THE INVESTIGATION OF NONLINEAR ABSORPTION OF STRONG ELECTROMAGNETIC WAVE IN GaAs/AlGaAs SEMI-PARABOLIC QUANTUM WELLS

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Abstract: *Quantum theory of nonlinear absorption of a strong electromagnetic wave (EMW) by confined electrons in asymmetric semi-parabolic quantum wells has been studied by using the quantum kinetic equation in assumption of electron - optical phonon scattering. The analytic expression of absorption coefficient is obtained in semi-parabolic quantum wells. The results in this case are compared with the case of the bulk semiconductors show the difference and the novelty of the results. The results numerically calculated for GaAs/AlGaAs in order to show the dependence of absorption coefficient on the photon energy of the temperature, the size of the asymmetric semi-parabolic quantum wells.*

Keywords: *Absorption coefficient, Quantum kinetic equation, asymmetric semi-parabolic quantum wells, electron - phonon scattering, electromagnetic wave.*

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1. INTRODUCTION

Quite recently, there has been considerable interest in the behavior of low-dimensional systems, since the motion of electrons is restricted,… The confinement potential of the system has changed the electron mobility significantly. The effect of electromagnetic wave absorption is an interesting topic for physicists. The absorption of electromagnetic wave in bulk semiconductor and low-dimensional system has been investigated [1]. The absorption of electromagnetic wave in two-dimensional system is calculated using the Kubo-Mori method [2],[3]. The absorption coefficient has been obtained using quantum kinetic method in low dimensional systems such as quantum wells, superlattices, quantum wires [4],[5]. In term of quantum wells, the potential confinement plays a crucial role in carrier characteristic such as the wave function and energy spectrum of the electron in the system. Considering each types of confinement potential, there are various researches and results obtained, such as infinite confinement potential [6],[7], parabolic confinement potential [8],[9]. Besides, the semi-parabolic confinement potential quantum wells have not been investigated clearly. Therefore, we choose the asymmetric semi-parabolic quantum wells to calculate the nonlinear absorption of a strong electromagnetic wave using quantum kinetic method. The paper is calculated in case of electron-optical phonon scattering with the close to threshold condition. That is still open for study.

In this paper we theoretically study the nonlinear absorption of strong electromagnetic wave (EMW) in an asymmetric semi-parabolic quantum well without magnetic field by using quantum kinetic equation method. We will consider the problem in case of the system is subjected to an EMW with the electric field vector $E = E_0 \sin\Omega t$ (E_0 and Ω are the amplitude and the frequency, respectively. The problem is considered in case of electron-optical phonon scattering with the close to threshold limitation. The purpose of this paper is to show the analytical expression of nonlinear absorption coefficient strong electromagnetic wave in semi-parabolic quantum wells. Numerical calculations are carried out with GaAs/AlGaAs semi-parabolic quantum wells.

The report is separated into four sections as follows: The first one is the introduction of the report. In the next section we construct the quantum kinetic equation for electrons confined in asymmetric semi-parabolic quantum wells. The analytical expression for the nonlinear absorption coefficient in the case of the electron – optical phonon interaction is also presented in Sec. II. The numerical results and brief review are indicated in Sec. III. Finally, conclusions are given in Sec. V.

2. THEORETICAL FRAMEWORKS

2.1. Electronic structure in an asymmetric semi-parabolic quantum wells

We consider an asymmetric semi-parabolic quantum wells subjected to the (x-y) plane with the infinite semi-parabolic confinement potential can be written as

$$V(z) = \begin{cases} 1/2\, m^* \omega_p^2 z^2, & 0 \le z \le L \\ \infty, & z < 0;\ z > L \end{cases}, \tag{1}$$

where m^* is the effective mass of the electron, ω_p is the frequency of the semi-parabolic potential in quantum wells and L is the well-width. z axis is the growth direction of the semi-parabolic quantum wells. The effective mass Hamiltonian for the electron in the system is given by $H_e = -\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + V(z).$ The eigenfuntions and eigenenergies can be obtained by solving the Schrodinger equation for H_e [10][11].

$$\begin{cases} \psi_{n,k_\perp}(z) = \varphi_n(z) U_c(r) \exp\left(ik_\perp r_\perp\right) \\ \varepsilon_{n,k_\perp} \quad = E_n + \hbar^2 / 2m^* . |k_\perp|^2 \end{cases}, \tag{2}$$

where $U_c(r)$ is the periodic part of the Bloch function, $k_\perp, r_\perp$ is the wave vector and coordinate in the (x-y) plane ($k_\perp = \sqrt{k_x^2 + k_y^2}$ and $r_\perp = \sqrt{r_x^2 + r_y^2}; r = \sqrt{r_\perp^2 + r_z^2}$). Besides, $\varphi_n(z)$ and E_n are the solutions of the one-dimensional Schrodinger equation $H_z\varphi_n(z) = E_n\varphi_n(z)$ as

$$\begin{cases} \varphi_n(z) = \sqrt{1/\beta\sqrt{\pi}2^{2n}(2n+1)!}\exp\left(-1/2\beta^2 z^2\right)H_{2n+1}(\beta z) \\ E_n \quad = \left(2n + 3/2\right)\hbar\omega_p \end{cases}, \tag{3}$$

where $\beta = \sqrt{m^*\omega_p/\hbar}$, $H_{2n+1}(\beta z)$ is the Hermite function and $n = 0,1,2,...$ refers to quantum index.

2.2. Quantum kinetic equation for electrons in asymmetric semi-parabolic quantum wells

The Hamiltonian of the electron – optical phonon system in asymmetric semi-parabolic quantum wells in the second quantization presentation can be written as:

$$\begin{aligned} H = &\sum_{n,\mathbf{k}_\perp} \varepsilon_{n,\mathbf{k}_\perp}\left(\mathbf{k}_\perp - \frac{e}{c}\mathbf{A}(t)\right)a^+_{n,\mathbf{k}_\perp}a_{n,\mathbf{k}_\perp} + \sum_{\mathbf{q}}\hbar\omega_{\mathbf{q}}\left(b^+_{\mathbf{q}}b_{\mathbf{q}} + \frac{1}{2}\right) \\ &+ \sum_{n,n'}\sum_{\mathbf{k}_\perp,\mathbf{q}} C(\mathbf{q})I_{n,n'}(q_z)a^+_{n,\mathbf{k}_\perp+\mathbf{q}}a_{n,\mathbf{k}_\perp}\left(b^+_{\mathbf{q}} + b_{\mathbf{q}}\right) \end{aligned} \tag{4}$$

where $\varepsilon_{n,\mathbf{k}_\perp}$ is energy of electron (2), $\mathbf{k}_\perp, \mathbf{q}$ respectively are wave vectors of electron, phonon, $|n,\mathbf{k}_\perp\rangle, |n',\mathbf{k}_\perp + \mathbf{q}\rangle$ are electron states before and after scattering, respectively. $a^+_{n,\mathbf{k}_\perp}, a_{n,\mathbf{k}_\perp}\left(b^+_{\mathbf{q}}, b_{\mathbf{q}}\right)$ are the creation and annihilation operators of the electron (phonon). $|C(q)|^2 = \frac{2\pi e^2\hbar\omega_o}{V_o\kappa\mathbf{q}^2}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)$ is the electron - optical phonon interaction constant. Here, e, ω_o, V_o, κ refer to the electron charge, optical phonon frequency, the normalized volume and deformation potential constant of the quantum wells, respectively. χ_∞ and χ_0 are the high and static-frequency dielectric constants. $\mathbf{A}(t)$ is the vector potential of an electromagnetic wave. $I_{n,n'}(q_z)$ is the form factor in the semi-parabolic quantum wells is given as follow

$$I_{n,n'}(q_z) \equiv I_{n,n'} = \frac{2}{L}\int_0^L \psi^*_n(z)\psi_{n'}(z)\exp(iq_z z)dz, \tag{5}$$

When a high-frequency electromagnetic wave is applied to the system in the y direction with electric field vector $\mathbf{E} = \mathbf{E}_0 \sin\Omega t$ (where $\mathbf{E}_0$ and Ω are the amplitude and the frequency of the electromagnetic wave), the quantum kinetic equation of average number of electron $f_{n,\mathbf{k}_\perp} = \left\langle a^+_{n,\mathbf{k}_\perp}a_{n,\mathbf{k}_\perp}\right\rangle_t$ is:

$$i\hbar\frac{\partial f_{n,\mathbf{k}_\perp}(t)}{\partial t}=\left\langle\left[a^+_{n,\mathbf{k}_\perp}a_{n,\mathbf{k}_\perp},H\right]\right\rangle_t, \tag{6}$$

Starting from the Hamiltonian (4) and using the commutative relations of the creation and the annihilation operators, we obtain the quantum kinetic equation for electrons in semi-parabolic quantum wells:

$$\begin{aligned}f_{n,\mathbf{k}_\perp}(t)=&\frac{1}{\hbar\Omega}\sum_{n,n'}\sum_{\mathbf{k}_\perp,\mathbf{q}}|C(\mathbf{q})|^2\sum_{k,\ell=-\infty}^{+\infty}J_{s+\ell}\left(\frac{e\mathbf{E}_0\cdot\mathbf{q}}{m\Omega^2}\right)J_s\left(\frac{e\mathbf{E}_0\cdot\mathbf{q}}{m\Omega^2}\right)\frac{\exp(-i\ell\Omega t)}{\ell}\\&\times\left[\frac{\bar f_{n,\mathbf{k}_\perp}N_\mathbf{q}-\bar f_{n',\mathbf{k}_\perp+\mathbf{q}}\left(N_\mathbf{q}+1\right)}{\varepsilon_{n',\mathbf{k}_\perp+\mathbf{q}}-\varepsilon_{n,\mathbf{k}\perp}-\hbar\omega_\mathbf{q}-\ell\hbar\Omega+i\hbar\delta}+\frac{\bar f_{n,\mathbf{k}_\perp}\left(N_\mathbf{q}+1\right)-\bar f_{n',\mathbf{k}_\perp+\mathbf{q}}N_\mathbf{q}}{\varepsilon_{n',\mathbf{k}_\perp+\mathbf{q}}-\varepsilon_{n,\mathbf{k}_\perp}+\hbar\omega_\mathbf{q}-\ell\hbar\Omega+i\hbar\delta}\right.\\&\left.+\frac{\bar f_{n',\mathbf{k}_\perp-\mathbf{q}}N_\mathbf{q}-\bar f_{n,\mathbf{k}_\perp}\left(N_\mathbf{q}+1\right)}{\varepsilon_{n,\mathbf{k}_\perp}-\varepsilon_{n',\mathbf{k}_\perp-\mathbf{q}}-\hbar\omega_\mathbf{q}-\ell\hbar\Omega+i\hbar\delta}-\frac{\bar f_{n',\mathbf{k}_\perp-\mathbf{q}}\left(N_\mathbf{q}+1\right)-\bar f_{n,\mathbf{k}_\perp}N_\mathbf{q}}{\varepsilon_{n,\mathbf{k}_\perp}-\varepsilon_{n',\mathbf{k}_\perp-\mathbf{q}}+\hbar\omega_\mathbf{q}-\ell\hbar\Omega+i\hbar\delta}\right]\end{aligned} \tag{7}$$

where $N_\mathbf{q}$ is the time-independent component of the phonon distribution function, $J_s(x)$ is the Bessel function and the quantity δ is infinitesimal and appears due to the assumption of an adiabatic interaction of the electromagnetic wave.

2.3. The nonlinear absorption coefficient of strong electromagnetic wave in asymmetric semi-parabolic quantum wells

The carrier current density formula in semi-parabolic quantum wells takes the form :

$$\mathbf{J}_\perp(t)=\frac{e\hbar}{m^*}\sum_{n,\mathbf{k}_\perp}\left[\mathbf{k}_\perp-\frac{e}{\hbar c}\mathbf{A}(t)\right]f_{n,\mathbf{k}_\perp}(t), \tag{8}$$

Because the motion of electrons is confined along the z direction in quantum wells, we only consider the in plane (x-y) current density vector of electrons $\mathbf{J}(t)$. By using the electron - optical phonon interaction factor in Eq. (4) and the Bessel function, from the expression for the current density vector Eq. (8) we establish the nonlinear absorption coefficient of the electromagnetic wave:

$$\alpha=\frac{8\pi}{c\sqrt{\chi_\infty}E_0^2}\left\langle\mathbf{J}(t)\cdot\mathbf{E}_0\sin\Omega t\right\rangle_t, \tag{9}$$

where $\delta(x)$ is the Dirac delta function. When the temperature of the system is high (T > 50K), the electron – optical phonon interaction is higher than other interactions. We consider the electron gas to be non-degenerate $\bar f_{n,k_\perp}=f_o^*\exp\left(\frac{\varepsilon_{n,k_\perp}}{k_BT}\right)$ with

$f_o^* = \dfrac{f_o e^{3/2}\pi^{3/2}\hbar^3}{V_o m^{*3/2}(k_B T)^{3/2}}$. Let assume that phonon is not dispersive means is the optical phonon frequency non-dispersion: $\hbar\omega_{\mathbf{q}} = \hbar\omega_0$ and $N_{\mathbf{q}} = \dfrac{k_B T}{\hbar\omega_0}$.

After some calculation, we obtain the expression for absorption coefficient:

$$\alpha = \frac{e^2 f_o^* \left(k_B T\right)^2}{4\pi c\sqrt{\chi_\infty}\rho\omega_p^2\Omega^3 m^{*2} d}\left(\frac{1}{\chi_\infty} - \frac{1}{\chi_0}\right)\sum_{n,n'}\left|I_{n,n'}\right|^2$$
$$\times\left\{\left[\exp\left(\frac{\hbar\Omega - \hbar\omega_0}{2k_B T}\right) - 1\right]\left[\frac{3e^2E_0^2 k_B T}{8m^{*2}\Omega^4}\left[\frac{\omega_0 - \Omega}{2k_B T} + \frac{\pi^2\left(n'^2 - n^2\right)}{2m^* d^2} + 1\right] + 1\right]\right. \quad , (10)$$
$$\left. + \left[\exp\left(\frac{\hbar\Omega + \hbar\omega_0}{2k_B T}\right) - 1\right]\left[\frac{3e^2E_0^2 k_B T}{8m^{*2}\Omega^4}\left[-\frac{\omega_0 + \Omega}{2k_B T} + \frac{\pi^2\left(n'^2 - n^2\right)}{2m^* d^2} + 1\right] + 1\right]\right\}$$

Eq. (10) is different compared to the symmetric parabolic quantum wells case, due to the analytical difference in eigenfuntions Ψ and eigenenergies ε which are characterized of the potential confinement in asymmetric semi-parabolic quantum wells. In the following, we will give physical conclusions to above results by carrying out a numerical evaluation and a graphic consideration using a computational method.

2.4. Numerical results and discussion

In this section, we give a deeper insight to the absorption coefficient for the case of a specific GaAs/AlGaAs asymmetric semi-parabolic quantum wells. For this section, the parameters used in computational calculations are as follows [2, 3].

2.4.1. The dependence of absorption coefficient on temperature

The dependence of absorption coefficient on temperature of the quantum well due to optical phonon scattering is illustrated in Fig.1. As can be seen from this figure that the absorption coefficient α depends significantly and non-linearly on temperature of the system. We can see that absorption coefficient increases gradually when temperature is low ($T < 100K$) that is a good agreement with the results obtained in case of parabolic quantum wells [12] or doped superlattices [3][6]. If temperature is high ($T > 100K$) the value of the absorption coefficient grows up linearly as the temperature increases. This is a difference compared to case of symmetric quantum wells while the absorption coefficient remains stably in high temperature domain. In addition, the absorption coefficient reaches a peak at 140K - 170K in case of doped superlattices [6]. Although the absorption coefficients both increase nonlinearly on temperature but different in value. That might because of the dissimilarity between analytical expression of the energy spectrums and wave functions of electrons in semi-parabolic quantum well and in others symmetrical quantum wells and two-dimensional systems.

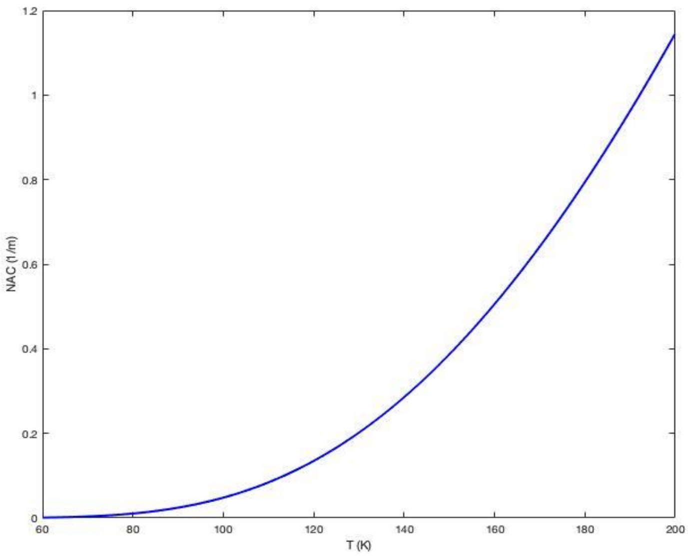

Fig. 1. The dependence of α on Temperature

2.4.2. The dependence of absorption coefficient on size of the quantum wells

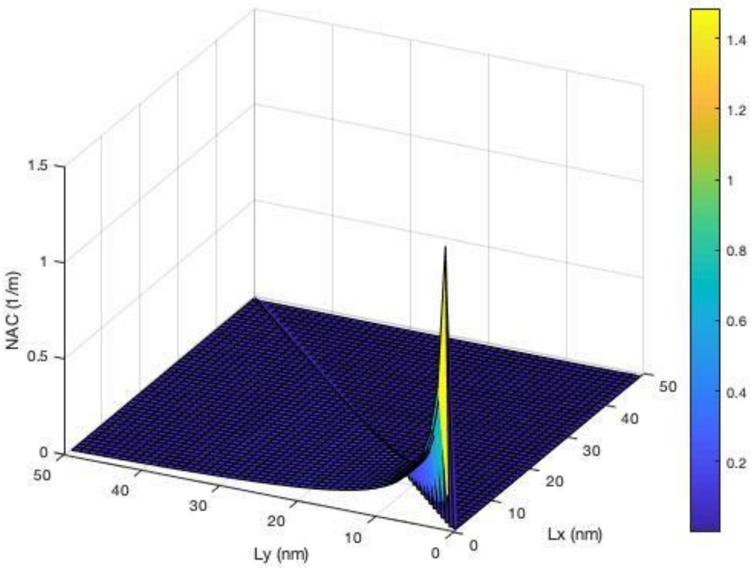

Fig. 2. The dependence of α on the well-width L.

In order to analyze the physical expression of absorption coefficient in quantum wells parameters, we investigate and graph the influence of absorption coefficient on the well-width L. In this case, we set the temperature at 100K and consider the square quantum well $L_x = L_y = L$. As can be seen from the graph, the absorption coefficient also depends nonlinear on well-width. The value of α is big when the quantum well is narrow (L < 10nm).

Then, the more well-width increases the more absorption coefficient decreases. This is due to the quantum size effect, when the size of the system increase to the bulk case, the quantum effect vanishes and it leads to the decrease in the value of coefficient. Besides, the EMW along y-axis makes the absorption coefficient depends strongly on well-width along direction y but not remarkable on well-width along x direction. This results would give a good suggestion for further experiments in the future. We can use this effect as one of the criteria for semi-parabolic quantum wells fabrication technology.

3. CONCLUSION

In this work, we have studied the nonlinear absorption coefficient of a strong electromagnetic wave in a GaAs/AlGaAs asymmetric semi-parabolic quantum well. By using the quantum kinetic equation method, we obtained the expression of the absorption coefficient due to the propagation of EMW in the system. Numerical results show that, towards lower temperature domain, the absorption coefficient remains stable and increases strongly when higher temperature ($T > 100K$). In addition, we investigate the impact of the size of quantum wells on absorption coefficient. The absorption coefficient decreases nonlinearly on the well-width which is a good agreement with the quantum size effect. Then, the influence of the electromagnetic wave is also mentioned with the difference in each direction of the well-width. These theoretical results show a potential of the asymmetric semi-parabolic quantum wells as a new two – dimensional material gas for designing optical devices as a promising alternative to other traditional semiconductors.

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NGHIÊN CỨU VỀ HẤP THỤ PHI TUYẾN SÓNG ĐIỆN TỪ MẠNH TRONG HỐ LƯỢNG TỬ BÁN PARABOL BẤT ĐỐI XỨNG GaAs/AlGaAs

Tóm tắt: *Nghiên cứu lý thuyết lượng tử hấp thụ phi tuyến sóng điện từ mạnh trong hố lượng tử bán parabol bất đối xứng bằng phương trình động lượng tử với giả thiết cơ chế tán xạ electron-phonon quang. Thu được biểu thức giải tích cho hệ số hấp thụ trong hố lượng tử bán parabol bất đối xứng. Các kết quả là mới mẻ và được so sánh với trường hợp trong bán dẫn thấp chiều truyền thống để thấy sự khác biệt. Kết quả giải tích cũng được đưa vào tính toán số với hố lượng tử GaAs/AlGaAs, thu được đồ thị sự phụ thuộc của hệ số hấp thụ phi tuyến vào nhiệt độ và độ rộng của hố lượng tử.*

Từ khóa: *Hệ số hấp thụ, phương trình động lượng tử, hố lượng tử bán parabol bất đối xứng, tán xạ electron-phonon, sóng điện từ.*