LOWER-SIZE FILTERS DEVELOPMENT FOR MONITORING MOVING INVERTED PENDULUMS

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ABSTRACT

This paper presents an investigation in monitoring an inverted pendulum mounted on a moving cart using a low-order filter approach. The inverted pendulum system is a classic benchmark for control system analysis and design due to its inherent instability. In this paper, a lower-size observer-based control system is developed based on theory of functional observer and full-state feedback controller form to stabilize and control the motion of the moving inverted pendulum. The designed observer offers a flexible and effective framework to reconstruct the unmeasured and acceptable variables, making it well-suited for implementation of feedback control with any strategy. The proposed control system has small size and very easy for implementation due to the filter estimate the control signals as a functional state variable rather than all state vector's information. The effectiveness of the proposed control method is verified by comprehensive simulations.

Keywords: *Lower-size Filter, Inverted pendulum, Control Input Estimation.*

1. INTRODUCTION

Applications of moving inverted pendulum model is important and used in scientific research as well as practical point of view [1]. A control system of a self-balancing inverted pendulum requires the development of pendulum model, controller design regime and mechanical installation [2], [3]. Based on the novel of controlling moving inverted pendulum model, various applications have been deployed such as self-balancing two-wheeled vehicles, anti-vibration of high-rise buildings, human robot, control systems of launching spaceship, balancing offshore drilling platforms [4] - [6].

In this research, we present a novel feedback control for moving inverted pendulum. With the availability of a detailed mathematical model of the studied system, a state feedback control law can be easily designed to stabilise the systems. However, any applicable state feedback control law need accessibility of all the state variables to generate a control input signal [7]. To handle this issue, full-order observer-based controllers where state observers were used to reconstruct the unmeasured states would be used. However, the control schemes require large amount of information in real-time and on-line. On the other hand, functional filters estimate linear functions of the state vector without estimating all the individual states and so lower the size and complexity of the designed filters [8]. Therefore, any designed state feedback control law can now be implemented in a simpler way by using a minimum-order filter. In our research, based on recent developments on reduced-order observers [9] - [11], a lower-size filter-based control regime is derived to manage the practical implementation of any given feedback control law. The control input signal can be reconstructed as the functional state variables.

2. SYSTEM DESCRIPTION AND PROBLEM STATEMENTS

In this paper, we consider a system of inverted pendulum mounted on a rectilinear vehicle (Fig. 1) including a moving carriage which only move on a horizontal axis, *Ox*, and a pendulum, be mounted on the body of the vehicle, which swing with a real time angle, *α(t).* In order to keep the pendulum balanced, a control force, $F(t)$, is applied to the vehicle. In Fig. 1, M, m, L are the mass of vehicle, mass of the pendulum and its length.

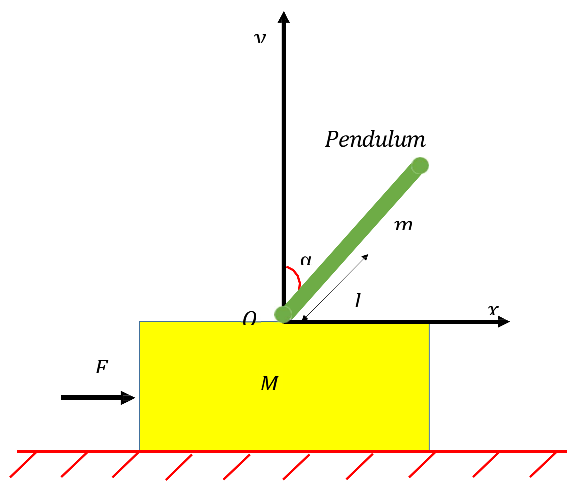


Fig. 1. *General model of a moving inverted pendulums*

By using some fundamental mathematical techniques, we obtain the dynamic description of the system

$$\begin{cases} (M+m)\ddot{x}(t) + ml\cos\big(\alpha(t)\big)\ddot{\alpha}(t) \\ \qquad +\mu\dot{x}(t) = \\ \quad ml\sin\big(\alpha(t)\big)\dot{\alpha}^2(t) + F(t), \\ (I+ml^2)\ddot{\alpha}(t) + m\ddot{x}l\cos\big(\alpha(t)\big) \\ \qquad = mgl\sin(\alpha)(t), \end{cases} \quad (1)$$

where I is moment of inertia corresponding to the center of the Pendulum, μ is the constant of of the friction force, $F_m = b\dot{x}(t)$.

By talking the linearisation for small value of α and with further calculations, we obtain

$$\begin{cases} \mathcal{Q}\ddot{x}(t) + m^2gl^2\alpha(t) + \mathcal{P}\mu\dot{x}(t) \\ \qquad = \mathcal{P}F(t) \\ \mathcal{Q}\ddot{\alpha}(t) - \mathcal{Z}\alpha(t) - ml\mu\dot{x}(t) \\ \qquad = -mlF(t) \end{cases} \quad (2)$$

where $\mathcal{Q} = (m + M)I + Mml^2, \mathcal{P} = I + ml^2, \mathcal{Z} = mgl(M + m)$.

Let we denote

$\Omega(t) = [x(t)\ \dot{x}(t)\ \alpha(t)\ \dot{\alpha}(t)]^T \in R^4$,

$y(t) = [x(t)\alpha(t)]^T \in R^2$ *and*

$u(t) = F(t) \in R$ are system state, output vectors and control signal respectively.

Accordingly, a state space representation of the studied systems can be expressed as follows

$$\begin{cases} \dot{\Omega}(t) = \mathcal{A}\Omega(t) + \mathcal{B}u(t) \\ \qquad yt = \mathcal{C}\Omega t \end{cases} \quad (3)$$

Where matrices $\mathcal{A} \in R^{4\times4}, \mathcal{B} \in R^{4\times1}, \mathcal{C} \in R^{2\times4}$ are

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{\mathcal{P}\mu}{\mathcal{Q}} & -\frac{mgl}{\mathcal{Q}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{ml\mu}{\mathcal{Q}} & \frac{\mathcal{Z}}{\mathcal{Q}} & 0 \end{bmatrix}$$

$$\mathcal{B} = \begin{bmatrix} 0 & -\frac{\mathcal{P}}{\mathcal{Q}} & 0 & -\frac{ml}{\mathcal{Q}} \end{bmatrix}^T$$

$$\mathcal{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Assume the feedback controller signal can be generate in the following form: $u(t) = K\Omega(t)$, the closed system is $\dot{\Omega}(t) = (A + BK)\Omega(t)$. There are many methods can be done to obtain matrix gain, K, such the system (3) is asymptotically stable. However, as we discussed in the Introduction, the proposed full-state feedback controller requires the available of all system state variables leading to the cost of implementation.

Motivated from the practical finding, the main purpose of this paper is to develop a reduced-order observer (filter) based feedback controller to maintain the vehicle and the pendulum at the desirable position.

3. SYSTEM MODEL AND FEEDBACK CONTROL SYSTEMS

Let we consider a lower-size filter in the following presentation

$$\begin{cases} \hat{u}(t) = \Upsilon\zeta(t) + \mathcal{L}y(t) \\ \dot{\zeta}(t) = \mathcal{N}\zeta(t) + \mathcal{M}y(t) + \mathcal{R}u(t) \end{cases} \quad (4)$$

where $\hat{u}(t)$ is the estimation of control signal $u(t), \zeta(t) \in R^r$ is the filter variable and $\Upsilon, \mathcal{L}, \mathcal{M}, \mathcal{R}$ are the filter's matrix with appropriate dimension.

Theorem 1 The controller signal estimation, $\hat{u}(t)$ converges asymptotically to the desirable control signal, $u(t) = K\Omega(t)$ if the following conditions hold

$\dot{\epsilon}(t) = \mathcal{N}\epsilon(t)$ is assymtotically stable,

$$\epsilon(t) = \zeta(t) - \mathcal{F}\Omega(t) \quad (5)$$

$$\mathcal{N}\mathcal{F} + \mathcal{M}\mathcal{C} - \mathcal{F}\mathcal{A} = 0; \mathcal{R} - \mathcal{F}\mathcal{B} = 0 \quad (6)$$

$$\mathcal{K} - \Upsilon\mathcal{F} - \mathcal{L}\mathcal{C} = 0. \quad (7)$$

Proof of Theorem 1: By some calculations, we can obtain $\dot{\epsilon}(t) = \mathcal{N}\epsilon(t) + (\mathcal{NF} + \mathcal{MC} - \mathcal{FA})\Omega(t) + (\mathcal{R} - \mathcal{FB})u(t) = 0$. Conditions (5)-(6) are satisfied, $\epsilon(t) \to 0, t \to \infty$.

We denote $e(t) = \hat{u}(t) - u(t) = \Upsilon\zeta(t) + \mathcal{LC}\Omega(t) - \mathcal{K}\Omega(t)$. Condition (7) holds, further we obtain, $e(t) = \Upsilon\big(\zeta(t) - \mathcal{F}\Omega(t)\big) \to 0$.

Proof is completed.

Accordingly, the filter based controller can be implemented in the following form

$$\begin{cases} u(t) = \Upsilon\zeta(t) + \mathcal{L}y(t) \\ \dot{\zeta}(t) = (\mathcal{N} + \mathcal{R}\Upsilon)\zeta(t) \\ \quad +(\mathcal{M} + \mathcal{R}\mathcal{L})y(t) \end{cases} \quad (8)$$

With the controller (8), the agumented closed-loop systems becomes

$$\dot{\mathcal{Y}}(t) = \mathcal{H}\mathcal{Y}(t) \quad (9)$$

where

$$\mathcal{H} = \begin{bmatrix} \mathcal{A} + \mathcal{BLC} & \mathcal{B}\Psi \\ (\mathcal{M} + \mathcal{RL})\mathcal{C} & (\mathcal{N} + \mathcal{R}\Upsilon) \end{bmatrix},$$

$$\mathcal{Y}(t) = \begin{bmatrix} \Omega(\mathrm{t}) \\ \zeta(\mathrm{t}) \end{bmatrix}.$$

Theorem 2: System (9) is asymtotically stable if exist matrices $\mathcal{R}, \mathcal{F}, \mathcal{L}, \mathcal{M}, \mathcal{N}, \Upsilon$ with appropritate dimension such the following conditions hold

$X(\mathcal{A} + \mathcal{BK})\&\mathcal{N}$ are hurwitz;

$\mathcal{NF} + \mathcal{MC} - \mathcal{FA} = 0$;

$\mathcal{R} - \mathcal{FB} = 0$; $\mathcal{K} - \Upsilon\mathcal{F} - \mathcal{LC} = 0$.

4. EFFECTIVENESS OF PROPOSED ALGORITHM

In this section, we undertake simulations to show the effectiveness of our proposed control scheme. The simulation data be taken from [1]- [6] is $M = 500, m = 50, b = 0.1, l = 3, I = 300$. According to the data, matrices $\mathcal{A}, \mathcal{B}$ in (3) can be obtained. In this simulation, three scenarios of the initial values of pendulum's angle and vehicle's position are

Scenario 1: - $\alpha_0 = 11^0 = 0.192$ rad and $x_0 = -3$m;

Scenario 2: - $\alpha_0 = 8^0 = 0.1396$ rad and $x_0 = -2$m;

Scenario 3: - $\alpha_0 = 5^0 = 0.083$ rad and $x_0 = -1$m .

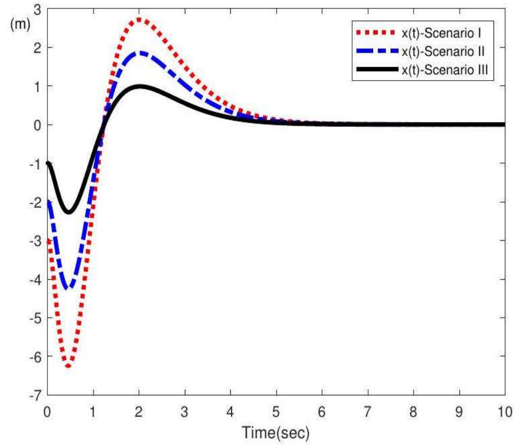
With the initial values of the pendulum's angle and vehicle's position, the system will be unstable without any control action. Our control purpose is to restore the system stability and to maintain $\alpha_0 = 0$ and $x_0 = 0$. At first, with the availability of the state space model (3), an optimal full state feedback controller [7] can be derived in the form of $\mathcal{K} = [-4097.4 \quad -7659.1 \quad 88467.9 \quad 50263.8]$

In this paper, the filter will estimate the functional state variable $\hat{u} = \mathcal{K}\Omega(t)$, hence the order of filter is only one. By solving condition (10), (see [8]-[10]), the filter's gain matrices can be obtained as follow:

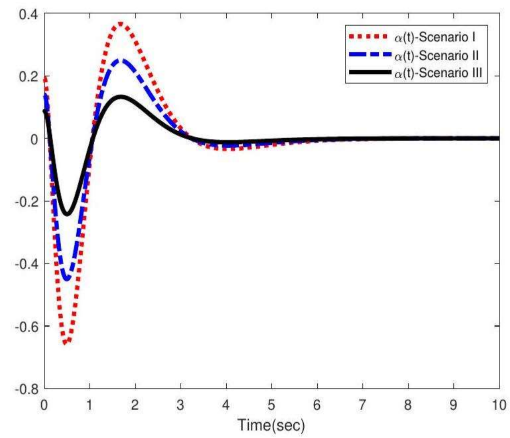
$\Upsilon = 1, \mathcal{N} = -2, \mathcal{R} = -9.3463$,

$\mathcal{L} = 10^5 \times [-0.1941 \quad 1.8900]$,

$\mathcal{M} = 10^4 \times [3.0629 \quad -4.9594]$.



(a) Vehicle position, $x(t)$



(b) pendulum's angle,$\alpha(t)$

Fig. 2. *The responses of closed-loop system: (a)$x(t)$; (b)$\alpha(t)$*

Fig. 2 demonstrates the responses of pendulum's angle, α(t) and vehicle's position, x(t) of the closed-loop system embedded the proposed filter based optimal controller. As can be seen that, for any initial values of α(t) and x(t), our controller can bring the pendulum's angle and vehicle's position back to the desirable values after some seconds of settling times, which verifies the effectiveness of our proposed control strategy.

CONCLUSION

In this paper, a lower-size filter-based control regime has been derived to stabilize the motion of the moving inverted pendulum. The solution has offered a flexible and versatile platform for the implementation of any full feedback control algorithm. Unlike conventional approach that relied on extensive state vector information, the proposed filter has estimated control signals as functional state variables, streamlining the implementation process, the use of a low-order filter combined with the functional observer framework provides a promising avenue for addressing complex control problems in various domains.

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