Soft binary piecewise intersection operation

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Abstract:

Soft set theory has emerged as a novel approach to modelling uncertainty, addressing a wide range of theoretical and practical problems. In this study, we define a new soft set operation called "soft binary piecewise intersection operation" and explore its fundamental algebraic properties by comparing them with the properties of the intersection operation in classical set theory. Many remarkable similarities have been observed between the intersection operation and the soft binary piecewise intersection operation. Additionally, we examine the distribution of the soft binary piecewise intersection operation over other types of soft set operations. By analysing the algebraic properties of the operation and its distribution rules, we demonstrate that the collection of soft sets over the universe, as well as the collection of soft sets with a fixed parameter set, alongside the soft binary piecewise intersection operation and other types of soft sets, forms several important algebraic structures such as hemirings, near-semirings, semirings, Boolean rings, Boolean algebras, De Morgan algebras, Kleene algebras, and Stone algebras.

Keywords: **conditional complements, soft binary piecewise intersection operation, soft set operations, soft sets.**

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1. Introduction

Interval mathematics, probability theory, and fuzzy set theory are frameworks used to model uncertainty, but each has its own limitations. D. Molodtsov, et al. (1999) [1] introduced Soft set (𝕊𝕊) theory as a new approach to model uncertainty and apply it to problems involving uncertainty. Since its inception, 𝕊𝕊 has been successfully applied to various areas of mathematics, including Riemann integration, game theory, probability theory, Perron integration process research, and measurement theory. The initial studies on 𝕊𝕊 operations were conducted by P.K. Maji, et al. (2023) [2] D. Pei, et al. (2005) [3], M.I. Ali, et al. (2009) [4] introduced numerous 𝕊𝕊 operations. A. Sezgin, et al. (2011) [5] investigated the fundamental properties of 𝕊𝕊 operations, showing their interrelations and defining the restricted symmetric difference of 𝕊𝕊s. M.I. Ali, et al. (2011) [6] thoroughly examined the algebraic structures of 𝕊𝕊s. Many researchers have since developed an interest in 𝕊𝕊 operations, further studying them in detail [7-18].

In recent years, new types of 𝕊𝕊 operations have been defined and investigated. Ö.F. Eren (2019) [19] defined and explored the properties of the soft binary piecewise difference operation in 𝕊𝕊s. A. Sezgin, et al. (2019) [20] introduced a new operation called the extended difference of 𝕊𝕊s. N.S. Stojanovic (2021) [21] defined and studied the extended symmetric difference of 𝕊𝕊s. N. Çağman (2021) [22] introduced two novel complement operations into the literature. A. Sezgin, et al. (2023) [23] expanded on these new binary set operations, defining several new operations. These operations were extended to 𝕊𝕊s by F.N. Aybek (2024) [24], who defined new restricted and extended 𝕊𝕊 operations. E. Akbulut (2024) [25], A.M. Demirci (2024) [26], and M. Sarıalioğlu (2024) [27] continued working on transforming extended operations in 𝕊𝕊s, focusing on complemented extended 𝕊𝕊 operations. Complementary soft binary piecewise symmetric difference was also explored [28].

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Any operation defined on a set forms an algebraic structure (mathematical system) along with the set. The most well-known algebraic structures in algebra are groups with one operation, rings with two operations, and fields. The aim of algebra is to classify these structures, and one of the key tasks in mathematics is to examine the properties of operations defined on a set. Two primary categories of $$ collections are studied in the context of $$s as algebraic structures: the first involves $$s with a fixed set of parameters, while the second considers $$s with various parameter sets. These collections exhibit different behaviours depending on the procedures applied. $$ operations are crucial in $$ theory, just as classical set operations are fundamental in classical algebra. Examining the algebraic structure of $$s in terms of new operations deepens our understanding of their algebraic framework.

This study aims to contribute to $$ theory by proposing a new $$ operation, the "soft binary piecewise intersection operation," and systematically investigating the algebraic structures associated with this and other $$ operations. The paper is organised as follows: Section 2 recalls the basic definitions relevant to $$s and algebraic structures. Section 3 presents the definition and examples of the new $$ operation, thoroughly examining its algebraic properties by comparing it with the intersection operation in classical set theory. Section 4 explores the distribution of the soft binary piecewise intersection operation over other types of $$ operations, such as restricted $$ operations, extended $$ operations, and soft binary piecewise operations. By considering the algebraic properties of $$ operations and their distribution rules, a detailed analysis is provided on the algebraic structures formed by $$s with these operations. We show that the collection of $$s over the universe and the collection of all $$s with a fixed parameter set, together with the soft binary piecewise intersection operation and other types of $$s, form several important algebraic structures. Section 5 discusses the significance of the findings and potential impacts on the subject. This study is crucial for both $$ theory and classical algebra, as exploring the algebraic structures of $$s in relation to novel $$ operations introduces new examples of algebraic structures and facilitates comprehension of their practical applications.

2. Preliminaries

In this section, some algebraic structures and basic concepts related to $$ theory are compiled and given.

Definition 2.1. Let U be the universal set, $\mathbb{E}$ be the parameter set, P(U) be the power set of U, and let $\mathbb{K} \subseteq \mathbb{E}$. A pair $(\mathbb{L},\mathbb{K})$ is called a $$ on U. Here, $\mathbb{L}: \mathbb{K} \to P(U)$ [1].

The set of all $$s over U is denoted by $S_{\mathbb{E}}(U)$. Let $\mathbb{K}$ be a fixed subset of E, then the set of all $$s over U with $\mathbb{K}$ is denoted by $S_{\mathbb{K}}(U)$.

Definition 2.2. Let $(\mathbb{L},\mathbb{K})$ be a $$ over U. If $\mathbb{L}(x)=\emptyset$ for all $x\in\mathbb{K}$, then $(\mathbb{L},\mathbb{K})$ is called a null $$ with respect to $\mathbb{K}$, denoted by $\emptyset_{\mathbb{K}}$. If $\mathbb{L}(x)=\emptyset$ for all $x\in \mathbb{E}$, then the $$ $(\mathbb{L},\mathbb{E})$ is called a null $$ with respect to $\mathbb{E}$, denoted by $\emptyset_{\mathbb{E}}$ [4].

Since the $$ is also a function, a $$ can be defined as $F: \emptyset \longrightarrow P(U)$. Such a $$ is called a null $$ and is denoted as $\emptyset_{\emptyset}$ [6].

Definition 2.3. Let $(\mathbb{L},\mathbb{K})$ be a $$ over U. If $\mathbb{L}(x)=U$ for all $x\in\mathbb{K}$, then the $$ $(\mathbb{L},\mathbb{K})$ is called an absolute $$ with respect to $\mathbb{K}$, denoted by $U_{\mathbb{K}}$. If $\mathbb{L}(x)=U$ for all $x\in \mathbb{E}$, then the $$ $(\mathbb{L},\mathbb{E})$ is called an absolute $$ with respect to $\mathbb{E}$, denoted by $U_{\mathbb{E}}$ [4].

Definition 2.4. Let $(Ł,K)$ and $(Ḃ,T)$ be SSs over U. If $K \subseteq T$ and $Ł(ɾ) \subseteq Ḃ(ɾ)$ for all $ɾ \in K$, then $(Ł,K)$ is said to be a soft subset of $(Ḃ,T)$, denoted by $(Ł,K) \tilde{\subseteq} (Ḃ,T)$.

If $(Ḃ,T)$ is a soft subset of $(Ł,K)$, then $(Ł,K)$ is said to be a soft superset of $(Ḃ,T)$, denoted by $(Ł,K) \tilde{\supseteq} (Ḃ,T)$. If $(Ł,K) \tilde{\subseteq} (Ḃ,T)$ and $(Ḃ,T) \tilde{\subseteq} (Ł,K)$, then $(Ł,K)$ and $(Ḃ,T)$ are called soft equal sets [3].

Definition 2.5. Let $(Ł,K)$ be a SS over U. The soft complement of $(Ł,K)$, denoted by $(Ł,K)^r = (F^r,K)$, is defined as follows: for all $ɾ \in K$, $F^r(ɾ) = U\text{-}Ł(ɾ)$ [4].

For two sets K and T, $K+Y = K' \cup T$ and $K\theta Y = K' \cap T'$ [22], $K*Y = K' \cup T'$, $K\,\gamma Y = K' \cap T$, $K\boldsymbol{\lambda} Y = K \cup T'$ [23].

Let "$\otimes$" be used to represent the set operations. Then,

Definition 2.6. Let $(Ł,K)$ and $(Ḃ, T)$ be two SSs over U. The restricted $\otimes$ operation of $(Ł,K)$ and $(Ḃ,T)$ is the SS $(Ɱ, Ᵽ)$, denoted by $(Ł,K) \otimes_R (Ḃ,T) = (Ɱ, Ᵽ)$, where $Ᵽ = K \cap T \neq \emptyset$ and for all $ɾ \in Ᵽ$, $Ɱ(ɾ) = Ł(ɾ) \otimes Ḃ(ɾ)$. Here, if $Ᵽ = K \cap T = \emptyset$, then $(Ł,K) \otimes_R (Ḃ,T) = \emptyset_\emptyset$ [4, 5, 24].

Definition 2.7. Let $(Ł,K)$ and $(Ḃ,T)$ be two SSs over U. The extended $\otimes$ operation $(Ł,K)$ and $(Ḃ,T)$ is the SS $(Ɱ, Ᵽ)$, denoted by $(Ł,K) \otimes_\varepsilon (Ḃ,T) = (Ɱ, Ᵽ)$, where $Ᵽ = K \cup T$, and for all $ɾ \in P$ [2, 4, 20, 21, 24].

$$Ɱ(ɾ) = \begin{cases} Ł(ɾ), & ɾ \in K - T \\ Ḃ(ɾ), & ɾ \in T - K \\ Ł(ɾ) \otimes Ḃ(ɾ), & ɾ \in K \cap T \end{cases}$$

Definition 2.8. Let $(Ł,K)$ and $(Ḃ,T)$ be two SSs over U. The complementary extended $\otimes$ operation $(Ł,K)$ and $(Ḃ,T)$ is the SS $(Ɱ, Ᵽ)$, denoted by $(Ł,K) \overset{*}{\otimes}_\varepsilon (Ḃ,T) = (Ɱ, Ᵽ)$, where $Ᵽ = K \cup T$, and for all $ɾ \in Ᵽ$ [25-27].

$$Ɱ(ɾ) = \begin{cases} Ł'(ɾ), & ɾ \in K - T \\ Ḃ'(ɾ), & ɾ \in T - K \\ Ł(ɾ) \otimes Ḃ(ɾ), & ɾ \in K \cap T \end{cases}$$

Definition 2.9. Let $(Ł,K)$ and $(Ḃ,T)$ be two SSs over U. The complementary soft binary piecewise $\otimes$ operation of $(Ł,K)$ and $(Ḃ,T)$ is the SS $(Ɱ,K)$, denoted by $(Ł,K) \overset{*}{\underset{\otimes}{\sim}} (Ḃ,T) = (Ɱ,K)$, where for all $ɾ \in K$ [28].

$$Ɱ(ɾ) = \begin{cases} Ł'(ɾ), & ɾ \in K - T \\ Ł(ɾ) \otimes (ɾ), & ɾ \in K \cap T \end{cases}$$

Definition 2.10. Let $(Ł,K)$ and $(Ḃ,T)$ be two SSs over U. The soft binary piecewise $\otimes$ operation of $(Ł,K)$ and $(Ḃ,T)$ is the SS $(Ɱ,K)$, denoted by $(Ł,K) \underset{\otimes}{\sim} (Ḃ,T) = (Ɱ,K)$, where for all $e \in K$ [19, 29-31].

$$Ɱ(ɾ) = \begin{cases} Ł(ɾ), & ɾ \in K - T \\ Ł(ɾ) \otimes Ḃ(ɾ), & ɾ \in K \cap T \end{cases}$$

For more about SSs, we refer to [32-34].

Definition 2.11. Let $(Ł,Ǩ)$ and $(Ɉ,Ŧ)$ be two $\mathbb{SS}$s on U. The cartesian product of $(Ł,Ǩ)$ and $(Ɉ,Ŧ)$ is the soft $(Ɱ,Ǩ\times Ŧ)$, denoted by $(Ł,Ǩ)\times(Ɉ,Ŧ)=(Ɱ,Ǩ\times Ŧ)$, where for all $(k,t)\in Ǩ\times Ŧ$, $Ɱ(Ǩ,Ŧ)=Ł(k)\times Ɉ(t)$ [35].

We refer to [36] for band, (bounded) semilattice, to [37] for left-right system and right-left system, to [38] for hemiring, to [39] for near-semiring (or seminearring), to [6, 40] for lattice, bounded lattice, complemented lattice, distributive lattice, Boolean, De Morgan, Kleene, Stone algebra, to [41] for MV-algebra, for more about $\mathbb{SS}$s, we refer to [42-56].

3. Soft binary piecewise intersection operation

Definition 3.1. Let $(Ł,Ǩ)$ and $(Ɉ,Ŧ)$ be $\mathbb{SS}$s over U. The soft binary piecewise intersection of $(Ł,Ǩ)$ and $(Ɉ,Ŧ)$ is the $\mathbb{SS}$ $(H,Ǩ)$, denoted by $(Ł,Ǩ)\tilde{\cap}(Ɉ,Y)=(Ɱ,Ǩ)$, where for all $ɾ\in Ǩ$,

$$Ɱ(ɾ)=\begin{cases} Ł(ɾ), & ɾ\in Ǩ-Ŧ \\ Ł(ɾ)\cap Ɉ(ɾ), & ɾ\in Ǩ\cap Ŧ \end{cases}$$

Example 3.2. Let $Ɇ=\{ɾ_1,ɾ_2,ɾ_3,ɾ_4\}$ be the parameter set $Ǩ=\{ɾ_1,ɾ_4\}$ and $Ŧ=\{ɾ_2,ɾ_3,ɾ_4\}$ be the subsets of $Ɇ$ and $U=\{ƀ_1,ƀ_2,ƀ_3,ƀ_4,ƀ_5,ƀ_6\}$ be the universe set. Let $(Ł,Ǩ)$ and $(Ɉ,Ŧ)$ be as follows:

$(Ł,Ǩ)=\{(ɾ_1,\{ƀ_2,ƀ_4,ƀ_6),(ɾ_4,\{ƀ_1,ƀ_2,ƀ_5,ƀ_6\})\}$

$(Ɉ,Ŧ)=\{(ɾ_2,\{ƀ_1,ƀ_2\}),(ɾ_3,\{ƀ_2,ƀ_3,ƀ_4,ƀ_5\}),(ɾ_4,\{ƀ_2,ƀ_3,ƀ_4\})\}$

Let $(Ł,Ǩ)\tilde{\cap}(Ɉ,Ŧ)=(Ɱ,Ǩ)$. Then,

$$Ɱ(ɾ)=\begin{cases} Ł(ɾ), & ɾ\in Ǩ-Ŧ \\ Ł(ɾ)\cap Ɉ(ɾ), & ɾ\in Ǩ\cap Ŧ \end{cases}$$

Since $Ǩ=\{ɾ_1,ɾ_4\}$, $Ǩ-Ŧ=\{ɾ_1\}$ and $Ǩ\cap Ŧ=\{ɾ_4\}$ so $Ɱ(ɾ_1)=Ł(ɾ_1)=\{ƀ_2,ƀ_4,ƀ_6\}$ $Ɱ(ɾ_4)=Ł(ɾ_4)\cap Ɉ(ɾ_4)=\{ƀ_1,ƀ_2,ƀ_5,ƀ_6\}\cap\{ƀ_2,ƀ_3,ƀ_4\}$. Thus,

$(Ł,Ǩ)\tilde{\cap}(Ɉ,Ŧ)=\{(ɾ_1,\{ƀ_2,ƀ_4,ƀ_6\}),(ɾ_4,\{ƀ_2\})\}$.

Theorem 3.3. Let $(Ł,Ǩ)$, $(Ɉ,Ŧ)$, $(Ɱ,£)$, $(Ɉ,Ǩ)$, $(Ɱ,Ǩ)$, $(Ǩ,Ç)$, and $(Ł,Ç)$ be $\mathbb{SS}$s over U.. Then,

1) Let $(Ł,Ǩ)$ and $(Ɉ,Ŧ)$ be two $\mathbb{SS}$s over U, then so is $(Ł,Ǩ)\tilde{\cap}(Ɉ,Ŧ)$.

2) If $Ǩ\cap Ŧ'\cap£=\emptyset$, then $[(Ł,Ǩ)\tilde{\cap}(Ɉ,Ŧ)]\tilde{\cap}(Ɱ,£)=(Ł,Ǩ)\tilde{\cap}[(Ɉ,Ŧ)\tilde{\cap}(Ɱ,£)]$.

3) $[(Ł,Ǩ)\tilde{\cap}(Ɉ,Ǩ)]\tilde{\cap}(Ɱ,Ǩ)=(Ł,Ǩ)\tilde{\cap}[(Ɉ,Ǩ)\tilde{\cap}(Ɱ,Ǩ)]$.

4) $(Ł,Ǩ)\tilde{\cap}(Ɉ,Ŧ)\neq(Ɉ,Ŧ)\tilde{\cap}(Ł,Ǩ)$ and $(Ł,Ǩ)\tilde{\cap}(Ɉ,Ǩ)=(Ɉ,Ǩ)\tilde{\cap}(Ł,Ǩ)$.

5) $(Ł,Ќ)\tilde{\cap}(Ł,Ќ)=(Ł,Ќ)$.

6) $(Ł,Ќ)\tilde{\cap}U_Ќ=U_Ќ\tilde{\cap}(F,Ќ)=(F,Ќ)$.

Theorem 3.3.1. By Theorem 3.3. (1), (2), (4), and (5), when $(Ł,Ќ)$, $(Ḃ,\bar{Ŧ})$ and $(Ꟈ,£)$ are the elements of the set $S_Ɇ(U)$, $(S_Ɇ(U),\tilde{\cap})$ is a non-commutative band.

Moreover, by Theorem 3.3 (1), (3), (4), (5), and (6), $(S_Ќ(U),\tilde{\cap})$ is a bounded semilattice, whose identity element is $U_Ќ$.

7) $(Ł,Ќ)\tilde{\cap}U_Ɇ=(F,Ќ)$.

8) $(Ł,Ќ)\tilde{\cap}Ø_Ø=(Ł,Ќ)$.

9) $Ø_Ø\tilde{\cap}(F,Ќ)=Ø_Ø$.

Theorem 3.3.2. From the properties of (1), (2), (8), and (9), when $(Ł,Ќ)$, $(Ḃ,\bar{Ŧ})$ and $(Ꟈ,£)$ are the elements of $S_Ɇ(U)$, $(S_Ɇ(U),\tilde{\cap})$ is an idempotent right-left system, where $Ќ\cap\bar{Ŧ}'\cap£=Ø$.

10) $(Ł,Ќ)\tilde{\cap}Ø_Ќ=Ø_Ќ\tilde{\cap}(Ł,Ќ)=Ø_Ќ$.

11) $(Ł,Ќ)\tilde{\cap}Ø_Ɇ=Ø_Ќ$.

12) $Ø_Ɇ\tilde{\cap}(Ł,Ќ)=Ø_E$.

13) $(Ł,Ќ)\tilde{\cap}(Ł,Ќ)^r=(Ł,Ќ)^r\tilde{\cap}(F,Ќ)=Ø_Ќ$.

14) $[(Ł,Ќ)\tilde{\cap}(Ḃ,\bar{Ŧ})]^r=(Ł,Ќ)^r\tilde{\cup}(Ḃ,\bar{Ŧ})^r$, $[(Ł,Ќ)\tilde{\cap}(Ḃ,Ќ)]^r=(Ł,Ќ)^r\tilde{\cup}(Ḃ,Ќ)^r$.

15) $(Ł,Ќ)\tilde{\cap}(Ḃ,Ќ)=U_Ќ\Leftrightarrow(Ł,Ќ)=(Ḃ,Ќ)=U_Ќ$.

16) $(Ł,Ќ)\tilde{\cap}(Ḃ,Ќ)=Ø_Ќ\Leftrightarrow(Ḃ,Ќ)\tilde{\subseteq}(Ł,Ќ)^r$ $((Ł,Ќ)\tilde{\subseteq}(Ḃ,Ќ)^r)$.

17) $Ø_Ќ\tilde{\subseteq}(Ł,Ќ)\tilde{\cap}(Ḃ,\bar{Ŧ})$ ve $Ø_Y\tilde{\subseteq}(Ḃ,\bar{Ŧ})\tilde{\cap}(Ł,Ќ)$.

18) $(Ł,Ќ)\tilde{\cap}(Ḃ,\bar{Ŧ})\tilde{\subseteq}U_Ќ$ ve $(Ḃ,\bar{Ŧ})\tilde{\cap}(Ł,Ќ)\tilde{\subseteq}U_Y$.

19) $(Ł,Ќ)\tilde{\cap}(Ḃ,\bar{Ŧ})\tilde{\subseteq}(Ł,Ќ)$ ve $(Ḃ,\bar{Ŧ})\tilde{\cap}(Ł,Ќ)\tilde{\subseteq}(Ḃ,\bar{Ŧ})$.

20) If $(Ł,Ќ)\tilde{\subseteq}(Ḃ,\bar{Ŧ})$, then $(Ł,Ќ)\tilde{\cap}(Ḃ,\bar{Ŧ})=(Ł,Ќ)$.

21) If $(Ł,K) \cong (Ḃ,K)$, then $(Ł,K)_{\cap}(Ṁ,Z) \cong (Ḃ,K)_{\cap}(H,Z)$ and $(Ṁ,Z)_{\cap}(Ł,K) \cong (Ṁ,Z)_{\cap}(Ḃ,K)$.

22) $(Ł,K)\tilde{\cap}(H,Z) \cong (Ḃ,K)\tilde{\cap}(H,Z)$ does not necessarily imply that $(Ł,K) \cong (Ḃ,K)$.

Similarly, $(Ṁ,Z)\tilde{\cap}(Ł,K) \cong (Ṁ,Z)\tilde{\cap}(Ḃ,K)$ does not necessarily imply that $(Ł,K) \cong (Ḃ,K)$.

Proof: Let $Ḙ=\{x_1,x_2,x_3,x_4,x_5\}$ be the parameter set, $K=\{x_1,x_3,x_5\}$ and $Z=\{x_1,x_3\}$ be two subsets of $Ḙ$, $U=\{b_1,b_2,b_3,b_4,b_5\}$ be the universal set. Let $(Ł,K)$, $(Ḃ,K)$, and $(Ṁ,Z)$ be $\mathbb{SS}$s over U as follows:

$(Ł,K)=\{(x_1,\{b_2,b_5\}),(x,b_2,b_5\}),(x_5,\{b_1,b_3,b_5\})\}$,

$(Ḃ,K)=\{(x_1,\{b_2\}),(x_3,\{b_1,b_2\}),(x_5,\{b_2\})\},(Ṁ,Z)=\{(x_1,\emptyset),(x_3,\emptyset)\}$.

Let $(Ł,K)\tilde{\cap}(Ṁ,Z)=(Ꝋ,K)$, then for all $Ǫ \in K\text{-}Z=\{x_5\}$, we have $Ꝋ(x_5)=Ꝋ(x_5)=(b_1,b_3,b_5\}$, for all $Ǫ \in K \cap Z=\{x_1,x\}$, $Ꝋ(e_1)=Ł(x_1) \cap Ṁ(x_1)=\emptyset$, $Ꝋ(x_3)=Ł(x_3) \cap Ṁ(x_3)=\emptyset$. Thus,

$(Ł,K)\tilde{\cap}(Ṁ,Z)=\{(x_1,\emptyset),(x_3,\emptyset),(x_5,(b_1,b_3,b_5\})\}$.

Let $(Ḃ,K)\tilde{\cap}(Ṁ,Z)=(T,K)$ then for all $Ǫ \in K\text{-}Z=\{x_5\}$, $T(x_5)=Ł(x_5)=(b_1,b_3,b_5\}$, for all $Ǫ \in K \cap Z=\{x_1,x_3\}$, where $T(x_1)=Ḃ(x_1) \cap Ṁ(x_1)=\emptyset$, $T(x_3)=Ł(x_3) \cap Ṁ(x_3)=\emptyset$. Thus,

$(Ḃ,K)\tilde{\cap}(Ṁ,Z)=\{(x_1,\emptyset),(x_3,\emptyset),(x_5,(b_1,b_3,b_5\})\}$.

Thus, it is observed that although $(Ł,K)\tilde{\cap}(Ṁ,Z) \cong (Ḃ,K)\tilde{\cap}(Ṁ,Z)$, $(Ł,K)\tilde{\cong}(Ḃ,K)$ does not satisfy.

Similarly, in the same example, it can be shown that $(Ṁ,Z)\tilde{\cap}(Ł,K) \cong (Ṁ,Z)\tilde{\cap}(Ḃ,K)$, but $(Ł,K)\tilde{\cong}(Ḃ,K)$ does not hold.

23) Let $(Ł,T)\tilde{\cong}(Ḃ,T)$ and $(K,Ç)\tilde{\cong}(L,Ç)$. Then, $(Ł,T)\tilde{\cap}(K,Ç)\tilde{\cong}(Ḃ,T)\tilde{\cap}(L,Ç)$ and $(K,Ç)\tilde{\cap}(Ł,T)\tilde{\cong}(L,Ç)\tilde{\cap}(Ḃ,T)$.

24) $(Ł,K)\tilde{\cap}[(F,K)\tilde{\cup}(Ḃ,K)]=(Ł,K)$ and $(F,K)\tilde{\cup}[(F,K)\tilde{\cap}(Ḃ,K)]=(F,K)$.

4. Distribution rules

The distributions of soft binary piecewise intersection operation over other $\mathbb{SS}$ operations are examined in this section.

Proposition 4.1. Let $(Ł,K)$, $(Ḃ,T)$, and $(Ṁ,£)$ be $\mathbb{SS}$s over U. Then,

1) $(Ł,K)\tilde{\cap}[(Ḃ,T) \cup_R (Ṁ,£]=[(Ł,K)\tilde{\cap}(Ḃ,T)] \cup_R [(Ł,K)\tilde{\cap}(Ṁ,£)]$.

2) If $K \cap (Y \Delta £)=\emptyset$, then $(Ł,K)\tilde{\cap}[(Ḃ,T) \cap_R (Ṁ,£)]=[(Ł,K)\tilde{\cap}(Ḃ,T)] \cap_R [(Ł,K)\tilde{\cap}(Ṁ,£]$.

3) $[(Ł,K) \cap_R (Ḃ,T)]\tilde{\cap}(Ṁ,£)=[(Ł,K)\tilde{\cap}(Ṁ,£)] \cap_R [(Ḃ,T)\tilde{\cap}(Ṁ,£)]$.

4) $[(Ł,Ќ) \cup_R (Ḃ,\overline{T})]\tilde{\cap}(Ṁ,£)=[(Ł,Ќ)\tilde{\cap}(Ṁ,£)]\cup_R[(Ḃ,\overline{T})\tilde{\cap}(Ṁ,£)]$.

5) $[(Ł,Ќ) \Delta_R (Ḃ,\overline{T})]\tilde{\cap}(Ṁ,£)=[(Ł,Ќ)\tilde{\cap}(Ṁ,£)]\Delta_R[(Ḃ,\overline{T})\tilde{\cap}(Ṁ,£)]$.

6) $[(Ł,Ќ) \gamma_R (Ḃ,\overline{T})]\tilde{\cap}(Ṁ,£)=[(Ł,Ќ)\tilde{\cap}(Ṁ,£)]\gamma_R[(Ḃ,\overline{T})\tilde{\cap}(Ṁ,£)]$.

7) $[(Ł,Ќ) \setminus_R (Ḃ,\overline{T})]\tilde{\cap}(Ṁ,£)=[(Ł,Ќ)\tilde{\cap}(Ṁ,£)]\setminus_R[(Ḃ,\overline{T})\tilde{\cap}(Ṁ,£)]$.

Corollary 4.1.1. $(S_Ɛ(U),\cap_R,\tilde{\cap})$ is an additive commutative, idempotent, semiring without zero and unity under certain conditions (UCC).

Corollary 4.1.2. $(S_Ќ(U),\cap_R,\tilde{\cap})$ is a commutative, idempotent semiring without zero but with unity.

Corollary 4.1.3. $(S_Ɛ(U),\cup_R,\tilde{\cap})$ is an additive commutative, idempotent, semiring without zero and unity UCC.

$(S_Ɛ(U),\cup_R,\tilde{\cap})$ is also an additive commutative, idempotent, (right) near-semiring with zero but without unity and zero-symmetric property UCC.

Corollary 4.1.4. $(S_Ќ(U),\cup_R,\tilde{\cap})$ is a commutative, idempotent hemiring with unity.

Corollary 4.1.5. $(S_Ɛ(U),\Delta_R,\tilde{\cap})$ is an additive commutative, multiplicative idempotent, (right) near-semiring with zero but without unity and zero-symmetric property UCC.

Corollary 4.1.6. $(S_Ќ(U),\Delta_R,\tilde{\cap})$ is a commutative, multiplicative idempotent hemiring with unity.

Corollary 4.1.7. $(S_Ќ(U),\Delta_R,\tilde{\cap})$ is a Boolean ring.

Corollary 4.1.8. $(S_Ќ(U),\emptyset_Ќ,U_Ќ,\cup_R,\tilde{\cap})$ is a Boolean Algebra, moreover $(S_Ќ(U),\emptyset_Ќ,U_Ќ,^c,\cup_R,\tilde{\cap})$ is a De Morgan Algebra, Ќleene Algebra and $(S_Ќ(U),\emptyset_Ќ,U_Ќ,^c,\cup_R,\tilde{\cap})$ a Stone Algebra.

Here note that since $(S_Ɛ(U),\tilde{\cap})$ is a non-commutative idempotent semigroup in $S_Ɛ(U)$, $\tilde{\cap}$ does not form a lattice together with $\cup_R$.

Corollary 4.1.9. $(S_Ќ(U),^r,\tilde{\cap},U_Ќ)$ is an MV-algebra.

Proposition 4.2. Let $(Ł,Ќ)$, $(Ḃ,\overline{T})$, and $(Ṁ,£)$ be $\mathbb{S}\mathbb{S}$s over U. Then,

1) $Ќ\cap(Y\Delta£)=\emptyset$ ise $(Ł,Ќ)\tilde{\cap}[(Ḃ,\overline{T})\cup_\varepsilon(Ṁ,£)]=[(Ł,Ќ)\tilde{\cap}(Ḃ,\overline{T})]\cup_\varepsilon[(Ł,Ќ)\tilde{\cap}(Ṁ,£)]$.

2) $(Ł,Ќ)\tilde{\cap}[(Ḃ,\overline{T})\cap_\varepsilon(Ṁ,£)]=[(Ł,Ќ)\tilde{\cap}(Ḃ,\overline{T})]\cap_\varepsilon[(Ł,Ќ)\tilde{\cap}(Ṁ,£)]$.

3) $[(Ł,Ќ)\Delta_\varepsilon(Ḃ,\overline{T})]\tilde{\cap}(Ṁ,£)=[(Ł,Ќ)\tilde{\cap}(Ṁ,£)]\Delta_\varepsilon[(Ḃ,\overline{T})\tilde{\cap}(Ṁ,£)]$.

4) $[(Ł,Ќ) \cup_\varepsilon (Ḑ,Ŧ)]\widetilde{\cap}(ϖ,£)=[(Ł,Ќ)\widetilde{\cap}(ϖ,£)]\cup_\varepsilon[(Ḑ,Ŧ)\widetilde{\cap}(ϖ,£)]$.

5) $[(Ł,Ќ)\cap_\varepsilon (Ḑ,Ŧ)]\widetilde{\cap}(ϖ,£)=[(Ł,Ќ)\widetilde{\cap}(ϖ,£)]\cap_\varepsilon[(Ḑ,Ŧ)\widetilde{\cap}(ϖ,£)]$.

6) $[(Ł,Ќ) \setminus_\varepsilon (Ḑ,Ŧ)]\widetilde{\cap}(ϖ,£)=[(Ł,Ќ)\widetilde{\cap}(ϖ,£)]\setminus_\varepsilon[(Ḑ,Ŧ)\widetilde{\cap}(ϖ,£)]$.

7) $[(Ł,Ќ) \gamma_\varepsilon (Ḑ,Ŧ)]\widetilde{\cap}(ϖ,£)=[(Ł,Ќ)\widetilde{\cap}(ϖ,£)]\gamma_\varepsilon[(Ḑ,Ŧ)\widetilde{\cap}(ϖ,£)]$.

The equations below are satisfied, where $Ќ\cap Ŧ\cap £=\emptyset$.

8) $[(Ł,Ќ) +_\varepsilon (Ḑ,Ŧ)]\widetilde{\cap}(ϖ,£)=[(Ł,Ќ)\widetilde{\cap}(ϖ,£)]+_\varepsilon[(Ḑ,Ŧ)\widetilde{\cap}(ϖ,£)]$.

9) $[(Ł,Ќ) \lambda_\varepsilon (Ḑ,Ŧ)]\widetilde{\cap}(ϖ,£)=[(Ł,Ќ)\widetilde{\cap}(ϖ,£)]\lambda_\varepsilon[(Ḑ,Ŧ)\widetilde{\cap}(ϖ,£)]$.

10) $[(Ł,Ќ) *_\varepsilon (Ḑ,Ŧ)]\widetilde{\cap}(ϖ,£)=[(Ł,Ќ)\widetilde{\cap}(ϖ,£)]*_\varepsilon[(Ḑ,Ŧ)\widetilde{\cap}(ϖ,£)]$.

11) $[(Ł,Ќ) \theta_\varepsilon (Ḑ,Ŧ)]\widetilde{\cap}(ϖ,£)=[(Ł,Ќ)\widetilde{\cap}(ϖ,£)]\theta_\varepsilon[(Ḑ,Ŧ)\widetilde{\cap}(ϖ,£)]$.

Corollary 4.2.1. $(S_Ɇ(U),\cap_\varepsilon,\widetilde{\cap})$ is an additive commutative, idempotent semiring without zero and without unity UCCs.

Also, $(S_Ɇ(U),\cap_\varepsilon,\widetilde{\cap})$ is an additive commutative, idempotent (right) near-semiring with zero but without zero-symmetric property and unity UCCs.

Corollary 4.2.2. $(S_Ќ(U), \cap_\varepsilon,\widetilde{\cap})$ is a commutative, idempotent semiring without zero but with unity.

$(S_Ќ(U),\setminus_\varepsilon,\widetilde{\cap}),(S_Ќ(U),\gamma_\varepsilon,\widetilde{\cap}),(S_Ќ(U),*_\varepsilon,\widetilde{\cap}),(S_Ќ(U),\theta_\varepsilon,\widetilde{\cap}),(S_Ќ(U),+_\varepsilon,\widetilde{\cap}),(S_Ќ(U),\lambda_\varepsilon,\widetilde{\cap})$ do not form a semiring, as the first operations are not associative in $S_Ќ(U)$.

Corollary 4.2.3. $(S_Ɇ(U),\cup_\varepsilon,\widetilde{\cap})$ is an additive commutative, idempotent semiring without zero and without unity UCCs.

Moreover, $(S_Ɇ(U),\cup_\varepsilon,\widetilde{\cap})$ is an additive commutative, idempotent (right) near-semiring with zero but without zero-symmetric property and unity UCCs.

Corollary 4.2.4. $(S_Ќ(U), \cup_\varepsilon ,\widetilde{\cap})$ is a commutative, idempotent hemiring with unity.

Corollary 4.2.5. $(S_E(U),\Delta_\varepsilon,\widetilde{\cap})$ is an additive commutative, multiplicative idempotent, (right) near-semiring with zero but without unity and zero symmetric property UCC.

Corollary 4.2.6. $(S_{\mathbb{E}}(U), \setminus_\varepsilon, \widetilde{\cap}), (S_{\mathbb{E}}(U), \gamma_\varepsilon, \widetilde{\cap}), (S_{\mathbb{E}}(U), +_\varepsilon, \widetilde{\cap}), (S_{\mathbb{E}}(U), \lambda_\varepsilon, \widetilde{\cap}), (S_{\mathbb{E}}(U), *_\varepsilon, \widetilde{\cap}), (S_{\mathbb{E}}(U), \theta_\varepsilon, \widetilde{\cap})$ are multiplicative idempotent (right) near-semiring with zero but without unity and without zero symmetric property UCCs.

A. Sezgin, et al. (2023) [24] showed that the first operations are associative in the collection $S_E(U)$ UCC.

Corollary 4.2.7. $(S_{\mathbb{K}}(U), \Delta_\varepsilon, \widetilde{\cap})$ is a commutative, multiplicatively idempotent hemiring with unity. $(S_{\mathbb{K}}(U), \Delta_\varepsilon, \widetilde{\cap})$ is also a Boolean ring.

Corollary 4.2.8. $(S_{\mathbb{K}}(U), \emptyset_{\mathbb{K}}, U_{\mathbb{K}}, \cup_\varepsilon, \widetilde{\cap})$ is a Boolean Algebra, moreover $(S_{\mathbb{K}}(U), \emptyset_{\mathbb{K}}, U_{\mathbb{K}}, {}^c, \cup_\varepsilon, \widetilde{\cap})$ is a De Morgan Algebra, Ƙleene Algebra and $(S_{\mathbb{K}}(U), \emptyset_{\mathbb{K}}, U_{\mathbb{K}}, {}^c, \cup_\varepsilon, \widetilde{\cap})$ a Stone Algebra.

Here note that since $(S_{\mathbb{E}}(U), \widetilde{\cap})$ is a non-commutative idempotent semigroup in $S_{\mathbb{E}}(U)$, $\widetilde{\cap}$ does not form a lattice together with $\cup_\varepsilon$.

Proposition 4.3. Let $(Ł, \mathbb{K})$, $(Ḍ, \overline{Ŧ})$, $(ϖ, £)$ be $\mathbb{SS}$s on U. Then,

1) If $\mathbb{K} \cap \overline{Ŧ} \cap £' = \emptyset$, $(Ł, \mathbb{K}) \widetilde{\cap} [(Ḍ, \overline{Ŧ}) \widetilde{\cup} (ϖ, £)] = [(Ł, \mathbb{K}) \widetilde{\cap} (Ḍ, \overline{Ŧ})] \widetilde{\cup} [(Ł, \mathbb{K}) \widetilde{\cap} (ϖ, £)]$.

2) If $\mathbb{K} \cap \overline{Ŧ}' \cap £ = \emptyset$, $(Ł, \mathbb{K}) \widetilde{\cap} [(Ḍ, \overline{Ŧ}) \widetilde{\cap} (ϖ, £)] = [(Ł, \mathbb{K}) \widetilde{\cap} (Ḍ, \overline{Ŧ})] \widetilde{\cap} [(ϖ, £) \widetilde{\cap} (Ł, \mathbb{K})]$.

3) $[(Ł, \mathbb{K}) \widetilde{\cup} (Ḍ, \overline{Ŧ})] \widetilde{\cap} (ϖ, £) = [(Ł, \mathbb{K}) \widetilde{\cap} (ϖ, £)] \widetilde{\cup} [(Ḍ, \overline{Ŧ}) \widetilde{\cap} (ϖ, £)]$.

4) $[(Ł, \mathbb{K}) \widetilde{\cap} (Ḍ, \overline{Ŧ})] \widetilde{\cap} (ϖ, £) = [(Ł, \mathbb{K}) \widetilde{\cap} (ϖ, £)] \widetilde{\cap} [(Ḍ, \overline{Ŧ}) \widetilde{\cap} (ϖ, £)]$.

5) $[(Ł, \mathbb{K}) \widetilde{\Delta} (Ḍ, \overline{Ŧ})] \widetilde{\cap} (ϖ, £) = [(Ł, \mathbb{K}) \widetilde{\cap} (ϖ, £)] \widetilde{\Delta} [(Ḍ, \overline{Ŧ}) \widetilde{\cap} (ϖ, £)]$.

6) $[(Ł, \mathbb{K}) \widetilde{\setminus} (Ḍ, \overline{Ŧ})] \widetilde{\cap} (ϖ, £) = [(Ł, \mathbb{K}) \widetilde{\cap} (ϖ, £)] \widetilde{\setminus} [(Ḍ, \overline{Ŧ}) \cap (ϖ, £)]$.

7) $[(Ł, \mathbb{K}) \widetilde{\gamma} (Ḍ, \overline{Ŧ})] \widetilde{\cap} (ϖ, £) = [(Ł, \mathbb{K}) \widetilde{\cup} (ϖ, £)] \widetilde{\gamma} [(Ḍ, \overline{Ŧ}) \widetilde{\cap} (ϖ, £)]$.

The equalities below are satisfied, where $\mathbb{K} \cap \overline{Ŧ} \cap £ = \emptyset$.

8) $[(Ł, \mathbb{K}) \widetilde{+} (Ḍ, \overline{Ŧ})] \widetilde{\cap} (ϖ, £) = [(Ł, \mathbb{K}) \widetilde{\cap} (ϖ, £)] \widetilde{+} [(Ḍ, \overline{Ŧ}) \widetilde{\cap} (ϖ, £)]$.

9) $[(Ł, \mathbb{K}) \widetilde{\lambda} (Ḍ, \overline{Ŧ})] \widetilde{\cap} (ϖ, £) = [(Ł, \mathbb{K}) \widetilde{\cap} (ϖ, £)] \widetilde{\lambda} [(Ḍ, \overline{Ŧ}) \widetilde{\cap} (ϖ, £)]$.

10) $[(Ł, \mathbb{K}) \widetilde{*} (Ḍ, \overline{Ŧ})] \widetilde{\cap} (ϖ, £) = [(Ł, \mathbb{K}) \widetilde{\cap} (ϖ, £)] \widetilde{*} [(Ḍ, \overline{Ŧ}) \widetilde{\cap} (ϖ, £)]$.

11) $[(Ł, \mathbb{K}) \widetilde{\theta} (Ḍ, \overline{Ŧ})] \widetilde{\cap} (ϖ, £) = [(Ł, \mathbb{K}) \widetilde{\cap} (ϖ, £)] \widetilde{\theta} [(Ḍ, \overline{Ŧ}) \widetilde{\cap} (ϖ, £)]$.

Corollary 4.3.1. $(S_{\mathbb{E}}(U),\tilde{\cap},\tilde{\cap})$ is an idempotent semiring without zero and unity UCCs.

Corollary 4.3.2. $(S_{\mathbb{K}}(U),\tilde{\cap},\tilde{\cap})$ is a commutative, idempotent semiring without zero but with unity.

Note that $(S_{\mathbb{K}}(U),\tilde{\setminus},\tilde{\cap}),(S_{\mathbb{K}}(U),\tilde{\gamma},\tilde{\cap}),(S_{\mathbb{K}}(U),\tilde{*},\tilde{\cap}),(S_{\mathbb{K}}(U),\tilde{\theta},\tilde{\cap}),(S_{\mathbb{K}}(U),\tilde{+},\tilde{\cap}),(S_{\mathbb{K}}(U),\tilde{\lambda},\tilde{\cap})$ do not form a semiring, as the first operations are not associative in $S_{\mathbb{K}}(U)$.

Corollary 4.3.3. $(S_{\mathbb{E}}(U),\tilde{\cup},\tilde{\cap})$ is an idempotent semiring without zero and unity UCCs.

Corollary 4.3.4. $(S_{\mathbb{K}}(U),\tilde{\cup},\tilde{\cap})$ is a commutative, idempotent hemiring with unity.

Corollary 4.3.5. $(S_{\mathbb{E}}(U),\tilde{\Delta},\tilde{\cap}),(S_{\mathbb{E}}(U),\tilde{\setminus},\tilde{\cap}),(S_{\mathbb{E}}(U),\tilde{\gamma},\tilde{\cap})$ are multiplicative idempotent, (right) near-semiring without zero and unity UCCs.

Corollary 4.3.6. $(S_{\mathbb{K}}(U),\tilde{\Delta},\tilde{\cap})$ is a commutative, multiplicatively idempotent hemiring with unity.

Corollary 4.3.7. $(S_{\mathbb{K}}(U),\tilde{\Delta},\tilde{\cap})$ is a Boolean ring.

Corollary 4.3.8. $(S_{\mathbb{E}}(U),\tilde{+},\tilde{\cap}),(S_{\mathbb{E}}(U),\tilde{*},\tilde{\cap}),(S_{\mathbb{E}}(U),\tilde{\theta},\tilde{\cap}),(S_{\mathbb{E}}(U),\tilde{\lambda},\tilde{\cap})$ are multiplicative idempotent (right) near-semiring without unity and zero UCCs.

Corollary 4.3.9. $(S_{\mathbb{K}}(U),\emptyset_{\mathbb{K}},U_{\mathbb{K}},\tilde{\cup},\tilde{\cap})$ is a Boolean Algebra, moreover $(S_{\mathbb{K}}(U),\emptyset_{\mathbb{K}},U_{\mathbb{K}},^{c},\tilde{\cup},\tilde{\cap})$ is a De Morgan Algebra, Kleene Algebra, and $(S_{\mathbb{K}}(U),\emptyset_{\mathbb{K}},U_{\mathbb{K}},^{c},\tilde{\cup},\tilde{\cap})$ is a Stone Algebra.

Here note that since $(S_{\mathbb{E}}(U),\tilde{\cap})$ is a non-commutative idempotent semigroup in the collection $S_{\mathbb{E}}(U)$, $\tilde{\cap}$ does not form a lattice together with $\tilde{\cup}$ in the collection $(S_{\mathbb{E}}(U)$.

5. Conclusions

When working with uncertain objects, parametric tools such as $\mathbb{SS}$s and soft operations are highly effective. Novel insights into the solution of parametric data issues come from the proposal of novel soft operations and the derivation of their algebraic properties and implementations. This study introduces a novel kind of $\mathbb{SS}$ operation in this regard by proposing a new $\mathbb{SS}$ operation called "soft binary piecewise intersection operation" and conducting a systematic investigation of algebraic structures connected with this new $\mathbb{SS}$ operation and other $\mathbb{SS}$ operations in the class of $\mathbb{SS}$s. Particularly, the algebraic properties of this new $\mathbb{SS}$ operation are thoroughly examined especially by comparing with the intersection operation in classical set theory, and the distributions of the soft binary piecewise intersection operation over other types of $\mathbb{SS}$ operations are handled. Considering the algebraic properties of the $\mathbb{SS}$ operations and distribution rules, a detailed analysis is provided on which algebraic structures the set of $\mathbb{SS}$s forms with these operations. We show that the collection of $\mathbb{SS}$s over the universe and the collection of all $\mathbb{SS}$s with a fixed parameter set together with the soft binary piecewise intersection operation and other types of $\mathbb{SS}$s form many

important algebraic structures such as semiring near-semiring, hemiring, Boolean ring, Boolean, De Morgan, Kleene, and Stone Algebra. Since we gain a detailed understanding of the use by studying the algebraic structures of $\mathfrak{SS}$s in relation to novel $\mathfrak{SS}$ operations as they also offer new examples of algebraic structures, this study contributes to the literature of not only $\mathfrak{SS}$ theory but also classical algebra. In future studies, other types of soft binary piecewise operations may be handled together with their properties and distributions to observe which algebraic structures they form in the class of soft set over the universe and in the class of $\mathfrak{SS}$s with a fixed parameter set. Besides new decision-making methods based on this new $\mathfrak{SS}$ operations may be proposed by considering the properties of the operation.

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