PHÂN TÍCH ỔN ĐỊNH PANEL TRỤ CÓ CHIỀU DÀY THAY ĐỔI THEO PHƯƠNG PHÁP GALERKIN – XẤP XỈ LIÊN TIẾP

STABILITY ANALYSIS OF CYLINDRICAL PANEL WITH VARIABLE THICKNESS BY USING THE GALERKIN – APPROXIMATE METHOD

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Ngày nhận bài: 29/8/2022
Ngày gửi phản biện: 14/9/2022
Ngày chấp nhận đăng: 30/9/2022

Tóm tắt:

Nội dung chính của bài báo là khảo sát chi tiết lực tới hạn của panel trụ với chiều dày thay đổi dựa trên lý thuyết biến dạng nhỏ và lý thuyết vỏ thoải. Phương pháp Galerkin – Xấp xỉ liên tiếp được sử dụng để xác định hệ số tới hạn của panel trụ có chiều dày thay đổi. Ảnh hưởng của thông số chiều dày thay đổi đến lực tới hạn được khảo sát. Từ đó thu được các công thức xác định lực tới hạn và các kết quả số được khảo sát cho panel trụ có biên tựa đơn chịu nén.

Từ khóa: *Panel trụ, phân tích ổn định, phân tích ổn định tuyến tính.*

Abstract:

The major objective of this paper is to investigate in detail the buckling of cylindrical panel of variable thickness, based on the small deflection theory and shallow shells theory. The Galerkin - Approximate method is in use to determine the critical load factor of cylindrical panel with variable thickness. The influence of the thickness non-uniformity parameter to the buckling load is investigated. General asymptotic formulae for the buckling load are derived and numerical results are investigated for compressive simply supported panels.

Keywords: *Cylidrical panel, stability analysis, linear buckling analysis.*

1. Introduction

Stability studies of thin shells and plates with variable thickness is a major branch of modern solid mechanics. Plates and shells as structural elements are seldom perfectly flat and of uniform thickness and the amount of variable thickness can affect the load carrying capacity of structures. In recent years, the study on stability of thin structural component with initial imperfections and variable thickness has attracted attention to many researchers [1-5]. Elishakoff et al [1] studied the effect of axisymmetric imperfections in the shape of the axisymmetric buckling mode on the buckling of cylindrical shells. Mateus et al [2] studied post-buckling behavior of corroded steel plates. Nguyen and Thach [3], investigated buckling analysis of perfect cylindrical panel with variable thickness. Nguyen and Tran [4] investigated the stability of thin rectangular plates with variable thickness on a basis of the theory of thin plates of small deflections. Yeh et al [5] treated chaotic and bifurcation dynamics for a simply supported rectangular plate of thermo-mechanical coupling in large deflection. Ye Zhiming [6] introduced the nonlinear analysis and optimization of shallow shell of variable thickness.

The major objective of this paper is to investigate in detail the buckling of cylindrical panel of variable thickness, based on the small deflection theory and shallow shells theory. The Galerkin - Approximate method is in use to determine the critical load factor of cylindrical panel with variable thickness. The influence of the thickness non-uniformity parameter to the buckling load is investigated. General asymptotic formulae for the buckling load are derived and numerical results are investigated for compressive simply supported panels.

2. Governing differential equations [1,3,7]

Consider a cylindrical panel with small thickness variation loaded in its mid-plane by uniform compression N_x^0 (Fig.1). As the cylindrical panel thickness is not uniform in the x direction and radius of cylindrical panel is R ,the dimensions of the sides in the x, y directions are a and b.

Assume that: $N_{ij}^0, M_{ij}^0, \varepsilon_{ij}^0, W^0 (i,j=1,2)$ are axial forces, moment, strain in the mid-plane, deflection in the fundamental pre-buckling state.

$N_{ij}^1, M_{ij}^1, \varepsilon_{ij}^1, W^1 (i,j=1,2)$ are axial forces, moment, strain in the mid-plane, deflection in the adjacent buckling state. We have the increments of solutions at buckling:

$$N_{ij} = N_{ij}^1 - N_{ij}^0, M_{ij} = M_{ij}^1 - M_{ij}^0 = M_{ij}^1$$
$$\varepsilon_{ij} = \varepsilon_{ij}^1 - \varepsilon_{ij}^0, W = W^1 - W^0 = W^1 \quad (1)$$

It is noted that in the buckling state: $M_{ij}^0 = 0, W^0 = 0$.

We shall use the basic general equations in term of increments to solve the stability problem of cylindrical panel compressed in the direction of long edges.

ISSN 2525-2615

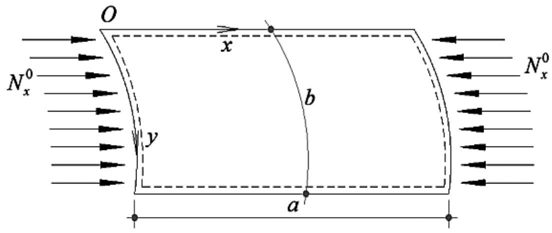

Fig.1. *Uniaxially compressed cylindrical panel*

The governing differential equations with variable thickness coefficients for the panels in general case are obtained as follows [3]:

$$h^2\nabla^2\nabla^2 F - 2h\frac{\partial h}{\partial x}\left(\frac{\partial^3 F}{\partial x^3}+\frac{\partial^3 F}{\partial x\partial y^2}\right)$$
$$+\left[2\left(\frac{\partial h}{\partial x}\right)^2 - h\frac{\partial^2 h}{\partial x^2}\right]\left(\frac{\partial^2 F}{\partial x^2}-\nu\frac{\partial^2 F}{\partial y^2}\right)$$
$$-2h\frac{\partial h}{\partial y}\left(\frac{\partial^3 F}{\partial y^3}+\frac{\partial^3 F}{\partial x^2\partial y}\right) \tag{2}$$
$$+\left[2\left(\frac{\partial h}{\partial y}\right)^2 - h\frac{\partial^2 h}{\partial y^2}\right]\left(\frac{\partial^2 F}{\partial y^2}-\nu\frac{\partial^2 F}{\partial x^2}\right)$$
$$+4(1+\nu)\frac{\partial h}{\partial x}\frac{\partial h}{\partial y}\frac{\partial^2 F}{\partial x\partial y} - 2(1+\nu)h\frac{\partial^2 h}{\partial x\partial y}\frac{\partial^2 F}{\partial x\partial y}$$
$$= Eh^3\left(-\frac{1}{R}\frac{\partial^2 W}{\partial x^2}\right)$$

$$\frac{Eh^3}{12(1-\nu^2)}\nabla^2\nabla^2 W + \frac{6Eh^2}{12(1-\nu^2)}\frac{\partial h}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}\right.$$
$$\left.+\frac{\partial^3 W}{\partial x\partial y^2}\right) + \frac{3Eh^2}{12(1-\nu^2)}\frac{\partial^2 h}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)$$
$$+\frac{6Eh}{12(1-\nu^2)}\left(\frac{\partial h}{\partial x}\right)^2\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)$$
$$+\frac{6Eh}{12(1-\nu^2)}\left(\frac{\partial h}{\partial y}\right)^2\left(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right)$$
$$+\frac{6Eh^2}{12(1-\nu^2)}\frac{\partial h}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\partial y}\right)$$
$$+\frac{3Eh^2}{12(1-\nu^2)}\frac{\partial^2 h}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right)$$
$$+\frac{Eh^2}{2(1+\nu)}\frac{\partial^2 h}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}+\frac{Eh}{(1+\nu)}\frac{\partial h}{\partial x}\frac{\partial h}{\partial y}\frac{\partial^2 W}{\partial x\partial y}$$
$$= -N_x^0\frac{\partial^2 W}{\partial x^2}+\frac{1}{R}\frac{\partial^2 F}{\partial x^2} \tag{3}$$

With: $$\nabla = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$$

Where W and F represent the displacement and the stress function, ν is Poisson's ratio, E is the modulus of elasticity, R is radius of cylindrical panel. Eq (2), Eq (3) constitute the governing differential equations for small deflections of cylindrical panel with variable thickness.

In Eqs (2-3), h is the cylindrical panel thickness, which is assumed here varying with sine function in x direction:

$$h(x) = h_0\left(1-\varepsilon\sin\frac{p\pi x}{a}\right);\ \varepsilon \ge 0 \tag{4}$$

where h_0 is the cylindrical panel thickness and ε, p are the non-dimensional parameters indicating the magnitude and wave of the thickness variation, respectively. When $x=0$ and $x=a$, one has $h(x)=h_0$, for the case $x=a/2$: one has $h(x)=h_0(1-\varepsilon)$ (Fig.2).

3. The Galerkin – Approximate method

Stress function and deflection function can be chosen:

$$W = \sum_{i=1}^{n} A_i w_i(x,y);\; F = \phi = \sum_{i=1}^{n} B_i \phi_i(x,y) \quad (5)$$

Eqs (2-3) can be rewritten:

$$\Phi = h^2\left(\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}\right)-2h\frac{dh}{dx}\left(\frac{\partial^3\phi}{\partial x^3}+\frac{\partial^3\phi}{\partial x\partial y^2}\right)+\left[2\left(\frac{dh}{dx}\right)^2-h\frac{d^2h}{dx^2}\right]\left(\frac{\partial^2\phi}{\partial x^2}-\nu\frac{\partial^2\phi}{\partial y^2}\right)+Eh^3\left(\frac{1}{R}\frac{\partial^2 W}{\partial x^2}\right)=0 \quad (6)$$

$$\Gamma = \frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+\frac{6Eh^2}{12(1-\nu^2)}\frac{dh}{dx}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)+\frac{3Eh^2}{12(1-\nu^2)}\frac{d^2h}{dx^2}\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)+\frac{6Eh}{12(1-\nu^2)}\left(\frac{dh}{dx}\right)^2\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)+N_x^0\frac{\partial^2 W}{\partial x^2}-\frac{1}{R}\frac{\partial^2\phi}{\partial x^2}=0 \quad (7)$$

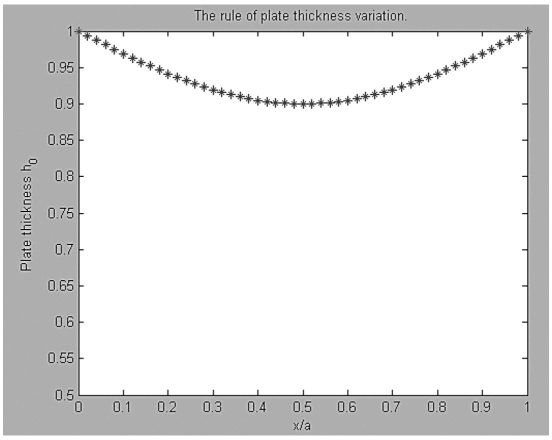

Fig.2. *Expression graph of thickness variation $h(x)$ when $\varepsilon=0.1$ and $p=1$*

Multiplying the resultant differential equation Φ by ϕ_i, Γ by w_i and intergrating over the area of the cylindrical panel, one has:

$$\iint_A \Phi\phi_i dxdy = 0 \quad i=1,2,...,n \quad (8)$$

$$\iint_A \Gamma w_i dxdy = 0 \quad i=1,2,...,n \quad (9)$$

From Eqs (8-9), the critical load is determined. In this paper, the expressions for buckling loads are determined only when $n=1$ using a single term stress and displacement series obtained in Eq (5).

Stress function and deflection function can be chosen satisfying the boundary conditions are simply supported as:

$$\phi = B_1\phi_1 = B_1\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$$
$$W = A_1 w_1 = A_1\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right) \quad (10)$$

When $n=1$, Eqs (8-9) becomes:

$$(a_1-a_2+a_3)B_1+a_4A_1=0$$
$$(b_1+b_2+b_3+b_4)A_1-b_5B_1+b_6A_1N_x^0=0 \quad (11)$$

Where:

ISSN 2525-2615

$$a_1 = \iint h^2\left(\frac{\partial^4\phi_1}{\partial x^4} + 2\frac{\partial^4\phi_1}{\partial x^2\partial y^2} + \frac{\partial^4\phi_1}{\partial y^4}\right)\phi_1 dxdy$$

$$a_2 = \iint 2h\frac{dh}{dx}\left(\frac{\partial^3\phi_1}{\partial x^3} + \frac{\partial^3\phi_1}{\partial x\partial y^2}\right)\phi_1 dxdy$$

$$a_3 = \iint\left[2\left(\frac{dh}{dx}\right)^2 - h\frac{d^2h}{dx^2}\right]\left(\frac{\partial^2\phi_1}{\partial x^2} - \nu\frac{\partial^2\phi_1}{\partial y^2}\right)\phi_1 dxdy$$

$$a_4 = \iint Eh^3\left(\frac{1}{R}\frac{\partial^2 w_1}{\partial x^2}\right)\phi_1 dxdy$$

$$b_1 = \iint \frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^4 w_1}{\partial x^4} + 2\frac{\partial^4 w_1}{\partial x^2\partial y^2} + \frac{\partial^4 w_1}{\partial y^4}\right)w_1 dxdy$$

$$b_2 = \iint \frac{6Eh^2}{12(1-\nu^2)}\frac{dh}{dx}\left(\frac{\partial^3 w_1}{\partial x^3} + \frac{\partial^3 w_1}{\partial x\partial y^2}\right)w_1 dxdy$$

$$b_3 = \iint \frac{3Eh^2}{12(1-\nu^2)}\frac{d^2h}{dx^2}\left(\frac{\partial^2 w_1}{\partial x^2} + \nu\frac{\partial^2 w_1}{\partial y^2}\right)w_1 dxdy$$

$$b_4 = \iint \frac{6Eh}{12(1-\nu^2)}\left(\frac{dh}{dx}\right)^2\left(\frac{\partial^2 w_1}{\partial x^2} + \nu\frac{\partial^2 w_1}{\partial y^2}\right)w_1 dxdy$$

$$b_5 = \iint \frac{\partial^2\phi_1}{R\partial x^2} w_1 dxdy$$

$$b_6 = \iint \frac{\partial^2 w_1}{\partial x^2} w_1 dxdy$$

From Eq (11), the critical load is determined:

$$N_x^0 = N^{cr} = -\frac{(b_1+b_2+b_3+b_4)(a_1-a_2+a_3)}{b_6(a_1-a_2+a_3)} + \frac{a_4 b_5}{b_6(a_1-a_2+a_3)} \quad (12)$$

Substituting Eq (10) into the parameters $a_1, a_2, ..., b_1, b_2, ...$ and take only to the first degree of ε, we obtain:

$$a_1 = \frac{\pi^3(a^2+b^2)^2 h_0^2(3\pi - 16\varepsilon)}{12a^3b^3}$$

$$a_2 = \frac{2\pi^3 h_0^2(a^2+b^2)\varepsilon}{3a^3b}$$

$$a_3 = \frac{2\pi^3 h_0^2(b^2-\nu a^2)\varepsilon}{3a^3b}$$

$$a_4 = \frac{\pi E h_0^3 b(8\varepsilon-\pi)}{4aR}$$

$$b_1 = \frac{\pi^3 E h_0^3(a^2+b^2)^2(\pi-8\varepsilon)}{48(1-\nu^2)a^3b^3}$$

$$b_2 = \frac{\pi^3 E h_0^3(a^2+b^2)\varepsilon}{6(1-\nu^2)a^3b}$$

$$b_3 = -\frac{\pi^3 E h_0^3(b^2+\nu a^2)\varepsilon}{6a^3b(1-\nu^2)}$$

$$b_4 = 0;\ b_5 = -\frac{\pi^2 b}{4aR};\ b_6 = -\frac{\pi^2 b}{4a}$$

Substituting $a_1, a_2, ..., b_1, b_2, ...$ into the Eq (12), the critical load is determined.

In this paper, the expression for buckling loads is determined only when $r = a/b = 1$, $\nu = 0.3$ and $n = p = 1$, using a single term displacement series obtained in Eq (5).

Eq (12) is given as the following:

$$N^{cr} = \frac{Eh_0^3(500\pi^3 + 20460\pi\varepsilon^2 - 6400\pi^2\varepsilon)}{b^2(1365\pi - 8463\varepsilon)} + \frac{Eh_0 b^2(4095\pi - 32760\varepsilon)}{12\pi^2 R^2(1365\pi - 8463\varepsilon)} \quad (13)$$

The buckling load factor of cylindrical panel with variable thickness is defined:

$$\lambda = \frac{N^{cr}}{N_0^{cr}} \quad (14)$$

Where:

N_0^{cr} is the buckling load of the cylindrical panel with constant thickness.

N^{cr} is the buckling load of the cylindrical panel with variable thickness.

In the case $\varepsilon = 0$ substituting into the Eq (13), the buckling load of the cylindrical panel with constant thickness ($h = h_0$) is given:

$$N_0^{cr} = 0.0253\frac{Eh_0 b^2}{R^2} + 3.6153\frac{Eh_0^3}{b^2} \quad (15)$$

And the buckling load factor:

$$\lambda = \frac{\dfrac{Eh_0^3(500\pi^3 + 20460\pi\varepsilon^2 - 6400\pi^2\varepsilon)}{b^2(1365\pi - 8463\varepsilon)}}{0.0253\dfrac{Eh_0 b^2}{R^2} + 3.6153\dfrac{Eh_0^3}{b^2}} + \frac{\dfrac{Eh_0 b^2(4095\pi - 32760\varepsilon)}{12\pi^2 R^2(1365\pi - 8463\varepsilon)}}{0.0253\dfrac{Eh_0 b^2}{R^2} + 3.6153\dfrac{Eh_0^3}{b^2}} \quad (16)$$

4. Numerical Analysis, Comparison and Discusion

In Eq (15), if $R \to \infty$ then the cylindrical panel with $r = a/b = 1$ will be the square plate, the same as the Timoshenko formula for a square plate is simply supported with constant thickness and $\nu = 0.3$ in form [7]:

$$N_0^{cr} = \frac{4\pi^2 Eh_0^3}{12b^2(1-\nu^2)} = 3.6153\frac{Eh_0^3}{b^2}$$

From Eq (16), when $R \to \infty$ the non-dimensional buckling load factor due to the thickness variation of the square plate the same as Nguyen and Tran in form [4]:

$$\lambda = \frac{(500\pi^3 + 20460\pi\varepsilon^2 - 6400\pi^2\varepsilon)}{3.6153(1365\pi - 8463\varepsilon)}$$

The effect of thickness variation parameter ε on the buckling load factor λ is studied. The following figures are presented for cylindrical panel with $r = a/b = 1$ and ν =0.3.

Buckling of the perfect cylindrical panel with variable thickness: the relationship between ε and λ is shown in Fig.3, in the case $R/b = 2$ and $b/h_0 = 30$.

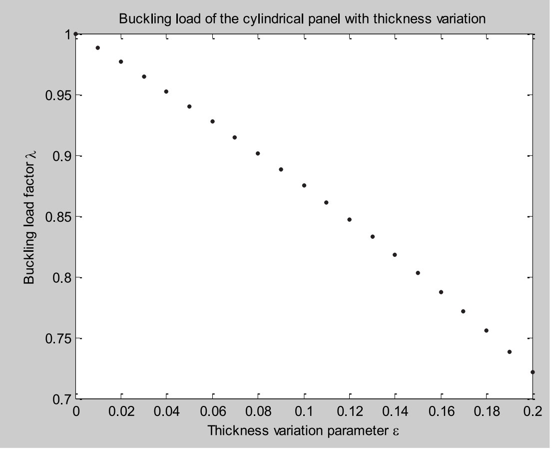

Fig.3. *Relation between ε and λ ($\frac{R}{b} = 2, \frac{b}{h_0} = 30$)*

The results obtained show that the effect of thickness variation occurs when ε is positive. Even if the amplitude of the thickness variation is as small as $\varepsilon = 0.1$, the buckling load factor of cylindrical panel $\lambda = 0.8754$ is reduced about 12% and in the case $\varepsilon = 0.2 \to \lambda = 0.7216$ is reducedabout 27.8% from its counterpart of the cylindrical panel with constant thickness, when

$$\frac{R}{b} = 2 \text{ and } \frac{b}{h_0} = 30 \text{ (Fig.3).}$$

From Eq (16), when $R/b = 2$, $b/h_0 = 30$ and ν =0.3 the non-dimensional buckling load factor due to the thickness variation of the perfect cylindrical panel with variable thickness

ISSN 2525-2615

there is a slight difference compared to Nguyen and Thach in form [3] , this is shown in Fig.4:

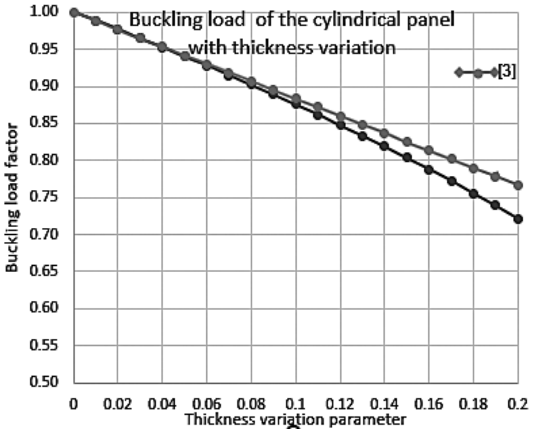

Fig.4. *Relation between* ε *and* λ ($\frac{R}{b}=2,\frac{b}{h_0}=30$ *) compared to [3]*

5. Conclusion

In this paper, based on the coupled governing stability equations for cylindrical panel in small deflection with variable thickness, a detailed study of the stability of the perfect cylindrical panel with thickness varying along only the x-axes with sine functions has been presented. The formulae for the buckling load have been derived using the Galerkin – Approximate method.

From the obtained results, the variable thickness with reduced thickness causes reduction of the load-carrying capacity of cylindrical panel structures. The influence of the thickness non-uniformity parameter to the buckling load is investigated. General asymptotic formulae for the buckling load are derived and numerical results are investigated for compressive simply supported panels.

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