

Stability analysis for vertical breakwater foundation according to ASD and LRFD

Dang Van Phu^{1,*}

¹ Faculty of Civil Engineering, Thuyloi University;

*Corresponding author: phudv@tlu.edu.vn

■ Received: 06/10/2025 ■ Revised: 24/10/2025 ■ Accepted: 28/11/2025

ABSTRACT

This paper analyzes the stability of breakwater foundations using two methods: the traditional allowable safety factor design (ASD) and the load factor and resistance factor method (LRFD), both based on reliability theory. Six vertical breakwater cross-sections were analyzed using the Bishop method to calculate the factor of safety (FS), with the basic load combination. Subsequently, the Monte Carlo simulation method was employed in reliability analysis to determine the load factors and resistance factors. All calculation steps are performed on the MATLAB platform. The analysis results show that the safety level of the stability of breakwater foundations using LRFD depends largely on the variation of the soil strength. When designing using LRFD with a target reliability index of 3.0, the design using ASD is safer than the LRFD design if the variation in soil strength is small. However, when the coefficient of variation of soil strength is great, the ASD approach is often less safe than the LRFD.

Keywords: Breakwater, ASD, LRFD, Reliability analysis, Slope stability

1. INTRODUCTION

A breakwater is an essential structure built near the coast to protect the shoreline, port area, and anchorage area from the impact of waves, currents, and other hydrological phenomena. With the formation of breakwaters, various types of structures have emerged, including rubble mound breakwaters, vertical wall breakwaters, composite breakwaters, sheet pile breakwaters, and semicircular breakwaters. Evaluating the stability of breakwater foundations is very necessary, one of which is the stability of breakwater foundations.

Research on breakwaters in the world has been carried out for several decades. Goda studied sea waves and the design of vertical wall breakwaters [1]. In this book, the author presents the calculation of wave forces acting on the vertical wall of breakwaters and the design of the core layer of breakwaters against the effect of sea waves. Also in 2000, Goda & Takagi used reliability theory to design caissons with optimal wave heights

[2]. Since the construction of breakwaters is very expensive, ensuring the safety of breakwaters in the face of wave pressure in storm scenarios is necessary. At that time, calculating the design according to reliability will help engineers achieve the desired safety. Oumeraci et al. also thoroughly studied the design of vertical wall breakwaters according to random theory [3]. In this study, a reliability theory design tool for calculating the design by partial factors was introduced. In Korea, the calculation and design of stability for breakwater foundations still employ the design method using allowable factors [4] and are gradually shifting toward a design method that uses load factors and resistance factors. In Japan, there is a standard for the design of port facilities, including breakwaters [5]. This standard is transitioning from the design guidance for breakwaters based on the partial factor design method (version from 2007 or earlier) to the load factor and resistance factor design method (from version 2018).

In Vietnam, several standards and studies on breakwaters have been conducted, ranging from investigations into the stability, efficiency, and sustainability of breakwaters to studies on new technologies and materials. TCCS 02: 2017/CHHVN, 2017 standard stipulates how to design breakwaters to resist the impact of waves, wind, currents, and other loads for port works [6]. This standard provides guidance for circular slip failure calculations for breakwater foundations using the traditional method. However, designing circular slip stability for breakwater foundations using the ASD method has the limitation of relying on a single safety factor, which leads to significantly different levels of actual reliability. In the problem of breakwater stability, the self-weight of the breakwaters and the soil mass often have low uncertainty and can be determined relatively accurately. In contrast, the wave loads and shear strength of the foundation soil often exhibit a high degree of uncertainty and are difficult to predict accurately. Meanwhile, ASD lacks a mechanism to calculate a higher level of safety for scenarios with dominant uncertain loads. Accordingly, structures designed according to the same FS standard have significantly different actual failure probabilities, creating uneven safety across the system.

To address the limitations of the ASD method, the LRFD method has gained wider use recently for bridge and civil engineering works. Regarding seaport works, the TCVN 11820-6, 2023 set includes 10 parts for "Seaport works - design requirements" developed by the Institute of Transport Science and Technology based on the reference to the technical standards for port and seaport works in Japan [7]. This standard includes guidelines for calculating breakwater stability using the partial load factor method (PFD) and the load factor and resistance factor method (LRFD). These two methods combine the limit state method and reliability theory in constructing appropriate coefficients for loads and

resistance. In particular, the design method based on load coefficients and resistance coefficients for circular sliding stability is the most suitable due to its advantages in application and closeness to reality. However, the application of LRFD in design still has many limitations in Vietnam because the parameters are being referenced from Japan and have not been built with specific characteristics in Vietnam. In addition, the research and calculation of load coefficients and resistance coefficients according to the target reliability index are still limited in Vietnam, and comparative studies between the two design methods have not been conducted. Therefore, in this study, a procedure for calculating load factors and resistance factors was proposed for some cases based on reliability theory with target reliability indices of 2.0 and 3.0 with some different values of cohesion coefficient and internal friction angle of soil layers (COV=10% and 20%). Then, the calculated coefficients were used to compare the design method based on load factors and resistance factors with the traditional design method for the circular sliding stability of a vertical breakwater foundation using a basic load combination.

2. METHODOLOGY

2.1. Deterministic analysis

There are several methods for calculating the factor of safety (FS) for breakwater foundations, similar to those used for slope stability, such as the limit equilibrium method and the finite element method. The limit equilibrium method (LEM) includes the Fellenius method, the Bishop method (BSM), the Spencer method, the Janbu method, and the Morgenstern-Price method [8]. In LEM, the sliding soil mass generated by the slipping surface is divided into vertical slices. Then, the force or moment equilibrium is used to calculate the minimum FS. This study focuses on the limit equilibrium methods and applies BSM to calculate the minimum factor of safety

for breakwater foundation stability. The factor of safety for breakwater foundation stability using BSM in the deterministic analysis is calculated by Equation (1).

$$FS = \frac{R}{Q} = \frac{\sum \left[\frac{\{c'b + (w' + q_{Ex}b) \tan \phi'\} \sec \theta}{1 + \tan \theta \tan \phi' / FS} \right]}{\sum \{(w + q_{Ex}b) \sin \theta\}} \quad (1)$$

In the above equation, c' is the soil cohesion at the bottom of the sliding arc; ϕ' is the internal friction angle of the soil based on the effective stress at the bottom of the sliding arc; w and w' are the total weight and effective weight of the soil in each segment; b is the width of the segment; θ is the inclination angle created by the bottom of the sliding arc at the segment under consideration and the horizontal direction; q_{Ex} is the additional load.

2.2 Reliability analysis

Many variables related to load and resistance parameters are often considered as uncertainties rather than deterministic quantities. These uncertainties are considered in the engineering problem and are referred to as random variables. Therefore, the design approach that takes random variables into account is called the reliability-based design approach. In this design approach, the reliability index or failure probability is usually estimated to assess the safety of the structure. These parameters can be calculated using several methods, including the Level 1 method, the Level 2 method, and the Monte Carlo simulation (MCS) method. The first two methods are more suitable for a problem with an explicit limit state function, and the reliability index β will be calculated. On the other hand, the MCS method can be applied to all problems, not only those with explicit but also those with implicit reliability functions. In the MCS method, the failure probability is calculated instead of the reliability index β .

The limit state function is shown below.

$$g = FS - 1 \quad (2)$$

The probability of failure is determined by the Equation (3).

$$P_f = \frac{1}{N_{MCS}} \sum_{i=1}^{N_{MCS}} I \left[\min_{j=1,2,\dots,N_{ss}} FS_j(X) < 1 \right] \quad (3)$$

where, $I[\cdot]$ is an index function, taken as 1 if $\min_{j=1,2,\dots,N_{ss}} FS_j(X) < 1$, and 0 otherwise.

Then, the reliability index β can be calculated using Equation (4).

$$\beta = -\Phi^{-1}(P_f) = \Phi^{-1}(1 - P_f) \quad (4)$$

2.3. Calculation of load factor and resistance factors

The load and resistance factor design (LRFD) method has been widely used due to its advantages over ASD. In LRFD, the uncertainties of loads and resistances are considered separately, which is suitable for reflecting the realism of random variables. Therefore, LRFD has recently been studied in the context of geotechnical engineering [9]. However, the application of LRFD in geotechnical engineering is challenging due to the high uncertainty associated with soil properties. In particular, the application of LRFD for breakwater foundations has received little attention, despite being a straightforward and simple method for design engineers. To apply the LRFD method, the load and resistance factors (LRFs) need to be calibrated according to the target failure probability or target reliability index.

The general conditions in design according to LRFD are defined as follows [9,10]:

$$(RF) \times R_n \geq \sum (LF)_i \times Q_{ni} \quad (5)$$

where RF and LF_i are the resistance factor and load factor, R_n and Q_{ni} are the nominal values of resistance and load term at the limit state.

Basically, the calculation of LRFs is performed according to the diagram shown in Figure 1.

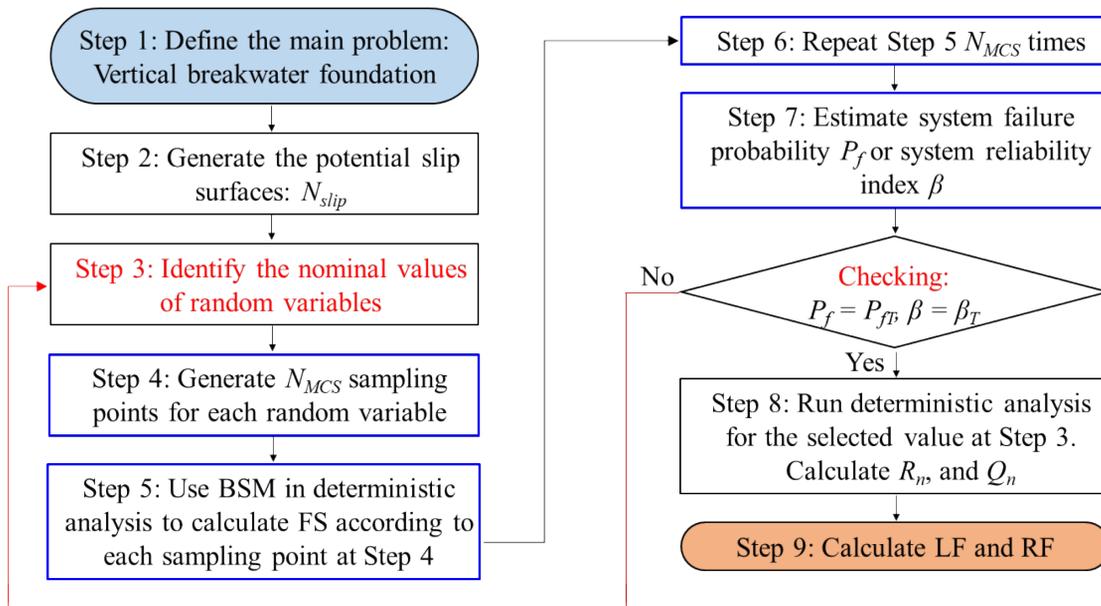


Figure 1. Diagram of calculating load and resistance factors

3. CASE STUDIES

Six vertical breakwater cross-sections were selected for study, as shown in Figs. 2 to 7. These six cross-sections have different

shapes and geological characteristics to provide a more comprehensive view of the application of ASD and LRFD methods.

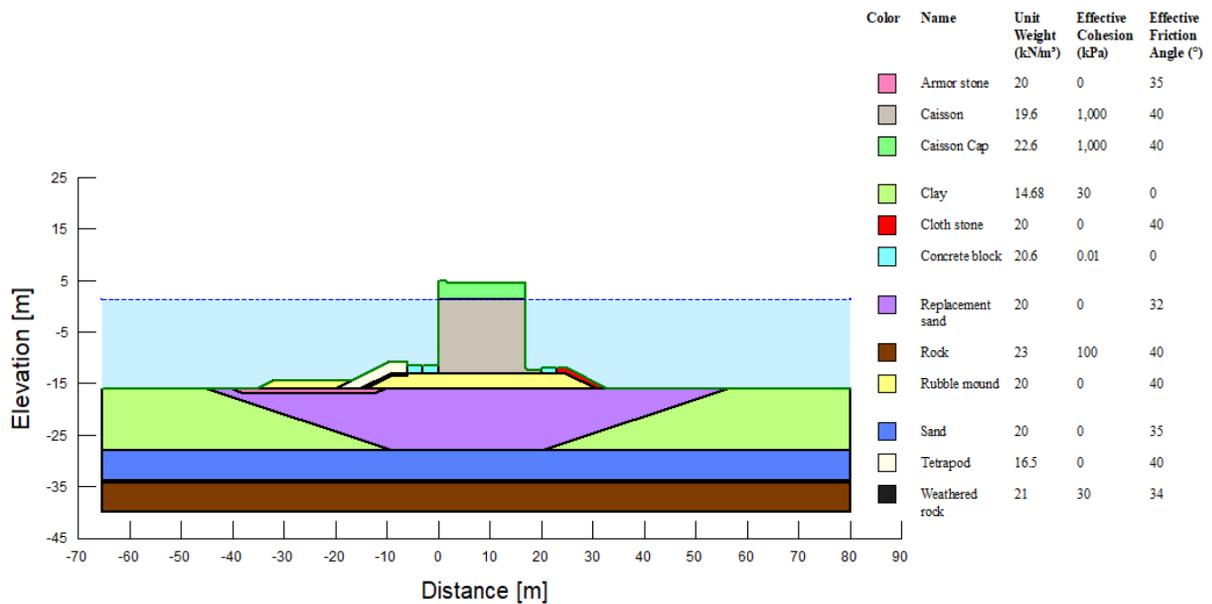


Figure 2. Cross-section 1

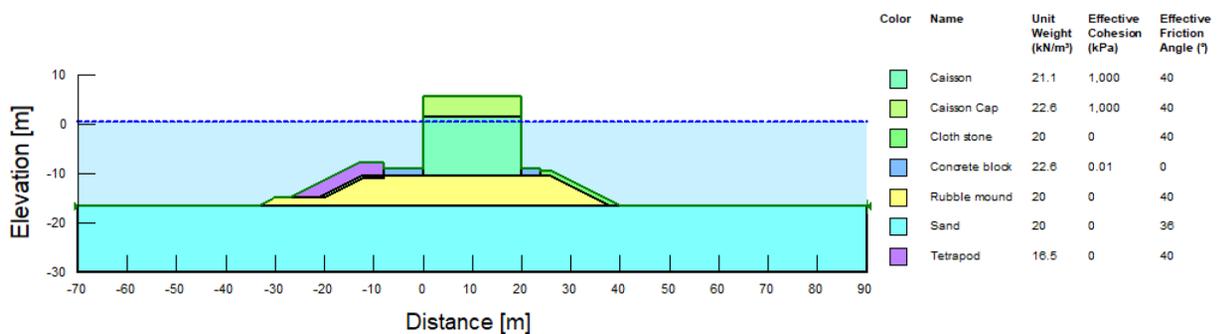


Figure 3. Cross-section 2

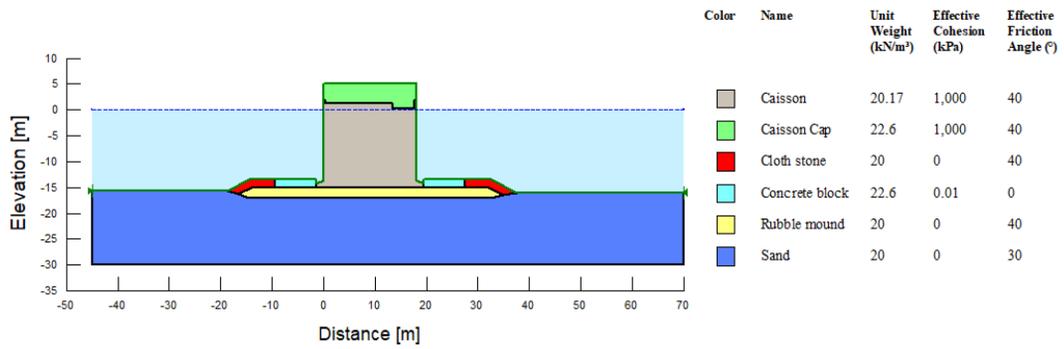


Figure 4. Cross-section 3

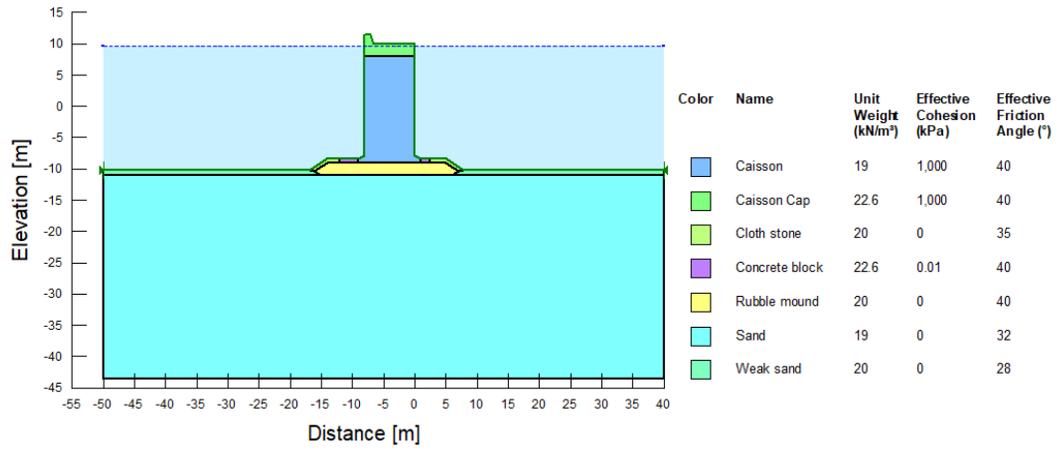


Figure 5. Cross-section 4

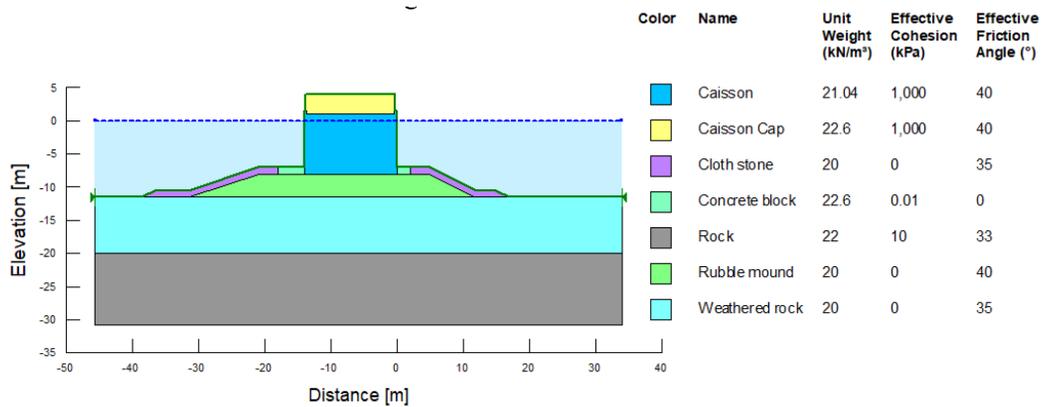


Figure 6. Cross-section 5

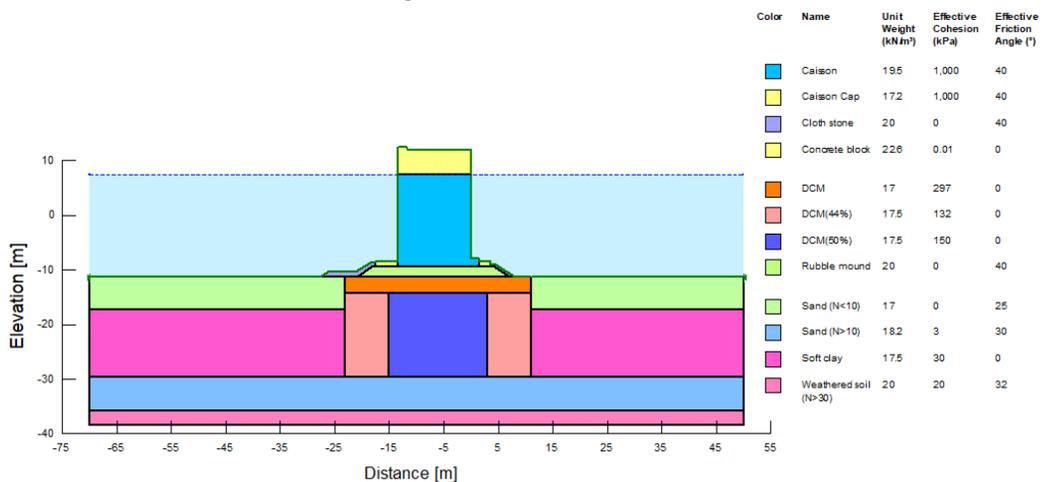


Figure 7. Cross-section 6

In this study, the bias factor is used instead of the expected value of the input random variables, as per the study by Allen et al. [1]. The bias factor is defined as the ratio between the measured value and the nominal value, as shown in Equation (9). In this equation, $\bar{\lambda}$ is the expectation of the bias factor, $\overline{X_m}$ is the expectation of the actual measured value, and X_n is the nominal value calculated from the design models. From here, the actual measured values of the input random variables can be determined as $X_m^i = \lambda_i \times X_n$.

$$\bar{\lambda} = \frac{\overline{X_m}}{X_n} \tag{6}$$

The statistical parameters of the random variables are given in Table 1. In the stability analysis of breakwater foundations, soil strength is an important factor, and the COV of its bias varies greatly. The range of COV values of soil strength studied by [11–13] has summarized that the COV of cohesion and friction angle are usually in the range of 10% to 70% and 4% to 20%. In this study, two COV values are considered, 10% and 20% for both cohesion and friction angle.

Table 1: Statisticalz parameters of random variables

Name	Symbol	Mean of bias	COV of bias	Distribution
Weight of plain concrete	W_c	1.02	2%	Normal
Weight of reinforced concrete	W_{rc}	0.98	2%	Normal
Weight of filling materials	W_f	1.02	4%	Normal
Tidal range–cross-section 1-3, 5	WL	1.00	20%	Normal
Tidal range–cross-section 4, 6	WL	1.00	5%	Normal
Tangent of friction angle	$\tan\phi'$	1.00	10%, 20%	Normal
Cohesion	c'	1.00	10%, 20%	Normal
Unit weight of soil	γ	1.00	$COV\gamma$	Normal

$COV\gamma = 2\%$ for clay and rock; 3% for sand and rubble

4. RESULTS

The results of the calculation of the load factor and resistance factor LRFs are shown in Table 2 and Table 3. In general, with a higher degree of uncertainty, i.e., a higher COV, the normalized resistance factor (nRF) decreases for the same value of β_T . Similarly, for a higher β_T , the value of nRF decreases for the same value of COV. It can be observed

that when applying reliability theory, the most unfavorable section among the six sections under consideration varies depending on the value of β_T and the coefficient of variation of the ground strength. For example, when β_T is 2.0 and $COV=10\%$, sections 4 and 6 are the most unfavorable, while when β_T is 2.0 and $COV=20\%$, only section 4 is the most unfavorable. When β_T is 3.0, if $COV=10\%$ then cross-section 3 is the most unfavorable, and if $COV=20\%$ then cross-section 4 is the most unfavorable.

Table 2: Results of load factor and resistance factor calibration with $\beta_T=2.0$

Cross-section	FS	COV = 10%				COV = 20%			
		β	LF	RF	nRF	β	LF	RF	nRF
1	1.846	2.010	1.044	0.885	0.848	1.996	1.022	0.681	0.667
2	2.316	2.001	1.057	0.871	0.824	2.002	1.064	0.661	0.621
3	2.543	2.008	1.051	0.842	0.801	1.999	1.052	0.634	0.603

Cross-section	FS	COV = 10%				COV = 20%			
		β	LF	RF	nRF	β	LF	RF	nRF
4	2.085	1.993	1.145	0.916	0.800	1.996	1.221	0.726	0.595
5	2.360	2.006	1.039	0.851	0.819	2.001	1.011	0.625	0.618
6	2.453	2.009	1.149	0.920	0.800	2.000	1.173	0.798	0.680

Table 3: Results of load factor and resistance factor calibration with $\beta_T=3.0$

Cross-section	FS	COV = 10%				COV = 20%			
		β	LF	RF	nRF	β	LF	RF	nRF
1	1.846	3.003	1.016	0.788	0.776	2.998	0.944	0.505	0.535
2	2.316	3.000	1.058	0.780	0.738	2.991	1.084	0.479	0.442
3	2.543	2.999	1.084	0.767	0.708	2.999	1.071	0.441	0.412
4	2.085	2.995	1.146	0.814	0.710	2.990	1.545	0.609	0.394
5	2.360	2.993	1.061	0.777	0.732	3.002	1.056	0.458	0.433
6	2.453	3.003	1.178	0.880	0.748	3.007	1.193	0.694	0.582

Figures 8 and 9 compare the traditional ASD and LRFD calculation methods. According to the traditional method, the value FS=1.3, as shown by the black dashed line, is often used to design the stability of the breakwater foundation with the basic load combination. This factor is used for Class II structures, as per TCCS 02: 2017/CHHVN, 2017. The six cross-sections used in this study are classified as Class II structures, with heights ranging from 5m to 25m, and a design safety factor of 1.30.

The red and purple solid lines represent the safety factors converted from nRF with $\beta_T = 3.0$ and $\beta_T = 2.0$. The green solid lines are the actual safety factors of the breakwater cross-sections.

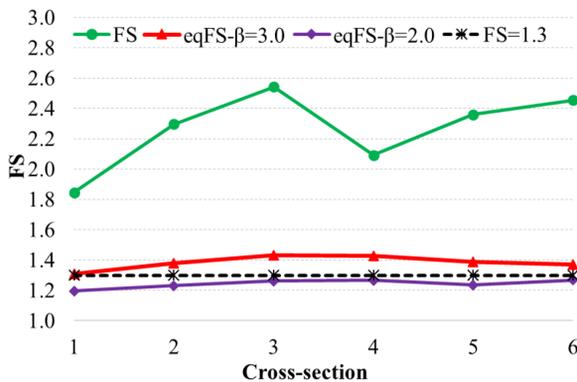


Figure 8. Comparison between design methods with COV=10%

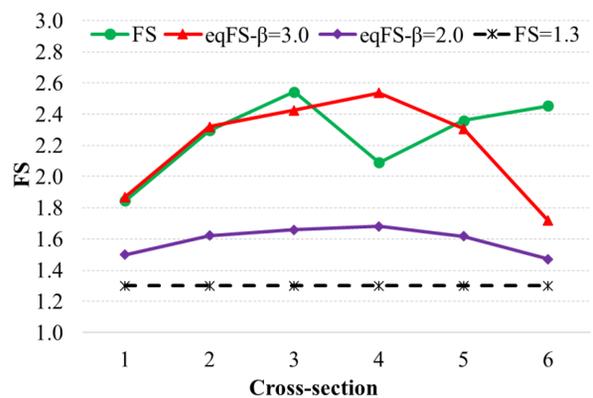


Figure 9. Comparison between design methods with COV=20%

According to the ASD method, the safety factor is determined as follows:

$$FS = \frac{R_n}{\sum Q_{ni}} \quad (7)$$

Combined with Equation (5), the resistance factor can be determined as follows:

$$RF \geq \frac{\sum LF_i Q_{ni}}{R_n} = \frac{\sum LF_i Q_{ni}}{FS \sum Q_{ni}} \quad (8)$$

Since only one load term is used, the normalized resistance factor nRF can be expressed as:

$$nRF = \frac{RF}{LF} \geq \frac{Q_n}{FS \times Q_n} = \frac{1}{FS} \quad (9)$$

Tables 4 and 5 re-evaluate the safety level according to the LRFD method compared with the current design according to the traditional method of the stability for breakwater foundation with COV=10% and 20%. If the inverse of the safety factor FS of the actual design is smaller than the resistance factor nRF, the design satisfies the condition of achieving the target reliability index as in Equation (12). It can be seen that, in most cases, the safety level of the actual design according to ASD is 40-100% higher than that of LRFD when the coefficient of variation (COV) is 10% with target reliability indices of 3.0 and 2.0. Typically, for cross-section 3,

the safety level of the actual design is higher at 77% with $\beta_T=3.0$ and 101.3% with $\beta_T=2.0$.

However, when the soil strength has a coefficient of variation (COV) of 20%, the actual safety level is significantly reduced compared to a COV of 10%, as shown in the Table 5. When designing according to the target reliability index of 3.0, some cases do not ensure the required conditions, such as cross-sections 1, 2, and 4. Cross-sections 3 and 5 have a safety level according to the actual design, which is quite close to the LRFD for $\beta_T = 3.0$, with a difference of less than 5%. However, cross-section 6 still maintains a fairly high safety level, even with a COV of 20%, which is higher than the design value of 42.8%.

Table 4: Re-evaluation by the LRFD method when COV=10%

Cross-section	FS	1/FS (ASD)	$\beta_T = 3.0$			$\beta_T = 2.0$		
			nRF (LRFD)	$\frac{nRF - (1/FS)}{(1/FS)} \times 100$	Check	nRF (LRFD)	$\frac{nRF - (1/FS)}{(1/FS)} \times 100$	Check
1	1.846	0.542	0.759	40.1%	OK	0.839	54.9%	OK
2	2.297	0.435	0.722	65.7%	OK	0.811	86.3%	OK
3	2.543	0.393	0.696	77.0%	OK	0.792	101.3%	OK
4	2.091	0.478	0.697	45.8%	OK	0.788	64.8%	OK
5	2.360	0.424	0.719	69.6%	OK	0.808	90.7%	OK
6	2.454	0.408	0.730	79.1%	OK	0.786	92.8%	OK

Table 5: Re-evaluation by the LRFD method when COV=20%

Cross-section	FS	1/FS (ASD)	$\beta_T = 3.0$			$\beta_T = 2.0$		
			nRF (LRFD)	$\frac{nRF - (1/FS)}{(1/FS)} \times 100$	Check	nRF (LRFD)	$\frac{nRF - (1/FS)}{(1/FS)} \times 100$	Check
1	1.846	0.542	0.533	-1.7%	NO	0.663	22.3%	OK
2	2.297	0.435	0.429	-1.5%	NO	0.615	41.2%	OK
3	2.543	0.393	0.412	4.8%	OK	0.603	53.3%	OK
4	2.091	0.478	0.392	-17.9%	NO	0.593	24.0%	OK
5	2.360	0.424	0.436	2.9%	OK	0.618	45.8%	OK
6	2.454	0.408	0.582	42.8%	OK	0.679	66.6%	OK

5. CONCLUSION

This paper analyzes the stability of a vertical breakwater foundation using basic load combinations with both the traditional method and the load factor and resistance

factor method. From the results of this paper, some conclusions can be drawn as follows:

- The higher the coefficient of variation of the soil strength, the lower the normalized resistance factor nRF.

– Similarly, as the target reliability index increases, the normalized resistance factor nRF decreases.

– When COV=10%, the actual design by the ASD method is more safety-oriented than LRFD, with target reliability indexes of 3.0 and 2.0.

– When COV=20%, the actual design by the ASD method, some cross-sections do not ensure the condition to achieve the target reliability index of 3.0.

From this, it can be seen that the design obtained by the traditional method, with a design value of FS = 1.3, often does not accurately reflect the reality of the parameter variations in the problem. This makes the safety level, when compared with the LRFD method, inconsistent when the coefficient of variation of the variables is different. However, to gain a complete, in-depth, and comprehensive understanding of the LRFD method applied to the stability of breakwater foundations, it is necessary to analyze a greater number of cross-sections with various load and geological conditions.

ACKNOWLEDGEMENTS

The author acknowledges Thuyloi University for funding this research.

REFERENCES

- [1] Allen, T. M., Nowak, A. S., & Bathurst, R. J. (2005). *Calibration to Determine Load and Resistance Factors for Geotechnical and Structural Design* (Issue September). Transportation Research Board. DOI: 10.17226/21978
- [2] Cherubini, C. (2000). *Reliability evaluation of shallow foundation bearing capacity on c' , ϕ' soils*. In Canadian Geotechnical Journal, 37(1), 264–269). DOI: 10.1139/cgj-37-1-264
- [3] Duncan, J. M., Wright, S. G., & Brandon, T. L. (2014). *Soil Strength and Slope Stability*. John Wiley & Sons, Inc., Hoboken, New Jersey.
- [4] Goda, Y. (2000). *Random Seas and Design of Maritime Structures*. In World Scientific, 4(1). World Scientific Publishing Co. Pte. Ltd., Singapore.
- [5] Goda, Y., & Takagi, H. (2000). *Reliability design method of caisson breakwaters with optimal wave heights*. Coastal Engineering Journal, 42(4), 357–387. DOI: 10.1016/S0578-5634(00)00018-3
- [6] Griffiths, D. V., Huang, J., & Fenton, G. A. (2009). *Influence of Spatial Variability on Slope Reliability Using 2-D Random Fields*. Journal of Geotechnical and Geoenvironmental Engineering, 135(10), 1367–1378. DOI: 10.1061/(ASCE)GT.1943-5606.0000099
- [7] KPHA. (2018). *Design Standard for Port and Harbor*.
- [8] Lin, P., & Bathurst, R. J. (2019). *Calibration of Resistance Factors for Load and Resistance Factor Design of Internal Limit States of Soil Nail Walls*. Journal of Geotechnical and Geoenvironmental Engineering, 145(1), 04018100. DOI: 10.1061/(asce)gt.1943-5606.0002005
- [9] Lin, P., & Liu, J. (2017). *Analysis of resistance factors for LFRD of soil nail walls against external stability failures*. Acta Geotechnica, 12(1), 157–169. DOI: 10.1007/s11440-016-0443-y
- [10] MLIT. (2009). *Technical Standard and Commentaries for Port and Harbor Facilities in Japan*.
- [11] Oumeraci, H., Kortenhaus, A., Allsop, W., de Groot, M., Crough, R., Vrijling, H., & Voortman, R. (2001). *Probabilistic design tools for vertical breakwaters*. In The National Audit of Violence (2003 - 2005).
- [12]. TCCS 02 : 2017/CHHVN. (2017). TCCS 02 : 2017/CHHVN: *Breakwater - Design Requirement*
- [13] TCVN 11820-6. (2023). TCVN 11820-6, 2023. *Marine Port Facilities - Design Requirements, Part 6: Breakwater*