In this article, some probabilistic models for AI are presented. Using Bayesian models and linear regression models we could make predictions in medicine as well as production and business. At the same time given models claim effective applications of AI technology.

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MỘT SỐ MÔ HÌNH XÁC SUẤT CHO TRÍ TUỆ NHÂN TẠO

Tóm tắt: *Trong bài báo này chúng ta sẽ trình bày một vài mô hình xác suất được áp dụng trong AI. Đó là các mô hình Bayes và hồi quy tuyến tính. Các ví dụ cũng được đưa ra để minh họa cho từng mô hình.*

Từ khóa: *Mô hình xác suất, trí tuệ nhân tạo, công thức Bayes, mạng Bayes, mô hình hồi quy tuyến tính.*

GAUGE SECTOR OF THE MINIMAL 3-3-1-1 MODEL

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Abstract: *In this work, we consider the gauge boson sector of the minimal 3-3-1-1 model. We identify the standard model gauge bosons and the new gauge particles. We prove that the number of the massive gauge bosons match those of the Goldstone bosons, leaving only photon and gluon massless. The dark matter vector is discussed.*

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1. INTRODUCTION

The standard model cannot explain the neutrino masses and the dark matter abundance of the universe [1]. The new approaches based on the 3-3-1-1 symmetry not only address such questions but also yield interested novel consequences [2, 3].

Yet, the gauge sector of the minimal 3-3-1-1 model was not investigated. In this work, we diagonalize the relevant gauge sector and prove that it contains appropriate gauge spectrum of the standard model. Additionally, the new gauge bosons are obtained, attracting attention at the current colliders.

In Sec. II, we introduce the minimal 3-3-1-1 model. In Sec. III, we diagonalize the gauge sector, identifying relevant mass spectrum. We conclude this work in Sec. IV.

2. CONTENT

2.1. Minimal 3-3-1-1 model

The gauge symmetry is given by,

$$SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes U(1)_N, \quad (1)$$

which is an extension of the 3-3-1 model [6–8] by supposing that $B - L$ closes algebraically with $SU(3)_L$ [2, 3]. The electric charge is obtained as

$$Q = T_3 - \sqrt{3}T_8 + X, \tag{2}$$

corresponding to the minimal fermion content of the minimal 3-3-1 model [6]. For the minimal fermion content, the baryon minus lepton number is embedded as

$$\boldsymbol{B - L = -\left(4/\sqrt{3}\right)T_8 + N} \tag{3}$$

After symmetry breaking, a matter parity arises to be

$$\mathrm{P} = (-1)^{3(\mathrm{B-L})+\mathbf{2s}} \tag{4}$$

Under the 3-3-1-1 symmetry, the mentioned fermion representations are

$$\psi_{aL} \equiv \begin{pmatrix} \nu_{aL} \\ e_{aL} \\ e^c_{aR} \end{pmatrix} \sim (1,3,0,-1/3), \quad \nu_{aR} \sim (1,1,0,-1), \tag{5}$$

$$Q_{\alpha L} \equiv \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ J_{\alpha L} \end{pmatrix} \sim (3,3^*,-1/3,-1/3), \quad Q_{3L} \equiv \begin{pmatrix} u_{3L} \\ d_{3L} \\ J_{3L} \end{pmatrix} \sim (3,3,2/3,1), \tag{6}$$

$$u_{aR} \sim (3,1,2/3,1/3), \quad d_{aR} \sim (3,1,-1/3,1/3), \tag{7}$$

$$J_{\alpha R} \sim (3,1,-4/3,-5/3), J_{3R} \sim (3,1,5/3,7/3), \tag{8}$$

which is sufficient for the anomaly cancellation [4, 5].

To break the 3-3-1-1 symmetry, we introduce the following scalars,

$$\eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^+ \end{pmatrix} \sim (1,3,0,2/3), \quad \rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^{++} \end{pmatrix} \sim (1,3,1,2/3), \tag{9}$$

$$\chi = \begin{pmatrix} \chi_1^- \\ \chi_2^{--} \\ \chi_3^0 \end{pmatrix} \sim (1,3,-1,-4/3), \quad \phi \sim (1,1,0,2), \tag{10}$$

with vacuum expectative values,

$$\langle\eta\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}, \quad \langle\rho\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix}, \quad \langle\phi\rangle = \frac{1}{\sqrt{2}}\Lambda \tag{11}$$

The total Lagrangian is given by

$$\begin{aligned}\mathcal{L} &= \sum_F \bar{F} i\gamma^\mu D_\mu F + \sum_S (D^\mu S)^\dagger (D_\mu S) - V + \mathcal{L}_Y \\ &-\frac{1}{4}(G_{i\mu\nu}G_i^{\mu\nu} + A_{i\mu\nu}A_i^{\mu\nu} + B_{\mu\nu}B^{\mu\nu} + C_{\mu\nu}C^{\mu\nu})\end{aligned} \tag{12}$$

where F and S indicate to the fermion multiplets and scalar multiplets, respectively. The covariant derivative takes the form,

$$D_\mu = \partial_\mu + ig_s t_i G_{i\mu} + igT_i A_{i\mu} + ig_X X B_\mu + ig_N N C_\mu, \tag{13}$$

where (t_i, T_i, X, N), (g_s, g, g_X, g_N), and (G_i, A_i, B, C) denote the generators, gauge couplingconstants, and gauge bosons of the 3-3-1-1 groups, respectively. The field strength tensors are

$$Gi\mu v = \partial\mu Giv - \partial vGi\mu - gsfijkGj\mu Gkv, \tag{14}$$

$$Ai\mu v = \partial\mu Aiv - \partial vAi\mu - gfijkAj\mu Akv, \tag{15}$$

$$B\mu v = \partial\mu Bv - \partial vB\mu, \qquad C\mu v = \partial\mu Cv - \partial vC\mu, \tag{16}$$

where f_{ijk} is the fine structure constant of SU(3) group.

Since the Yukawa Lagrangian and scalar potential are not investigated in this work, they are being skipped.

2.2. Gauge spectrum

In this section, let us consider the gauge boson spectrum. The gauge bosons obtain masses when the scalar fields develop VEVs. Therefore, the mass Lagrangian of the gauge bosons is given by

$$\mathcal{L}_{mass}^{gause} = \sum_{S=\eta,\rho,\chi,\phi} (D_\mu\langle S\rangle) + (D^\mu\langle S\rangle) \tag{17}$$

Where

$$\begin{aligned} D_\mu\langle\eta\rangle &= (\partial_\mu + ig_s G_{i\mu}t_i + igA_{i\mu}T_i + ig_X B_\mu X + ig_N C_\mu N)\langle\eta\rangle \\ &= \frac{igu}{2\sqrt{2}}\begin{pmatrix} A_{3\mu} + \frac{1}{\sqrt{3}}A_{8\mu} + \frac{4}{3}t_N C_\mu \\ \sqrt{2}W_\mu^- \\ \sqrt{2}X_\mu^+ \end{pmatrix}, \end{aligned} \tag{18}$$

Where $t_N = \frac{g_N}{g}$ (19)

And

$$W_\mu^\pm = \frac{A_{1\mu} \mp iA_{2\mu}}{\sqrt{2}}, \quad X_\mu^\pm = \frac{A_{4\mu} \pm iA_{5\mu}}{\sqrt{2}}, \quad Y_\mu^{\pm\pm} = \frac{A_{6\mu} \pm iA_{7\mu}}{\sqrt{2}}$$

$$D_\mu\langle\rho\rangle = \frac{igv}{2\sqrt{2}} \begin{pmatrix} \sqrt{2}W_\mu^+ \\ -A_{3\mu} + \frac{1}{\sqrt{3}}A_{8\mu} + 2t_X B_\mu + \frac{4}{3}t_N C_\mu \\ \sqrt{2}Y_\mu^{++} \end{pmatrix}, \quad (20)$$

in which $\boldsymbol{t_X = \frac{g_X}{g}}$

$$D_\mu\langle\chi\rangle = \frac{igw}{2\sqrt{2}} \begin{pmatrix} \sqrt{2}X_\mu^- \\ \sqrt{2}Y_\mu^{--} \\ -\frac{2}{\sqrt{3}}A_{8\mu} - 2t_X B_\mu - \frac{8}{3}t_N C_\mu \end{pmatrix} \quad (21)$$

$$D_\mu\langle\phi\rangle = \sqrt{2}igt_N C_\mu \Lambda. \quad (22)$$

Combining (17), (18), (20), (21) and (22), we have

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{gause}} &= \frac{g^2u^2}{8}[2W_\mu^+W^{\mu-} + 2X_\mu^+X^{\mu-} + (A_{3\mu} + \frac{1}{\sqrt{3}}A_{8\mu} + \frac{4}{3}t_N C_\mu)^2] \\ &+ \frac{g^2v^2}{8}[2W_\mu^+W^{\mu-} + 2Y_\mu^{++}Y^{\mu--} + (-A_{3\mu} + \frac{1}{\sqrt{3}}A_{8\mu} + 2t_X B_\mu + \frac{4}{3}t_N C_\mu)^2] \\ &+ \frac{g^2w^2}{8}[2X_\mu^+X^{\mu-} + 2Y_\mu^{++}Y^{\mu--} + (-\frac{2}{\sqrt{3}}A_{8\mu} - 2t_X B_\mu - \frac{8}{3}t_N C_\mu)^2] \\ &+ 2g^2t_N^2\Lambda^2 C_\mu C^\mu \\ &= \mathcal{L}_{mass}^{changed} + \mathcal{L}_{mass}^{neutral} \end{aligned} \quad (23)$$

The mass Lagrangian of the charged scalars is written as

$$\begin{aligned} \mathcal{L}_{\text{mass}}^{\text{gause}} &= \frac{g^2u^2}{8}(2W_\mu^+W^{\mu-} + 2X_\mu^+X^{\mu-}) \\ &+ \frac{g^2v^2}{8}(2W_\mu^+W^{\mu-} + 2Y_\mu^{++}Y^{\mu--}) \\ &+ \frac{g^2w^2}{8}(2X_\mu^+X^{\mu-} + 2Y_\mu^{++}Y^{\mu--}) \\ &= \frac{g^2}{4}(u^2+v^2)W_\mu^+W^{\mu-} + \frac{g^2}{4}(u^2+w^2)X_\mu^+X^{\mu-} + \frac{g^2}{4}(v^2+w^2)Y_\mu^{++}Y^{\mu--} \end{aligned} \quad (24)$$

The gauge bosons $W^\pm$, $X^\pm$ and $Y^{\mp\mp}$ by themselves are physical fields with the respective masses,

$$m_W^2 = \frac{g^2}{4}(u^2+v^2) \quad m_X^2 = \frac{g^2}{4}(u^2+w^2), \quad m_Y^2 = \frac{g^2}{4}(v^2+w^2) \quad (25)$$

Note that the mass of W boson implies $u^2 + v^2 = v_{\rm w}^2 =$ (246 GeV)^2. Due to the conditions, $w \gg u \sim v$, we have $m_W \ll m_X \simeq m_Y \simeq$ TeV. The X and Y are new gauge bosons with the large masses given in w scale.

The mass Lagrangian of the neutral scalar fields is written as

$$\begin{aligned}\mathcal{L}_{\rm mass}^{\rm neutral} &= (u^2+v^2)\frac{g^2}{8}A_{3\mu}^2 + (u^2+v^2+4w^2)\frac{g^2}{24}A_{8\mu}^2 \\ &+(v^2+w^2)\frac{g^2}{2}t_X^2B_\mu^2 + (u^2+v^2+4w^2+9\Lambda^2)\frac{2g^2}{9}t_N^2C_\mu^2 \\ &+(u^2-v^2)\frac{g^2}{4\sqrt{3}}A_{3\mu}A_{8\mu} - v^2\frac{g^2}{2}t_XA_{3\mu}B_\mu + (u^2-v^2)\frac{g^2}{3}t_NA_{3\mu}C_\mu \\ &+(v^2+2w^2)\frac{g^2}{2\sqrt{3}}t_XA_{8\mu}B_\mu + (u^2+v^2+4w^2)\frac{g^2}{3\sqrt{3}}t_NA_{8\mu}C_\mu \\ &+(v^2+2w^2)\frac{2g^2}{3}t_Xt_NB_\mu C_\mu \\ &= \frac{1}{2}\begin{pmatrix} A_{3\mu} & A_{8\mu} & B_\mu & C_\mu\end{pmatrix} M^2 \begin{pmatrix} A_3^\mu \\ A_8^\mu \\ B^\mu \\ C^\mu\end{pmatrix}.\end{aligned} \quad (26)$$

The squared mass matrix of neutral gauge bosons is found to be,

$$M^2 = \begin{pmatrix} (u^2+v^2)\frac{g^2}{4} & (u^2-v^2)\frac{g^2}{4\sqrt{3}} & -v^2\frac{g^2}{2}t_X & (u^2-v^2)\frac{g^2}{3}t_N \\ (u^2-v^2)\frac{g^2}{4\sqrt{3}} & (u^2+v^2+4w^2)\frac{g^2}{12} & (v^2+2w^2)\frac{g^2}{2\sqrt{3}}t_X & (u^2+v^2+4w^2)\frac{g^2}{3\sqrt{3}}t_N \\ -v^2\frac{g^2}{2}t_X & (v^2+2w^2)\frac{g^2}{2\sqrt{3}}t_X & (v^2+w^2)g^2t_X^2 & (v^2+2w^2)\frac{2g^2}{3}t_Xt_N \\ (u^2-v^2)\frac{g^2}{3}t_N & (u^2+v^2+4w^2)\frac{g^2}{3\sqrt{3}}t_N & (v^2+2w^2)\frac{2g^2}{3}t_Xt_N & (u^2+v^2+4w^2+9\Lambda^2)\frac{4g^2}{9}t_N^2 \end{pmatrix}. \quad (27)$$

The neutral gauge bosons ($A_{3\mu}$,$A_{8\mu}$,B_μ,C_μ) mix via the mass matrix M^2. It is easily checked that M^2 has a zero eigenvalue (the photon mass) and respective eigenstate independent of VEVs as follows,

$$m_A^2 = 0, \quad A_\mu = \frac{t_X}{\sqrt{1+4t_X^2}}A_{3\mu} - \frac{\sqrt{3}t_X}{\sqrt{1+4t_X^2}}A_{8\mu} + \frac{1}{\sqrt{1+4t_X^2}}B_\mu \quad (28)$$

q where, $t_X = g_X/g = s_W/\sqrt{1-4s_W^2}$, with $s_W = e/g$ is the sine of the Weinberg angle. The Eq (28) can be rewritten as

$$m_A^2 = 0, \quad A_\mu = \frac{t_X}{\sqrt{1+4t_X^2}}A_{3\mu} - \frac{\sqrt{3}t_X}{\sqrt{1+4t_X^2}}A_{8\mu} + \frac{1}{\sqrt{1+4t_X^2}}B_\mu \quad (28)$$

$$A_\mu = s_WA_{3\mu} + c_W(-\sqrt{3}t_WA_{8\mu} + \sqrt{1-3t_W^2}B_\mu \quad (29)$$

while the terms in the parentheses are the weak hypercharge, $\boldsymbol{Y = -\sqrt{3}t_W A_{8\mu} + \sqrt{1-3t_W^2}B_\mu}$ *(or* $\boldsymbol{Y = -\sqrt{3}T_8 + X}$*)*

The remaining states of the boson in the standard model Z and the new bosons Z′ and C are

$$Z_\mu = c_W A_{3\mu} - s_W(-\sqrt{3}t_W A_{8\mu} + \sqrt{1-3t_W^2}B_\mu \tag{30}$$

$$Z'_\mu = \sqrt{1-3t_W^2}A_{8\mu} + \sqrt{3}t_W B_\mu), \tag{31}$$

$$Z_N = C_\mu. \tag{32}$$

The mass matrix M^2 can be diagonalized via several steps. In the first step, we change the basis to *$(A_{3\mu}, A_{8\mu}, B_\mu, C_\mu) \to (A, Z, Z^0, C)$*,

$$\begin{pmatrix} A_{3\mu} \\ A_{8\mu} \\ B_\mu \\ C_\mu \end{pmatrix} = U_1 \begin{pmatrix} A \\ Z \\ Z' \\ C \end{pmatrix}, \quad U_1 = \begin{pmatrix} s_W & c_W & 0 & 0 \\ -\sqrt{3}s_W & \sqrt{3}s_W t_W & \sqrt{1-3t_W^2} & 0 \\ c_W\sqrt{1-3t_W^2} & -s_W\sqrt{1-3t_W^2} & \sqrt{3}t_W & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{33}$$

In this new basis, the mass matrix M^2 becomes

$$M'^2 = U_1^T M^2 U_1 = \begin{pmatrix} 0 & 0 \\ 0 & M_s'^2 \end{pmatrix} \tag{34}$$

Where

$$M_s'^2 = U_1^T M^2 U_1 = \begin{pmatrix} m_Z^2 & m_{ZZ'}^2 & m_{ZC}^2 \\ m_{ZZ'}^2 & m_{Z'}^2 & m_{Z'C}^2 \\ m_{ZC}^2 & m_{Z'C}^2 & m_C^2 \end{pmatrix} = g^2 \times$$

$$\begin{pmatrix} \frac{u^2+v^2}{4c_W^2} & \frac{(1-4s_W^2)u^2-(1+2s_W^2)v^2}{4\sqrt{3}c_W^2\sqrt{1-4s_W^2}} & \frac{(u^2-v^2)t_N}{3c_W} \\ \frac{(1-4s_W^2)u^2-(1+2s_W^2)v^2}{4\sqrt{3}c_W^2\sqrt{1-4s_W^2}} & \frac{(1-4s_W^2)^2u^2+(1+2s_W^2)^2v^2+4c_W^4w^2}{12(1-4s_W^2)c_W^2} & \frac{t_N[(1-4s_W^2)u^2+(1+2s_W^2)v^2+4c_W^2w^2]}{3\sqrt{3}c_W\sqrt{1-4s_W^2}} \\ \frac{(u^2-v^2)t_N}{3c_W} & \frac{t_N[(1-4s_W^2)u^2+(1+2s_W^2)v^2+4c_W^2w^2]}{3\sqrt{3}c_W\sqrt{1-4s_W^2}} & \frac{4}{9}(u^2+v^2+4w^2+9\Lambda^2)t_N^2 \end{pmatrix} \tag{35}$$

In the limit, $u, v \ll \Lambda, w$, we have $m_Z^2, m_{ZZ'}^2, m_{ZC}^2 \ll m_{Z'}^2, m_{Z'C}^2, m_C^2$. Hence, in the second step, the mass matrix M^{02} can be diagonalized by using seesaw formula to separate the light state (Z) from the heavy states (Z^0,C). We denote the new basis as (A, Z_1, Z'_1, C_1) so that A,Z_1 are physical fields and decoupled while the rest mix,

$$\begin{pmatrix} A \\ Z \\ Z' \\ C \end{pmatrix} = U_2 \begin{pmatrix} A \\ Z_1 \\ Z'_1 \\ C_1 \end{pmatrix}, \quad M''^2 = U_2^T M'^2 U_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_{Z_1}^2 & 0 \\ 0 & 0 & M_s''^2 \end{pmatrix}, \tag{36}$$

Where

$$M_s''^2 = \begin{pmatrix} m_{Z'}^2 & m_{Z'C}^2 \\ m_{Z'C}^2 & m_C^2 \end{pmatrix}, \quad m_{Z_1}^2 \simeq m_Z^2 - \mathcal{E} \begin{pmatrix} m_{ZZ'}^2 \\ m_{ZC}^2 \end{pmatrix} \tag{37}$$

The m_{Z1} is the mass of Z_1 light state, while $M_s''^2$ is the mass sub-matrix of Z'_1, C_1 heavy states. By the virtue of seesaw approximation, we have

$$U_2 \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \mathcal{E} \\ 0 & -\mathcal{E}^T & 1 \end{pmatrix}, \quad \mathcal{E} \equiv \begin{pmatrix} m_{ZZ'}^2 & m_{ZC}^2 \end{pmatrix} \begin{pmatrix} m_{Z'}^2 & m_{Z'C}^2 \\ m_{Z'C}^2 & m_C^2 \end{pmatrix}^{-1} \tag{38}$$

The E is a two-component vector given by

$$\mathcal{E}_1 \simeq -\frac{\sqrt{1-4s_W^2}\{4s_W^2 w^2(u^2+v^2) + 3\Lambda^2[-u^2+v^2+2(2u^2+v^2)s_W^2]\}}{4\sqrt{3}(1-s_W^2)^2 w^2 \Lambda^2} \ll 1 \tag{39}$$

$$\mathcal{E}_2 \simeq \frac{s_W^2(u^2+v^2)}{4(1-s_W^2)^{3/2} t_N \Lambda^2} \ll 1, \tag{40}$$

which are suppressed at the leading order $u, v \ll \Lambda, w$. When neglecting the mixing of Z,Z^0 and C, m_{Z1} ' m_Z, we can identify $Z_1 \equiv Z, Z'_1 \equiv Z'$ and $C_1 \equiv C$. For the final step, it is easily to diagonalize M''^2 (or $M_s''^2$) to obtain two remaining physical states, denoted by Z_2 and Z_N, such that

$$\begin{pmatrix} A \\ Z_1 \\ Z'_1 \\ C_1 \end{pmatrix} = U_3 \begin{pmatrix} A \\ Z_1 \\ Z_2 \\ Z_N \end{pmatrix}, \quad U_3 \simeq \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_\xi & -s_\xi \\ 0 & 0 & s_\xi & c_\xi \end{pmatrix}, \tag{41}$$

$$M'''^2 = U_3^T M''^2 U_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m_{Z_1}^2 & 0 & 0 \\ 0 & 0 & m_{Z_2}^2 & 0 \\ 0 & 0 & 0 & m_{Z_N}^2 \end{pmatrix}. \tag{42}$$

The mixing angle and new masses are given by

$$t_{2\xi} = \frac{-2m_{Z'C}^2}{m_C^2 - m_{Z'}^2} \simeq -\frac{8}{\sqrt{3}} \frac{\sqrt{1-4s_W^2}\, c_W t_N w^2}{12(1-4s_W^2)t_N^2\Lambda^2 + \left[\frac{16}{3}(1-4s_W^2)t_N^2 - c_W^2\right] w^2} \tag{43}$$

$$m_{Z_2}^2 \simeq \frac{g^2}{2}\left(4(\Lambda^2+\frac{4}{9}w^2)t_N^2+\frac{w^2c_W^2}{3(1-4s_W^2)} -\sqrt{\left[4(\Lambda^2+\frac{4}{9}w^2)t_N^2-\frac{w^2c_W^2}{3(1-4s_W^2)}\right]^2+\frac{64}{27}\frac{c_W^2t_N^2w^4}{1-4s_W^2}}\right), \quad (44)$$

$$m_{Z_N}^2 \simeq \frac{g^2}{2}\left(4(\Lambda^2+\frac{4}{9}w^2)t_N^2+\frac{w^2c_W^2}{3(1-4s_W^2)} +\sqrt{\left[4(\Lambda^2+\frac{4}{9}w^2)t_N^2-\frac{w^2c_W^2}{3(1-4s_W^2)}\right]^2+\frac{64}{27}\frac{c_W^2t_N^2w^4}{1-4s_W^2}}\right). \quad (45)$$

The Z_2 and Z_N are heavy particles with the masses in w scale.

The physical fields are related to the gauge states as

$$\begin{pmatrix} A_{3\mu} \\ A_{8\mu} \\ B_\mu \\ C_\mu \end{pmatrix} = U \begin{pmatrix} A \\ Z_1 \\ Z_2 \\ Z_N \end{pmatrix}, \quad U = U_1U_2U_3, \quad (46)$$

Where

$$U = \begin{pmatrix} s_W & c_W & c_W(\mathcal{E}_1c_\xi+\mathcal{E}_2s_\xi) & c_W(\mathcal{E}_2c_\xi-\mathcal{E}_1s_\xi) \\ -\sqrt{3}s_W & \sqrt{3}s_Wt_W-\mathcal{E}_1\sqrt{1-3t_W^2} & \sqrt{3}(\mathcal{E}_1c_\xi+\mathcal{E}_2s_\xi)s_Wt_W+c_\xi\sqrt{1-3t_W^2} & \sqrt{3}(\mathcal{E}_2c_\xi-\mathcal{E}_1s_\xi)s_Wt_W-s_\xi\sqrt{1-3t_W^2} \\ c_W\sqrt{1-3t_W^2} & -\sqrt{3}\mathcal{E}_1t_W-s_W\sqrt{1-3t_W^2} & \sqrt{3}c_\xi t_W-(\mathcal{E}_1c_\xi+\mathcal{E}_2s_\xi)s_W\sqrt{1-3t_W^2} & -\sqrt{3}s_\xi t_W-(\mathcal{E}_2c_\xi-\mathcal{E}_1s_\xi)s_W\sqrt{1-3t_W^2} \\ 0 & -\mathcal{E}_2 & s_\xi & c_\xi \end{pmatrix}.$$

In the limit,$\{u^2,v^2\}/\{w^2,\Lambda^2\} \ll 1$ and means that the standard model Z boson by itself is a physical field Z ' Z_1 and do not mix with the new gauge bosons $Z_{2,N}$.

The ρ-parameter (or $\Delta\rho \equiv \rho-1$ used below) that is due to the contribution of the new physics comes from the tree-level mixing of Z with Z^0 and C, which can be evaluated as [?]

$$(\Delta\rho)_{tree} = \frac{m_W^2}{c_W^2m_{Z_1}^2}-1=\frac{m_Z^2}{m_Z^2-\mathcal{E}(m_{ZZ'}^2\ m_{ZC}^2)^T}-1\simeq\frac{\mathcal{E}(m_{ZZ'}^2\ m_{ZC}^2)^T}{m_Z^2} \quad (47)$$

Here, notice that $m_W = c_W m_Z$ and $m^2_Z \sim m^2_{ZZ}0 \sim m^2_{ZC}$. We get the deviation as

$$(\Delta\rho)_{tree} \simeq \frac{[(1-4s_W^2)u^2-(1+2s_W^2)v^2]^2}{4c_W^4v_w^2w^2}+\frac{s_W^4v_w^2}{c_W^4\Lambda^2} \quad (48)$$

We study the two (u,w) parameters in two cases $\Lambda \gg w$ and $\Lambda \sim$ w, both cases are being limited by the experimental Δρ parameters $0.00016 \le \Delta\rho \le 0.00064$. The case $\Lambda \gg w$ yields results similar to the 3-3-1 model [6]. In the case $\Lambda \sim$ w, we choose Λ = 2w, to the new physics scale w > 1.4 TeV.

3. CONCLUSION

We have found that the W,Z_1,A fields are the standard model gauge bosons. The fields Z_2,Z_N and X,Y are new gauge bosons. There are 9 massive gauge bosons matching 9

Goldstone bosons in the scalar sector. Constraint from the rho parameter yields that $\Lambda \sim w \sim$ TeV. Therefore, the new gauge bosons such as Z_2,Z_N and X,Y can be investigated by the current colliders.

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CƠ CHẾ GAUGE CỦA MÔ HÌNH 3-3-1-1 TỐI THIỂU

Tóm tắt: *Trong bài báo này, chúng tôi nhận diện mô hình hạt gauge chuẩn và các hạt gauge mới. Ta có thể phân biệt được hạt gauge chuẩn và hạt gauge mới. Chúng tôi chứng minh rằng số lượng của hạt gauge trùng với số lượng của hạt Goldstone, chỉ để lại hạt photon và gluon không khối lượng do đó vector vật chất tối được thảo luận.*

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Từ khoá: *Vật chất tối, mô hình 3-3-1-1.*