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Cracking moment of rectangular reinforced concrete sections corresponding to the bi-linear stress-strain diagram of concrete

Tính toán mô men kháng nứt của tiết diện dầm bê tông cốt thép ứng với mô hình ứng xử phi tuyến song tuyến tính của bê tông

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ABSTRACT

The determination of the cracking moment is an important content when designing reinforced concrete structures working in aggressive environments or having advanced requirements on the impermeability and durability. This paper aims to establish a formula that allows to more precisely determine the cracking moment of rectangular reinforced concrete sections by considering the non-linear behavior of concrete simplified as the bi-linear stress-strain diagram according to TCVN 5574 - 2018. The analytical formula has been experimentally validated with most errors less than 7%. The paper has also calculated the cracking moment of reinforced concrete sections corresponding to different reinforcement ratios, the calculation results have showed that: (1) the cracking moment increases linearly as a function of tensile reinforcement ratio and of compressive reinforcement ratio; (2) the tensile reinforcement ratio has a great influence on the cracking moment while the influence of compressive reinforcement ratio is insignificant; (3) the approximate method presented in TCVN 5574 – 2018 significantly underestimates the cracking moment of the section, especially in the case of high tensile reinforcement ratio.

Keywords: Reinforced concrete; cracking moment; stress-strain diagram; nonlinear behavior; bi-linear diagram.

TÓM TẮT

Việc xác định mô men kháng nứt là một nội dung quan trọng khi thiết kế kết cấu bê tông cốt thép làm việc trong môi trường xâm thực hoặc có những yêu cầu nâng cao về khả năng chống thấm và độ bền lâu. Bài báo nghiên cứu nhằm mục đích thiết lập công thức cho phép xác định một cách chính xác hơn mô men kháng nứt của tiết diện bê tông cốt thép hình chữ nhật có xét tới ứng xử phi tuyến của bê tông được đơn giản hóa dưới dạng biểu đồ ứng suất - biến dạng hai đoạn thẳng theo TCVN 5574 - 2018. Công thức thiết lập đã được kiểm chứng thực nghiệm với hầu hết các sai số nhỏ hơn 7%. Bài báo cũng tính toán mô men kháng nứt của tiết diện bê tông cốt thép ứng với các hàm lượng cốt thép khác nhau, kết quả tính toán chỉ ra rằng: (1) mô men kháng nứt tăng tuyến tính với hàm lượng cốt thép chịu kéo và hàm lượng cốt thép chịu nén; (2) hàm lượng cốt thép chịu kéo có ảnh hưởng lớn đến mô men kháng nứt trong khi ảnh hưởng của hàm lượng cốt thép chịu nén là không đáng kể; (3) phương pháp tính toán mô men kháng nứt trình bày trong TCVN 5574 - 2018 đánh giá thấp đáng kể mô men kháng nứt của tiết diện, đặc biệt trong trường hợp hàm lượng cốt thép chịu kéo lớn.

Từ khóa: Bê tông cốt thép; mô men kháng nứt; biểu đồ ứng suất - biến dạng; ứng xử phi tuyến; biểu đồ hai đoạn thẳng.

1. INTRODUCTION

The calculation and control of the formation or expansion of concrete cracks is one of the contents that need to be carried out in the design process of reinforced concrete structures. For conventional reinforced concrete structures, the design standard TCVN 5574 - 2018 [1] allows the appearance of concrete cracks with limited crack width depending on the reinforcement type and the permeability limitation requirements of the structure. For reinforced concrete structures working in aggressive environments or having advanced requirements on the impermeability and durability, the design standard TCVN 5574 - 2018 [1] do not allow the appearance of concrete cracks. In this case, the structure must be designed in such a way that the bending moment caused by the factored loads does not exceed the cracking moment of the section.

The determination of the cracking moment of reinforced concrete sections according to the American standard ACI 318 [2] and the European standard EC2 [3] is relatively simple, based on the following assumptions: (1) the plane cross section remains plane after deformation; (2) the concrete and reinforcement behave elastically until the concrete tensile stress reaches the concrete tensile strength. The Vietnamese standard TCVN 5574 - 2018 [1] allows determining the cracking moment in a manner similar to the ACI 318 [2] and EC2 [3] but introducing a correction factor to take into account the nonlinear behavior of tensile concrete. Apart from this simple method, TCVN 5574 – 2018 [1] also allows determining the cracking moment more precisely based on the nonlinear stress-strain diagram of concrete which can be simplified by two or three straight segments.

Many studies on the cracking moment of reinforced concrete structures have shown that the simplified methods presented in design standards significantly underestimate the cracking moment compared to experimental and numerical results [4, 5, 6, 7]. This is due, on the one hand, to the fact that the design standards [1, 2, 3] have not reasonably taken into account the nonlinear behavior of concrete when calculating the cracking moment. In fact, the concrete begins to crack when the its tensile strain reaches a limit value, the tensile stress in the concrete then also reaches the concrete tensile strength. However, when assuming that the concrete behaves elastically until it cracks when the concrete stress at the tensile edge reaches the concrete tensile strength as assumed in the design standards [2, 3], the concrete tensile strain is still small compared to the limit value which causes the cracking of the concrete. Although the design standard TCVN 5574 – 2018 [1] has provided a correction factor to take into account the nonlinear behavior of tensile concrete, however, using the same correction value for all concrete strength levels is inaccurate [7]. The underestimation of cracking moment, on the other hand, also comes from the fact that the design standards have ignored the influence of reinforcement. Indeed, the tensile reinforcement has the effect of reducing the stress and strain of the tensile concrete zone. Therefore, the higher the reinforcement ratio, the higher the cracking moment [4 - 8].

Several theoretical studies have been carried out in order to more accurately estimate the cracking moment of reinforced concrete sections considering the nonlinear behavior of concrete. Valivonis and Skuturna [9] have proposed an analytical formula for estimating the cracking moment of reinforced concrete structures strengthened with carbon fibre laminates. However, the authors have not presented in detail the method of establishing the formula and there are significant errors between the calculated and experimental results. Dao and Tung [7] have established the cracking moment formula based on bi-linear stress-strain diagram of concrete according to the TCVN 5574 - 2018 [1]. However, the authors have not considered the influence of tensile and compressive reinforcement. In addition, the strain from which the tensile stress in concrete reaches the tensile strength corresponding to bi-linear stress-strain model is specified as 0.008% [1], which is not the elastic strain calculated based on the tensile strength and elastic modulus of concrete. Hiep and Duy [8] have also established the cracking moment formula taking into account the nonlinear behavior of concrete according to the TCVN 5574 - 2018 [1]. However, instead of taking the elastic modulus of concrete, the authors have taken the converted deformation modulus when calculating the compressive stress of concrete. Indeed, the converted deformation modulus is used to calculate the ultimate internal forces of reinforced concrete sections, taking into account the nonlinear behavior of compresive concrete by simplifying the entire stress-strain curve of compressive concrete as bilinear diagram. When calculating the cracking moment, the compressive stress in the concrete is still very small and therefore the compresive concrete zone still behaves elastically as instructed in TCVN 5574 - 2018 [1]. In addition, the authors [8] have replaced the tensile strains of the concrete with the values, this makes the established formula lose its generality.

This paper aims to establish a formula to more accurately determine the cracking moment of rectangular reinforced concrete sections based on the bi-linear stress-strain diagram of concrete taking into account the influence of both tensile and compressive reinforcement. The paper also compares the cracking moment determined by this method with the cracking moment calculated by simple approximate method presented in TCVN 5574 - 2018 [1]. The influence of tensile and compressive reinforcement ratios on the cracking moment has also been highlighted in the paper.

2. DETERMINATION OF CRACKING MOMENT BASED ON THE BI-LINEAR STRESS-STRAIN DIAGRAM OF CONCRETE

The determination of cracking moment of a rectangular reinforced concrete section taking into account the nonlinear behavior of concrete is carried out based on the assumptions shown in Figure 1, including:

- The plane cross section remains plane after deformation (Figure 1.a);
- The tensile and compressive reinforcement behave elastically (the stress is proportional to the strain);
- The concrete zone in compression behaves elastically (the stress is proportional to the strain), the stress diagram in this zone is triangular (Figure 1.b);
- The concrete zone in tension behaves nonlinearly, the stress diagram in this zone is simplified as a trapezium (Figure 1.b);
- The concrete tensile strains ε_{bt1}, ε_{bt2} (Figure 1.a) corresponding to the bi-linear stress-strain model according to the TCVN 5574 – 2018 [1] are equal to ε_{bt1} = 0.008%, ε_{bt2} = 0.015%;
- The tensile stress of concrete just before cracking reaches R_{bt} (concrete tensile strength used in the serviceability limit state design);
- The cross section is in equilibrium.

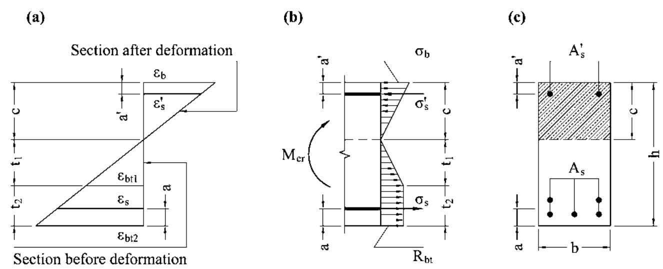

Figure 1. The assumptions used to determine the cracking moment: (a) strain diagram; (b) stress diagram; (c) cross section.

From the plane cross-section assumption (Figure 1.a), based on the similar triangles principle, we can determine the following quantities:

- The strain at the compresive concrete edge

$$\varepsilon_b = \left(\frac{c}{h-c}\right)\varepsilon_{bt2} = \left(\frac{\xi}{1-\xi}\right)\varepsilon_{bt2} \qquad (1)$$

▪ The strain at the center of gravity of tensile reinforcement

$$\varepsilon_s = \left(\frac{h-c-a}{h-c}\right)\varepsilon_{bt2} = \left(1-\frac{a/h}{1-\xi}\right)\varepsilon_{bt2} \quad (2)$$

▪ he strain at the center of gravity of compressive reinforcement

$$\varepsilon_s^{'} = \left(\frac{c-a^{'}}{h-c}\right)\varepsilon_{bt2} = \left(\frac{\xi-a^{'}/h}{1-\xi}\right)\varepsilon_{bt2} \quad (3)$$

▪ The height of tensile concrete zone having triangular stress distribution

$$t_1 = (h-c)\frac{\varepsilon_{bt1}}{\varepsilon_{bt2}} = h(1-\xi)\frac{\varepsilon_{bt1}}{\varepsilon_{bt2}} \quad (4)$$

▪ The height of tensile concrete zone having uniform stress distribution

$$t_2 = (h-c)\left(1-\frac{\varepsilon_{bt1}}{\varepsilon_{bt2}}\right) = h(1-\xi)\left(1-\frac{\varepsilon_{bt1}}{\varepsilon_{bt2}}\right) \quad (5)$$

Where

H is the height of cross section;

c is the height of concrete zone in compression;

a is the distance from the center of gravity of the tensile reinforcement to the tensile concrete edge;

a′ is the distance from the center of gravity of the compressive reinforcement to the compressive concrete edge;

ξ is the relative height of the compressive concrete zone compared to the section height;

$$\xi = \frac{c}{h} \quad (6)$$

From the assumption that the concrete in compression and the reinforcement (both in tension and in compression) behave elastically, we can determine the stress in the concrete and the reinforcement as follows:

▪ The stress in concrete at the compresive edge

$$\sigma_b = E_b\varepsilon_b = \left(\frac{\xi}{1-\xi}\right)E_b\varepsilon_{bt2} \quad (7)$$

▪ The stress in tensile reinforcement at their center of gravity

$$\sigma_s = E_s\varepsilon_s = \left(1-\frac{a/h}{1-\xi}\right)n_s E_b\varepsilon_{bt2} \quad (8)$$

▪ The stress in compressive reinforcement at their center of gravity

$$\sigma_s = E_s\varepsilon_s = \left(1-\frac{a/h}{1-\xi}\right)n_s E_b\varepsilon_{bt2} \quad (9)$$

where

E_b is the modulus of elasticity of concrete;

E_s, $E_s^{'}$ are, respectively, the elastic modulus of the tensile reinforcement and of the compressive reinforcement;

n_s, $n_s^{'}$ are, respectively, the ratios of elastic modulus of the tensile reinforcement and of the compressive reinforcement to the elastic modulus of concrete;

$$n_s = \frac{E_s}{E_b} \quad (10)$$

$$n_s^{'} = \frac{E_s^{'}}{E_b} \quad (11)$$

The height c or relative height ξ = c / h of the compressive concrete zone must satisfy the forces balance condition along the member axis. From the stress distribution diagram in concrete and reinforcement shown in Figure 1.b, we can express the forces balance condition as follows:

$$0.5\sigma_b cb + \sigma_s^{'}A_s^{'} = 0.5R_{bt}t_1 b + R_{bt}t_2 b + \sigma_s A_s \quad (12)$$

where

A_s is the cross-sectional area of the tensile reinforcement;

$A_s^{'}$ is the cross-sectional area of the compressive reinforcement;

b is the cross-sectional width.

By substituting the formulas (4), (5), (7), (8) and (9) into the forces balance equation (12), we can express the latter as the following quadratic equation of ξ:

$$k_2\xi^2 + k_1\xi + k_0 = 0 \quad (13)$$

Where

$$k_2 = 0.5 - \left(1-0.5\frac{\varepsilon_{bt1}}{\varepsilon_{bt2}}\right)\frac{R_{bt}}{E_b\varepsilon_{bt2}} \quad (14)$$

$$k_1 = n_s\mu_s + n_s^{'}\mu_s^{'} + 2\left(1-0.5\frac{\varepsilon_{bt1}}{\varepsilon_{bt2}}\right)\frac{R_{bt}}{E_b\varepsilon_{bt2}} \quad (15)$$

$$k_0 = -\left(1-\frac{a}{h}\right)n_s\mu_s - \frac{a^{'}}{h}n_s^{'}\mu_s^{'} - \left(1-0.5\frac{\varepsilon_{bt1}}{\varepsilon_{bt2}}\right)\frac{R_{bt}}{E_b\varepsilon_{bt2}} \quad (16)$$

$$\mu_s = \frac{A_s}{bh} \quad (17)$$

$$\mu_s^{'} = \frac{A_s^{'}}{bh} \quad (18)$$

The relative height of compressive concrete zone, ξ, is the positive solution of quadratic equation (13):

$$\xi = \frac{-k_1 + \sqrt{k_1^2 - 4k_2k_0}}{2k_2} \quad (19)$$

From the moments equilibrium condition about the neutral axis (zero deformation, distance c from the compresive edge of the section), we can determine the cracking moment of the section by the following formula:

$$M_{cr} = \left[\begin{array}{l} 2\sigma_b\xi^2 + R_{bt}(1-\xi)^2\left(3-\frac{\varepsilon_{bt1}^2}{\varepsilon_{bt2}^2}\right) + \\ 6\sigma_s^{'}\mu_s^{'}\left(\xi-\frac{a^{'}}{h}\right) + 6\sigma_s\mu_s\left(1-\xi-\frac{a}{h}\right)\end{array}\right]\frac{bh^2}{6} \quad (20)$$

where σ_b, σ_s and $\sigma_s^{'}$ are, respectively, the stresses in concrete (7), in tensile reinforcement (8) and in compressives reinforcement (9).

3. DETERMINATION OF CRACKING MOMENT BY THE SIMPLE AND APPROXIMATE METHOD

To simplify the calculation process, TCVN 5574 - 2018 [1] allows determining the cracking moment of reinforced concrete sections by the simple and approximate formula. This formula has a similar form to those presented in the American standard ACI 318 [2] and the European standard EC2 [3], which are established based on the assumption that the materials (concrete and reinforcement) behave elastically until the concrete reaches its tensile strength. However, TCVN 5574 - 2018 [1] has introduced a correction coefficient to consider the nonlinear behavior of the tensile concrete zone, and then the cracking moment of the reinforced concrete section can be determined by the following formula:

$$M_{cr} = \gamma \frac{I_{red}}{y_t} R_{bt} \quad (21)$$

where

y_t is the distance from the center of gravity of the "converted section" (reinforcement is converted to concrete) to the tensile edge of the section

$$y_t = \frac{0.5h + \mu_s(n_s - 1)a + \mu_s'(n_s' - 1)(h - a')}{1 + \mu_s(n_s - 1) + \mu_s'(n_s' - 1)} \quad (22)$$

$$I_{red} = \left[\frac{bh^3}{12} + bh(y_t - 0.5h)^2 + A_s(n_s - 1)(y_t - a)^2 + A_s'(n_s' - 1)(h - y_t - a')^2 \right] \quad (23)$$

I_{red} is the moment of inertia of the "converted" section"

γ is the correction factor taking into account the nonlinear behavior of the tensile concrete zone before cracking, γ = 1.30 for rectangular section or for T-shaped sections with compressive flange (Appendix G, TCVN 5574 - 2018 [1]);

In the case of ignoring the concrete area occupied by the reinforcement, we can replace the quantities n_s - 1 and n_s' -1 in the formulas (22) and (23) by n_s and n_s', respectively. Additionally, to further simplify the calculation process, TCVN 5574 – 2018 [1] also allows determining the moment of inertia of the "converted section" without including the reinforcement (considering A_s = 0, A_s' = 0), we can then take $y_t = 0.5h$ and $I_{red} = bh^3/12$.

4. RESULTS AND DISCUSSION

To verify the analytical formulas, we have compared the cracking moment calculated according to the formulas (20) and (21) with the experimental results in the literature [7, 10 – 13]. The cross-sectional dimensions, mechanical properties of the concrete and reinforcement as well as the calculated and experimental cracking moments are presented in Table 1. It can be noted that the approximate formula (21) significantly underestimates the cracking moment compared to formula (20) and compared to experiment. This can be explained by the fact that the approximate formula (21) has initially established based on the assumption of elastic materials, then adjusted with a coefficient taking into account the nonlinear behavior of concrete. However, this correction factor is still small and does not accurately consider the nonlinear behavior of concrete [7]. It can also be noted that the cracking moments estimated by the formula (21) present small errors compared to the experimental results, most of the errors are less than 7% except the beam of section 120 x 204 mm [13] which presents an error of 17%. This means that the analytical formula (21) established based on bilinear stress-strain model can satisfactorily estimate the cracking moment of reinforced concrete sections.

Table 1. Comparison of the cracking moment calculated by the formulas (20) and (21) with the experimental results.

Ref.	b	h	Reinforcement		a, a′	E_b	E_s, E_s'	R_{bt}	M_{cr} (kNm)		
	(mm)	(mm)	Ten.	Com.	(mm)	(GPa)	(GPa)	(MPa)	Exp.	(20)	(21)
[7]	200	300	2d12	2d8	30	32	205.5	2.92	13.72	13.85	12.14
	200	300	2d16	2d8	30	32	202.5	2.92	15.34	14.59	12.61
	200	300	4d16	2d8	30	32	202.5	2.92	16.29	16.29	13.70
[10]	150	150	2d12	-	16	30	200	2.19	2.50	2.36	1.84
[11]	120	220	2d10	-	30	30	200	2.01	3.38	3.47	2.72
[12]	100	100	2d8	-	20	33	213	2.36	0.66	0.70	0.56
[13]	120	204	2d8	-	25	25	205	1.50	2.74	2.28	1.74
	118	202	2d8	2d8	25	25	205	1.50	2.17	2.26	1.73
	120	202	2d12	2d8	25	25	205	1.50	2.57	2.66	1.90

To study the influence of tensile and compressive reinforcement ratios on the cracking moment of reinforced concrete sections, we have calculated the cracking moment M_{cr} according to the formula (20) established based on bilinear stress-strain model as well as according to the approximate formula (21). The calculation parameters are summarized in Table 2.

Table 2. The parameters used in the calculation of cracking momet of rectangular reinforced concrete section.

Meaning	Symbol	Unit	Value
Grade of compressive strength of concrete	GR_b	-	B25
Grade of tensile reinforcement	GR_s	-	CB400V
Grade of compressive reinforcement	GR_s'	-	CB400V
Width of cross section	b	(mm)	250
Height of cross section	h	(mm)	500
Distance from the center of gravity of tensile and compressive reinforcement to the tensile and compressive edge of the section, respectively	a a′	(mm) (mm)	60 40
Modulus of elasticity of concrete	E_b	(GPa)	30
Modulus of elasticity of tensile reinforcement	E_s	(GPa)	200
Modulus of elasticity of compressive reinforcement	E_s'	(GPa)	200
Tensile strength of concrete (serviceability limit state)	R_{bt}	(MPa)	1.55
Ratio of tensile reinforcement	μ_s	(%)	0.0 ÷ 3.0
Ratio of compressive reinforcement	μ_s'	(%)	0.0 ÷ 1.5

Figure 2 shows the variation of the cracking moment M_{cr} calculated according to (20) and (21) as a function of tensile reinforcement ratio in the case of ignoring the influence of compressive reinforcement ($\mu'_s = 0$). It can be noted that the cracking moment M_{cr} calculated by both methods increases linearly with the tensile reinforcement ratio μ_s, the increase is significant, especially when calculating according to the method based on bilinear stress-strain model (the cracking moment increases more than twice when μ_s increases from 0% to 3%). The linear increase in cracking moment with the tensile reinforcement ratio has been reported in the literature [8]. This can be explained by the fact that the higher the tensile reinforcement ratio, the lower the stress and strain of tensile concrete and therefore the larger moment is required for the tensile strain of concrete to reach the limit value that causes concrete cracking.

It can also be observed in the Figure 2 that the approximate method (21) significantly underestimates the cracking moment compared to formula (20) established based on the bilinear stress-strain model, especially in the case of high tensile reinforcement ratio. The difference in cracking moment between the two methods is acceptable when the tensile reinforcement ratio is low (21% when $\mu_s = 0$), the difference is larger as the tensile reinforcement ratio is higher (44% when $\mu_s = 3\%$). The increase in cracking moment deviation between the two methods may arise from the fact that the formula (21) underestimates the stress and strain of the tensile reinforcement. Indeed, we have found that the stress in the tensile reinforcement calculated based on the bilinear stress-strain diagram of concrete is more than twice as high as the stress calculated based on the assumption that concrete behaves elastically. Because the cracking moment increases with the product of stress and ratio of tensile reinforcement, the higher the tensile reinforcement ratio, the more formula (21) underestimates the cracking moment compared to formula (20).

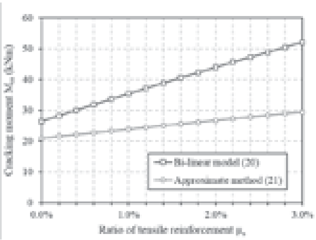

Figure 2. Variation of the cracking moment as a function of tensile reinforcement ratio in the case of ignoring the influence of compressive reinforcement ($\mu'_s = 0$).

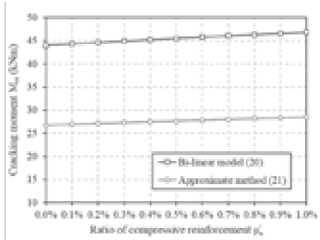

Figure 3. Variation of the cracking moment as a function of compressive reinforcement ratio in the case of tensile reinforcement ratio $\mu_s = 2\%$.

Figure 3 shows the variation of the cracking moment M_{cr} as a function of compressive reinforcement ratio μ'_s calculated according to (20) and (21) in the case of tensile reinforcement ratio $\mu_s = 2\%$. It can be noted that the cracking moment M_{cr} calculated by both methods also increases linearly with the compressive reinforcement ratio μ'_s. However, the increase is very slow, which shows that the influence of compressive reinforcement is insignificant compared to that of tensile reinforcement (Figure 2). This can be explained, from the point of view of elastic analyse used to establish formula (21), by the fact that both tensile and compressive reinforcement increase the moment of inertia of the section in the same way. However, the distance between the neutral axis and the tensile edge of the section decreases with the tensile reinforcement ratio but increases with the compressive reinforcement ratio. This leads to the cracking moment increasing rapidly with the tension reinforcement ratio but increasing slowly with the compressive reinforcement ratio.

5. CONCLUSION

The paper has established the formulas that allow determining the cracking moment of rectangular reinforced concrete sections, taking into account the nonlinear behavior of concrete simplified as bi-linear stress-strain diagram according to TCVN 5574 – 2018 [1]. The proposed formula has been validated experimentally with most errors below 7%. The paper has also performed detailed calculations of the cracking moment corresponding to different ratios of the tensile reinforcement and of the compressive reinforcement, compared the cracking moments obtained by the method established based on the bi-linear model and by the approximate method. The calculation results have shown that:

- The cracking moment increases linearly as a function of the tensile and compressive reinforcement ratios;
- The simple and approximate method presented in TCVN 5574 – 2018 [1] significantly underestimates the cracking moment compared to the method based on bi-linear stress-strain diagram. The difference between the two methods can be acceptable when the tensile reinforcement ratio is low (21% when $\mu_s = 0$), however this difference is larger and larger as the tensile reinforcement ratio is higher (44% when $\mu_s = 3\%$);
- The influence of compressive reinforcement is insignificant compared to that of tensile reinforcement.

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