$ | Motor back emf constant | 0.00767 (V.s/rad) |
| $K_g$ | Gear ratio | 70 |
| $K_t$ | Motor torque constant | 0.00767 (N.m/A) |
| $J_m$ | Motor inertia | 3.87 ×10$^{-7}$ (kg.m$^2$) |
| $J_{eq}$ | Total system inertia | 0.0023 (kg.m$^2$) |
| $T_m$ | Motor torque | (N.m) |
| $T_l$ | Load torque | (N.m) |
| $\eta_g$ | Gearbox efficiency | 0.85 |
| $\eta_m$ | Motor efficiency | 0.87 |
| $B_{eq}$ | Viscous damping coefficient | 4 ×10$^{-3}$ |

With system's parameter values presented above, one gets $p_1$ = 37.5147, $p_2$ = 487.5357, $p_3$ = 66.6224, $p_4$ = 842.8414.

# 3. Active Disturbance Rejection Control

### 3.1 General ideas

The concept of ADRC was pioneered by J. Han [5]. A second order plant is considered here to establish the concept of ADRC:

$$\ddot{y}(t) = f\left(t, \dot{y}, y, \omega\right) + b_0.u(t) \quad (17)$$

where *u* is the control input, *y* is the output and *ω* is the disturbance. According to Han, the generalized term $f\left(t, \dot{y}, y, \omega\right)$ is insignificant while only its real-time estimate $\hat{f}$ is important. Therefore, an extended state observer (ESO) is constructed to provide $\hat{f}$ such that we can compensate the impact of $f\left(t, \dot{y}, y, \omega\right)$ on the model by means of disturbance rejection. This allows the control law to be constructed as:

$$u = \frac{u_0 - \hat{f}}{b_0} \quad (18)$$

to reduces the plant in (17) to a form of:

$$\ddot{y}(t) \simeq u_0 \quad (19)$$

which can be easily controlled. In general, this concept is applicable to higher order systems. It requires little knowledge of the plant, the only thing required is the knowledge of the order of the plant and the approximate value of parameter $b_0$. The convergence of ESO is extensively discussed in [17].

### 3.2 The Extended State Observer and the Control Law

The ESO was originally proposed by J. Han [5] and made practical by the tuning method proposed by Gao [7], which simplified its implementation and made the design transparent to engineers. The main idea is to use an augmented state space model of equation (17) that includes $f\left(t, \dot{y}, y, \omega\right)$ (from now on *f* is used to denote $f\left(t, \dot{y}, y, \omega\right)$ where applicable) as an additional state. In particular, let $x_1 = y$, $x_2 = \dot{y}$ and $x_3 = f$.

The augmented state space form of equation (17) is

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{pmatrix} = \underbrace{\begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}}_{A}\begin{pmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{pmatrix} + \underbrace{\begin{pmatrix}0\\ b_0\\ 0\end{pmatrix}}_{B}.u(t) + \begin{pmatrix}0\\ 0\\ 1\end{pmatrix}\dot{f}(t)$$

$$y(t) = \underbrace{\begin{pmatrix}1 & 0 & 0\end{pmatrix}}_{C}\begin{pmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{pmatrix} \quad (20)$$

The state observer can be formulated as:

$$\begin{pmatrix}\dot{\hat{x}}_1(t)\\ \dot{\hat{x}}_2(t)\\ \dot{\hat{x}}_3(t)\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\hat{x}_1(t)\\ \hat{x}_2(t)\\ \hat{x}_3(t)\end{pmatrix} + \begin{pmatrix}0\\ b_0\\ 0\end{pmatrix}.u(t) + \begin{pmatrix}l_1\\ l_2\\ l_3\end{pmatrix}\left(y(t) - \hat{x}_1(t)\right)$$

$$= \underbrace{\begin{pmatrix}-l_1 & 1 & 0\\ -l_2 & 0 & 1\\ -l_3 & 0 & 0\end{pmatrix}}_{A-LC}\begin{pmatrix}\hat{x}_1(t)\\ \hat{x}_2(t)\\ \hat{x}_3(t)\end{pmatrix} + \underbrace{\begin{pmatrix}0\\ b_0\\ 0\end{pmatrix}}_{B}.u(t) + \begin{pmatrix}l_1\\ l_2\\ l_3\end{pmatrix}.y(t) \quad (21)$$

where $l_1$, $l_2$ and $l_3$ are observer parameters to be determined, provides an estimate of the state of equation (20). $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$ will track *y*, $\dot{y}$ and *f* respectively.

Then the control law

$$u = \frac{u_0 - \hat{x}_3}{b} \text{ with } u_0 = K_P.(r - \hat{x}_1) - K_D.\hat{x}_2 \quad (22)$$

reduces equation (17) to equation (23), that is:

$$\ddot{y}(t) \simeq u_0 = K_P(r(t) - y(t)) - K_D.\dot{y}(t) \quad (23)$$

where $r$ is the set point.

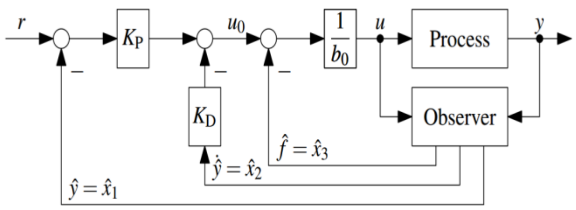


**Fig. 3** *ADRC for a second order plant*

Taking the Laplace Transform of (23), one has the close-loop transfer function as follows:

$$G_{cl}(s) = \frac{Y(s)}{R(s)} \simeq \frac{K_P}{s^2 + K_D s + K_P} \quad (24)$$

### 3.3 Tuning procedure

According to [18], the ADRC's parameters can be chosen to tune the closed loop to a critically damped behavior and a desired 2% settling time $T_{settle}$. The tuning procedure is summarized as follows:

- Get the desired settling time $T_{settle}$.
- Choose $K_P$ and $K_D$ to get a negative-real double pole, $s_{1/2}^{CL} = s^{CL}$ :
- $K_p = \left(s^{CL}\right)^2$, $K_D = -2.s^{CL}$ with $s^{CL} = -\frac{6}{T_{settle}}$

  (25)
- Since the observer dynamics must be fast enough, the observer poles $s_{1/2}^{ESO}$ must be placed left of the close-loop pole $s^{CL}$, for suggestion:
- $s_{1/2}^{ESO} = s^{ESO} \approx (3...10).s^{CL}$ (26)
- The observer parameters can be computed from its characteristic polynomial:

$$\det\left(sI - (A - LC)\right) = s^3 + l_1 s^2 + l_2 s + l_3 \quad (27)$$

$$\overset{!}{=} \left(s - s^{ESO}\right)^3$$

Then

$$l_1 = -3.s^{ESO}, l_2 = 3.\left(s^{ESO}\right)^2, l_3 = \left(s^{ESO}\right)^3 \quad (28)$$

## 4. Application of ADRC for FLM

### 4.1 Controller design

Applying the ADRC theory presented in previous section, we rewrite Equation (15) as follows:

$$\ddot{\theta}(t) = f(t) + b_0.u(t)$$

Where $f(t) = -p_1\dot{\theta}(t) + p_2\alpha(t)$ known as disturbance and $u(t)$ is input control as $V_m$. We get:

$$b_0 = p_3 = 66.6224$$

Choose: $T_{Settle} = 1\ (s)$. Hence, $s^{CL} \approx \frac{-6}{T_{Settle}} = -6$

$$K_P = (s^{CL})^2 = 36,\ K_D = -2s^{CL} = 12$$

$$s^{ESO} = 10s^{CL} = -60$$

$$l_1 = -3s^{ESO} = 180$$

$$l_2 = 3(s^{ESO})^2 = 10800$$

$$l_3 = -(s^{ESO})^3 = 216000$$

$$A - LC = \begin{bmatrix} -l_1 & 1 & 0 \\ -l_2 & 0 & 1 \\ -l_3 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -180 & 1 & 0 \\ -10800 & 0 & 1 \\ -216000 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 66.6224 \\ 0 \end{bmatrix};\ L = \begin{bmatrix} 180 \\ 10800 \\ 216000 \end{bmatrix}$$

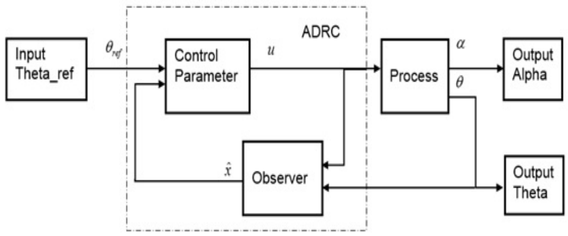


**Fig. 4** *ADRC for FLM*

### 4.2 Simulation

From the calculated data in Section 2 and control parameters of the ADRC, use the Matlab-Simulink software to simulate controller and we achieve the results as shown below.

Fig. 5, Fig.6 and Fig. 7 show the simulation results of the ADRC controller for FLM. The results show that the link position $\theta$ reaches the desired value with settling time of 1s and an overshoot is of 2%. The maximum oscillation amplitude of the link's tip is about 1 degree.

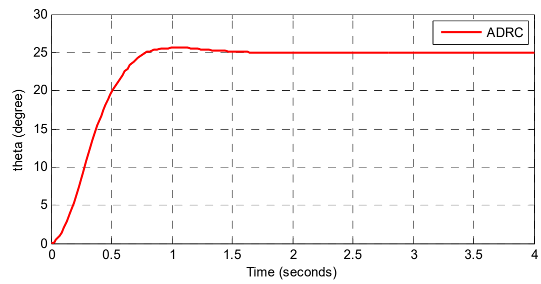


**Fig. 5** *Output angular displacement*

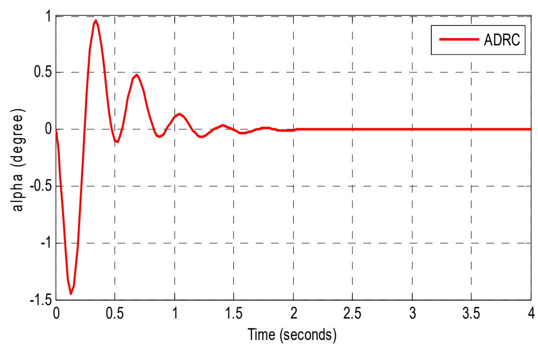


**Fig. 6** *Tip displacement*

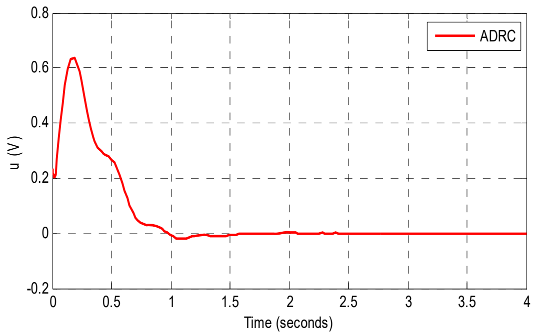


**Fig. 7** *Control input*

In order to test the robustness of the system, Fig. 8 and Fig. 9 show the evolution of $\theta$ and $\alpha$ when we increase link mass M from 0.065kg to 0.13 and 0.195kg and the ADRC parameters as in section 4.1. The link still arrives the defined position with settling time is of 1s. In fact, changing link mass M can be considered as varying stiffness when link geometry is constant. Hence, the oscillation amplitude of the link's tip decreased due to increased stiffness.

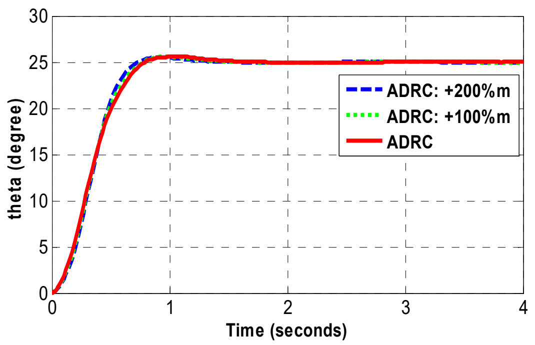


**Fig. 8** *Output angular displacement under variations of the mass of link*

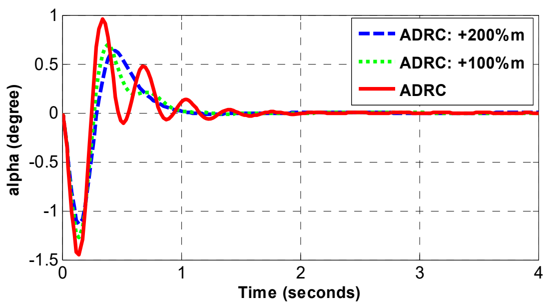


**Fig. 9** *Tip displacement under variations of the mass of link*

## 5. Concluding remarks

This paper has initially approached the position control problem of 1DOF-FLM based on Active Disturbance Rejection Control. From the positive performances in term of reference tracking and vibration reduction of the closed-loop system, one can observe that the use of ADRC method has advantages such as less dependence on the modeling and simple implementation. ADRC can be considered as a promising practical method, not only for robotic engineering, but also for many other systems that share the flexibility nature such as crane systems and liquid transfer process. The future work will dedicate to apply ADRC control for multi-link manipulators.

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