



Research Article

# BUILDING AN INVENTORY OF LEARNING MATERIALS EXPLOITING MOBILE AUGMENTED REALITY TECHNOLOGY FOR SPATIAL GEOMETRY

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## ABSTRACT

*The application of information technology in teaching mathematics in the world as well as in Vietnam has become popular, this article, therefore, aims at building an inventory of learning materials exploiting augmented reality technology on smartphones for spatial geometry. The technological solution used in this study is GeoGebra 3D Calculator because of its free-of-charge, suitability with high schools in Vietnam and Ho Chi Minh City University of Education, and its existing co-users. This study is completed with an ebook containing 123 visual objects of all the current exercises in the Grade 11 Geometry Textbook. The ebook has been introduced to the user community through YouTube and Facebook with a desire of creating a visual teaching medium for teachers and supporting students to visualise spatial geometry objects.*

**Keywords:** Mobile Augmented Reality; GeoGebra; Spatial Geometry

## 1. Background

Technology is rapidly affecting every aspect of our lives, and our way of learning is not exceptional (Firmin, & Genesi, 2013). It has opened up numerous chances to use a variety of new technology tools for teaching and learning systems. Research on technology-enhanced learning has increasingly focused on emergent technologies such as augmented reality, ubiquitous learning (u-learning), mobile learning (m-learning), and games for improving the satisfaction and experiences of users in enriched learning environments. These studies take advantage of technological innovations in hardware and software for mobile devices and their increasing popularity among people to place students

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at the center of the learning process (Bacca et al., 2014) and to create unprecedented opportunities to the education systems with its capabilities to integrate, enhance, and interact with each other over a wide geographic distance in a way to achieve the learning objectives (Majumdar, 2015). It has the potential to transform the nature and process of learning environments.

Technology has been used in mathematics teaching and learning since the introduction of simple four-function calculators in the 1970s (Bennison, & Goos, 2010). Since then, parallel to the development of technology, the application of information technology in mathematics is increasingly important and developed with the appearance of many modern technological devices supporting the teaching of mathematics such as GeoGebra and Wolfram alpha. The National Council of Teachers of Mathematics (NCTM, 2000) provides a vision for technology implementation in the mathematics classroom centered on the notion that technology has the potential to enhance mathematics learning, support mathematics teaching effectively, and influence what mathematics is taught. Therefore, one of the most important tasks in mathematics teaching and learning today is the revision of curricula and teaching methods to use technology effectively (Fey, 1989).

In the current curriculum, the Ministry of Education and Training (2006) has confirmed that the proper use of teaching devices will help to enhance the positive effects of teaching methods. Simultaneously, the Ministry of Education and Training also encourages teachers to create extra teaching aids suitable for their learning content and to exploit information technology in mathematics teaching in schools. Next, in the new general education curriculum in mathematics, the Ministry of Education and Training (2018) enhances the role of teaching equipment in mathematics education with the introduction of the capacity to use tools and means of teaching as one of the five components of mathematical competency that teachers need to shape and develop for students.

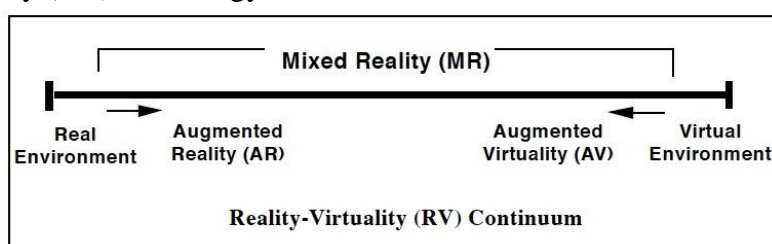
Many studies have shown that spatial geometry is a learning content causing many difficulties for students (Mammana, & Villani, 1998). In particular, teaching spatial geometry in Vietnam requires students to be proficient in the synthesis method (Le, 2017) in which visualizing 3D geometric objects is an essential step. Vietnamese students often start to solve spatial geometry problems from 2D drawings on a flat surface (paper or board). However, the 2D drawings are unable to show all the properties of the spatial geometry (3D) (Tang, 2014). This is an obstacle to judging the spatial properties required for argument and proof.

In this context, we are interested in building an inventory of drawings of the most intuitive textbook exercises to assist teachers and students in spatial geometry teaching and learning.

## 2. Solution

### 2.1. Selecting augmented reality technology

Traditionally, to visually represent spatial geometrical objects without “breaking” the third dimension of space, teachers often use real models and objects. However, these teaching facilities do not often meet the complex configuration of exercises in practical teaching in Vietnam. Therefore, in this study, we are interested in the application of information technology achievements to represent 3D objects. Specifically, we choose augmented reality (AR) technology, which has received a lot of attention in recent years.



**Figure 1.** Reality–Virtuality Continuum (Milgram, & Kishimo, 1994)

The augmented reality can be considered to lie on a “Reality–Virtuality Continuum” (Figure 1) between the real environment and virtual environment (Milgram & Kishimo, 1994). The term “Augmented Reality” was coined by a Boeing researcher named Tom Caudell in 1990. The technology was known much earlier than that. Particularly, in 1968, Ivan Sutherland and Bob Sproull invented a device that is hung from the ceiling and by putting it on, users would see lots of graphics generated by computers as if they were entering an alternative world. At that time, the technology used in this device was impractical, which led to it not being widely used. Until 1974, Myron Krueger introduced Videoplace, a project combining a projection system and video cameras that produced shadows on the screen. This setup made the user feel as though they were interacting with the environments. In 1992, the first fully real operational augmented reality system was created by Louis Rosenburg. It was called Virtual Fixtures, a complicated robotic system to deal with the lack of high-speed 3D graphics processing power to improve working efficiency and productivity. This system was an early version of what most AR systems currently do nowadays. In 2000, Hirokazu Kato of the Nara Institute of Science and Technology in Japan created and released software called ARToolKit, which marked a big development of augmented reality technology. Through this software, one could capture real-world actions and overlay it with virtual objects with the help of video tracking. This significantly influences what we experience today in all flash-based augmented reality apps. Augmented reality has come a long way since its early conception. Until now, its advancements have been even more promising and potential to apply in many fields such as games (e.g., Pokemon Go) and furniture (e.g., IKEA Place) (Isberto, 2018).

The definition of augmented reality is perceived differently among researchers. However, the most commonly accepted definition is a combination of three main features: integration of real objects and virtual objects, real-time interaction, accurate 3D registration of real and virtual objects (Koutromanos et al., 2016). In recent years, along with the spread and the gradual popularity of mobile devices (smartphones), people talk more about mobile augmented reality. It is AR that you can take with you wherever you go, which means that the hardware required to implement an AR application is something that you take with you wherever you go (Craig, 2013).

With the development of technology, devices equipped with AR are becoming more and more popular. AR is applied to many fields and provides many benefits for everyone. In education, AR assists educators and learners in their teaching and learning process (Cheah et al., 2014). AR has been proved to have great impacts on students' academic performance. Omar et al. (2019) show that AR helps students to improve their visualization skills. The difference of using the AR application is significant, proving that AR is an effective tool in education. Also, the feedback of students shows that AR helps with the visualization process the most which is true because it is the nature of AR to help users manipulate the virtual object in the real environment. Huang et al. (2015) also state that AR helps young children to inspect 3D objects from many angles; hence, improves their understanding of the concept. AR also has a great impact on developing children's creativity and imagination. It is inferred from the study that with the help of AR, children's concentration and memorization ability have been boosted. Lin et al. (2015) show that AR helps to improve students' spatial ability, especially students with low academic achievement. AR has the potential to further engage and motivate learners in discovering resources and applying them to the real world from a variety of diverse perspectives that have never been implemented in the real world (Lee, 2012). The use of AR applications in learning has been proved to bring many benefits such as attention, engagement, interest, motivation, satisfaction, knowledgeable comprehension, academic achievement, knowledge retention, enjoyment, and anatomy (Saltan, & Arslan, 2017).

## **2.2. *Selecting GeoGebra 3D Calculator application***

Technically, in order to build an inventory of learning materials about the 3D geometrical objects based on mobile augmented reality technology, many applications can be considered. In this study, we select GeoGebra 3D Calculator application.

GeoGebra was developed from Markus Hohenwarter's master thesis at the University of Salzburg in 2002. The first version was very simple with a few tools and options, working on 2D geometry. After the software was published to the Internet, many teachers were enthusiastic and contacted Hohenwarter for permission to use and share it. Being inspired, he continued developing GeoGebra. After years of development, nowadays, GeoGebra is dynamic mathematics software for all levels of education that

brings together geometry, algebra, spreadsheets, graphing, statistics, and calculus in one easy-to-use package. GeoGebra has a rapidly expanding community of millions of users located in just about every country. GeoGebra has also received many educational software awards (MERLOT Classic Award 2013, Microsoft Partner of the Year Award 2015, Archimedes 2016). For version 4.0 (in 2014), users can use GeoGebra on smartphones, both iOS and Android operating systems. In 2015, version 5.0 allows users to create 3D geometrical objects and rotate this object to visualize the depth of the drawing with the “Rotate 3D Graphics View” feature. This version has later been added a tool allowing users to choose the type of the projection (in 3D Graphics View Style Bar). One of the types of projection is “Projection for glasses”, which means you have to wear 3D glasses when you see the objects and that makes the objects become more like real. In September 2019, GeoGebra enabled AR in the app 3D Calculator. Users can place 3D objects as real objects on many surfaces and go around to see them at different angles.

The selection of GeoGebra 3D Calculator application in this study is based on the following reasons:

(1) GeoGebra is free. This means a lot for of a developing country as regards the copyright and financial constraints.

(2) For students, GeoGebra is not too new. Indeed, in the textbook “Informatics for Secondary School – Volume 2”, GeoGebra has been included in the “Learning software”, the lesson “Learning to draw dynamic geometry with GeoGebra” (Figure 2). Thus, students that have been taught IT in schools have had the opportunity to access this software.

(3) In an undergraduate program of Mathematics Teacher Education at the Ho Chi Minh City University of Education, GeoGebra is taught in the module “Applying information technology in teaching Mathematics” to help students embrace new technologies in math education. This is one of the required modules with two credits. With what is equipped in this module, students will be the first users to access, exploit the resource, and continue to spread it during their practicum at high schools and after graduation.

(4) These days, GeoGebra has developed a huge user community. On the [geogebra.org](http://geogebra.org) website, users are connected and shared through an inventory with over one million of free activities, simulations, exercises, lessons, and games for math and science. Based on this developed platform, our inventory can be spread quickly and widely, thereby contributing more to the community.

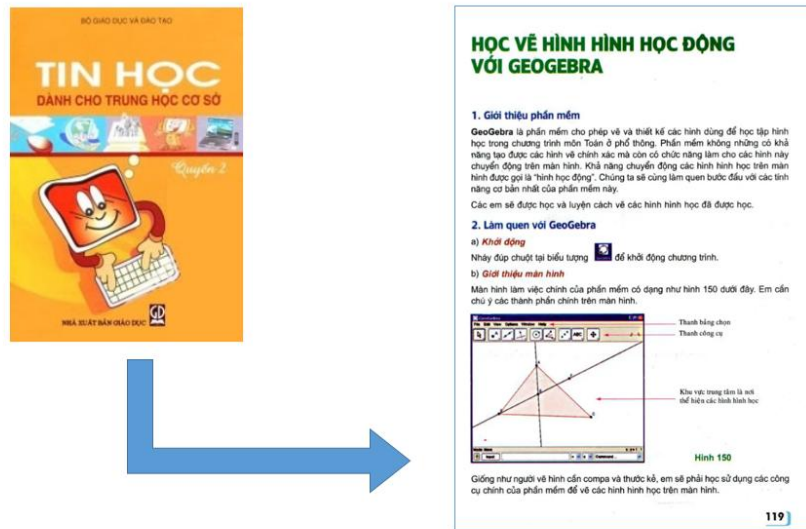


Figure 2. An excerpt introducing GeoGebra in the textbook “Informatics for Secondary School – Volume 2”

### 3. Implementation

The construction and spread of the learning repository is implemented in three stages. (Figure 3)

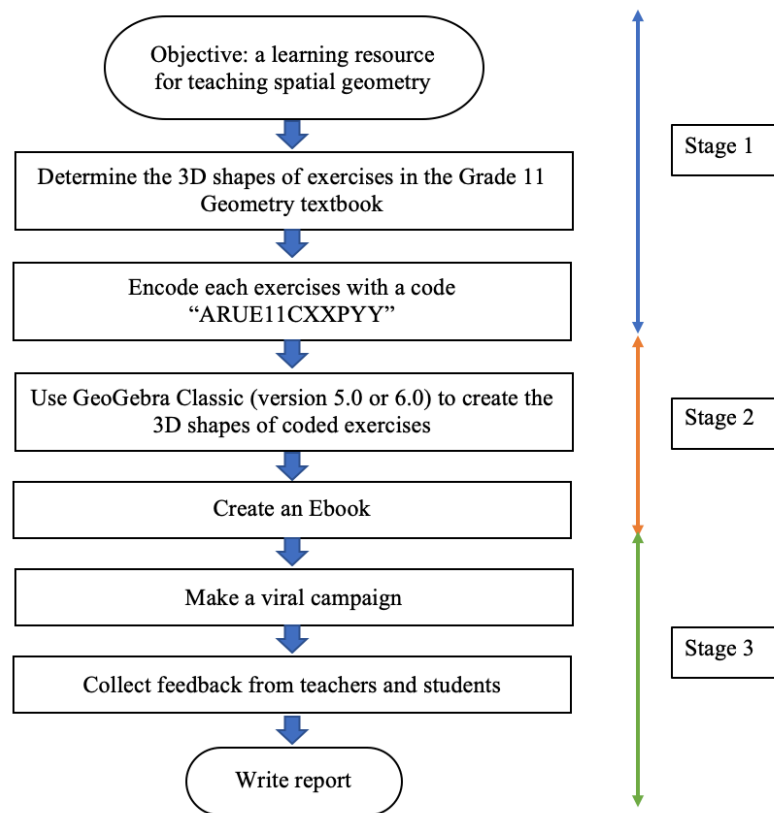


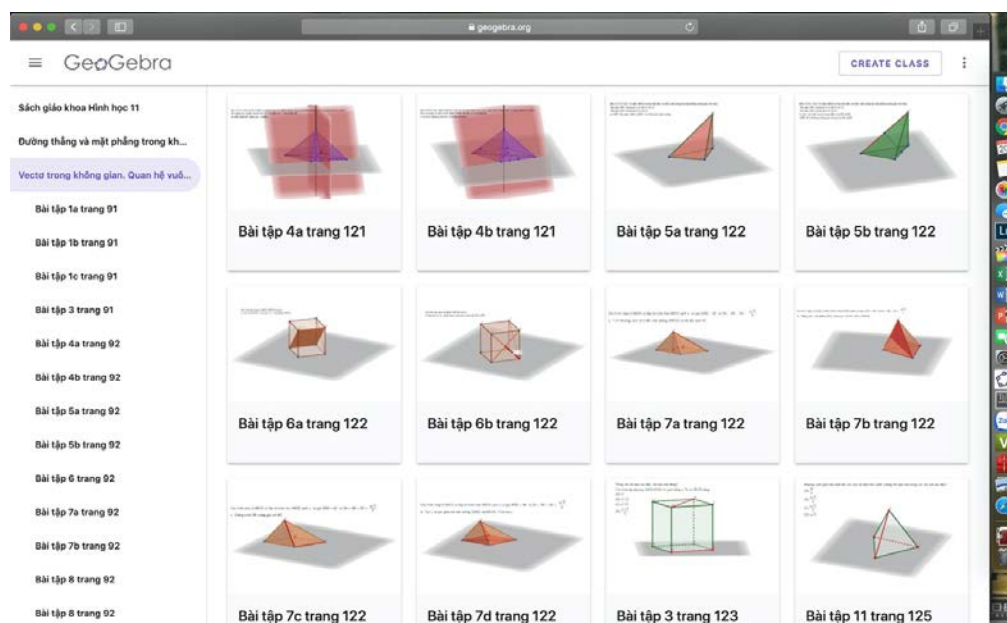
Figure 3. Three stages of implementation

Stage 1: Determine the 3D shapes of exercises in the Grade 11 Geometry Textbook. Then we encode each exercise with a code “ARUE11CxPy”. Here, ARUE represents the project name “Augmented Reality in the University of Education”, “11C” is a symbol to identify those exercises of the textbook Geometry 11 (standard), “x” represents the ordinal of the exercise and “y” is the book page number on which it is available. For example, the 3D shape of exercise 5 on page 126 of the textbook would have the code “ARUE11C5P126”.

Stage 2: We use GeoGebra Classic (version 5.0 or 6.0) to create the 3D objects of coded exercises in Stage 1. Subsequently, all of the files are examined by the research team. The final product files will be collected into an ebook and posted on the “Classroom Resources” of the website <https://www.geogebra.org/materials>. This is a contribution to the community: a shared (open) database of GeoGebra users which is free for teaching and learning spatial geometry.

Stage 3: After the ebook is ready to use, the research team has made an instruction video to help teachers and students understand how to use it, which is posted online to social media like Facebook and Youtube. Simultaneously, during the period of pedagogical experiment, we also use the ebook (through the 3D calculator app) in real teaching situation at several high schools in Ho Chi Minh City to collect and survey feedback from teachers and students.

#### 4. Result



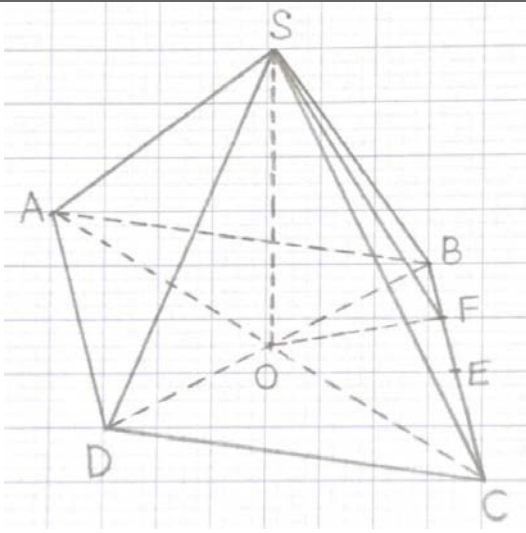
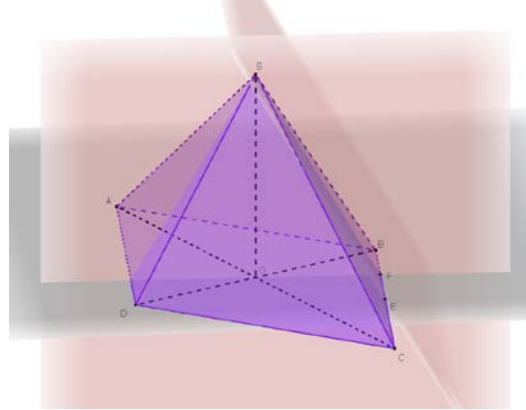
*Figure 4. Ebook about learning materials on GeoGebra website*

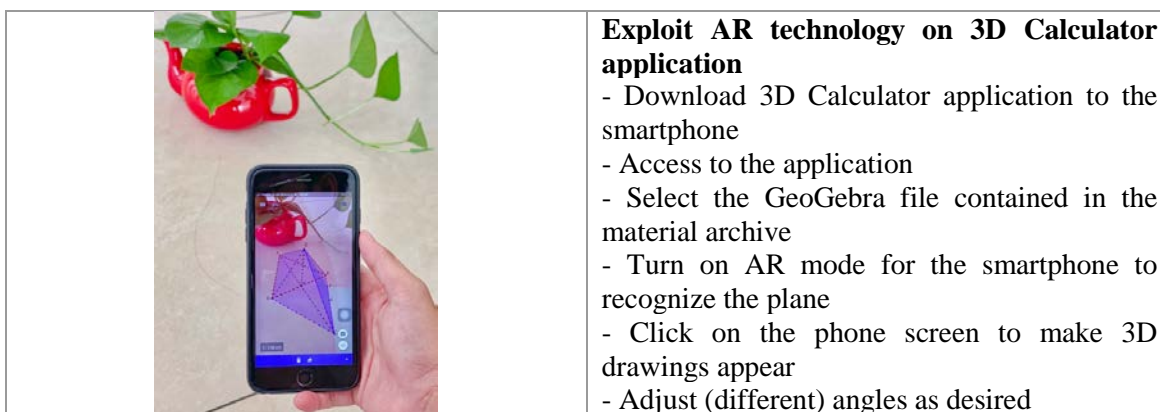
In Stage 1, we identified 123 questions in the Grade 11 Geometry Textbook (standard version) that need to be drawn. In Stage 2, we created 123 3D objects of these exercises and packaged them into the “Grade 11 Geometry Textbook” ebook which was

uploaded to the “Classroom Resources” of the GeoGebra website (Figure 4). Users can search for drawings by entering the assignment number and the corresponding page into the search box in the 3D Calculator application. Below, we illustrate the drawing case of exercise 4a on page 121 in the paper-pen environment, in GeoGebra Classic 5 and in the 3D Calculator application (Table 1).

*Exercise 4a (page 121): The pyramid  $S.ABCD$  has the base  $ABCD$  of the rhombus with side  $a$  and the angle  $BAD$  of  $60^\circ$ .  $O$  is the intersection point of  $AC$  and  $BD$ .  $SO$  is perpendicular to the base and  $SO = 3a/4$ .  $E$  is midpoint of  $BC$ ,  $F$  is midpoint of  $BE$ . Prove the plane  $(SOF)$  perpendicular to the plane  $(SBC)$ .*

**Table 1.** Representation in different environment of 3D object of the exercise 4a on page 121

Perspective representation	Construction
	<p><b>Draw the picture on the paper</b></p> <ul style="list-style-type: none"> <li>- Draw parallelogram <math>ABCD</math>, in which side <math>AB</math> is drawn with dashed lines</li> <li>- Draw two diagonal lines <math>AC</math>, <math>BD</math> with dashed lines and call <math>O</math> as their intersection point</li> <li>- Take the point <math>S</math> on the vertical line passing through <math>O</math></li> <li>- Connect <math>SA</math>, <math>SB</math>, <math>SC</math>, <math>SD</math> with solid lines and <math>SO</math> with dash lines</li> <li>- Let <math>E</math> be the midpoint of <math>BC</math> and <math>F</math> the midpoint of <math>BE</math></li> <li>- Connect <math>SF</math> by solid lines and <math>OF</math> by dashed lines</li> </ul>
	<p><b>Create 3D object in GeoGebra Classic 5</b></p> <ul style="list-style-type: none"> <li>- Construct an equilateral triangle <math>ABD</math></li> <li>- Let <math>O</math> be the midpoint of <math>BD</math></li> <li>- Construct <math>C</math> to be symmetrical with <math>A</math> through <math>O</math></li> <li>- Draw a line through <math>O</math> perpendicular to the plane <math>(ABCD)</math> and the center surface <math>O</math> with radius <math>3/4 AB</math>. Let <math>S</math> be their intersection point</li> <li>- Connect <math>SA</math>, <math>SB</math>, <math>SC</math>, <math>SD</math> and <math>SO</math></li> <li>- Let <math>E</math> be the midpoint of <math>BC</math> and <math>F</math> the midpoint of <math>BE</math></li> <li>- Draw two planes <math>(SBC)</math> and <math>(SOF)</math></li> </ul>



To solve this exercise, or more accurately, to solve the type of tasks “Prove two planes perpendicular”, students need to find a line that lies in one plane and perpendicular to the other. Finding this line on drawings in the paper-pen environment is very difficult because 2D drawings do not express the “perpendicular” between the line and the plane. However, with the help of augmented reality technology (3D Calculator application), students can observe 3D objects from many angles and can then judge and choose the desired line (in this case, that is BC).

In stage 3, to spread the inventory of 3D drawings in the community, we created an instruction video on how to exploit the inventory on the 3D Calculator application and posted it on YouTube and Facebook.

## 5. Conclusion

From the situation that high school students and teachers often encounter many difficulties in learning spatial geometry, we have created an inventory of 3D geometric objects in the exercises of Grade 11 Geometry Textbook that users can exploit on the GeoGebra 3D Calculator smartphone’s augmented reality application. This study contributes to pointing out the contributions and positive effects of smartphones in education. For teachers (and also student teachers in practicum), this is a modern and convenient teaching medium that supports the illustration of 3D objects which are difficult to imagine in the paper-pen environment and also cause troubles for students to make judgments and arguments in proof of spatial geometry. For students, this is a useful learning tool that motivates them in learning spatial geometry.

This research just aims at creating a learning inventory of drawings to support visual observation. In practice, how do teachers and students rate it? Is it useful and necessary? This needs to be clarified through a survey that we plan to conduct in the future. On the other hand, a more important question to be asked is how this learning inventory can be used to improve the quality of education, for example, to develop the capacity to use teaching tools and facilities for students as defined in the general education curriculum in Mathematics in 2018.

❖ **Conflicts of Interest:** Authors have no conflict of interest to declare.

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**XÂY DỰNG KHO HỌC LIỆU DẠY HỌC HÌNH HỌC KHÔNG GIAN  
THEO HƯỚNG KHAI THÁC CÔNG NGHỆ  
THỰC TẠI ẢO TĂNG CƯỜNG TRÊN DI ĐỘNG**

**Võ Văn Nghĩa, Võ Văn Hóa, Đặng Vũ Quang Thịnh, Đoàn Cao Khả,  
Trương Ngọc Huy, Nguyễn Thị Thuỳ Linh, Phan Văn Đức, Nguyễn Tổng Công Minh,  
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**TÓM TẮT**

Trong xu hướng tăng cường ứng dụng công nghệ thông tin trong dạy học Toán trên thế giới cũng như tại Việt Nam, bài viết quan tâm đến việc xây dựng một kho học liệu dạy học hình học không gian theo hướng khai thác công nghệ thực tại ảo tăng cường trên điện thoại di động. Giải pháp công nghệ được quan tâm trong nghiên cứu này là ứng dụng GeoGebra 3D Calculator do được phân phối miễn phí, phù hợp với thực tiễn trường phổ thông cũng như tại Trường Đại học Sư phạm Thành phố Hồ Chí Minh, nên có một cộng đồng người dùng đông đảo. Nghiên cứu hoàn thành với một ebook chứa 123 visual objects của tất cả các bài tập trong Sách giáo khoa Hình học 11 hiện hành. Ebook này đã được giới thiệu với cộng đồng người dùng qua Youtube và Facebook với mong muốn đóng góp một phương tiện dạy học trực quan cho giáo viên và hỗ trợ học sinh hình dung các đối tượng hình học không gian.

**Từ khóa:** thực tại ảo tăng cường trên di động; GeoGebra; Hình học không gian