

## FAST INTEGRAL TERMINAL SLIDING MODE CONTROL FOR AN UNDERACTUATED QUADROTOR WITH THE EXTERNAL DISTURBANCE AND INPUT SIGNAL DELAY

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ARTICLE INFO		ABSTRACT
<b>Received:</b>	<b>04/6/2024</b>	This paper is invested to study trajectory tracking problem for Quadrotor unmanned aerial vehicle (UAV) subject to the external disturbance and input time delay. In this study, the Quadrotor with six degrees of freedom is considered as a highly nonlinear system, based on the Euler-Newton formulation, a math model of Quadrotor consists of translational and rotational systems is given. According to Pade approximation, a study is given about reducing time input delay. Then, a new robust control based on Integral Fast Terminal Sliding Mode Control (IFTSM) algorithm is proposed for both translational and rotational systems of quadrotor. The IFTSM control law: (i.) fast convergence errors; (ii.) chattering reduction; (iii.) to address the external disturbance and input time delay by its robustness. In addition, using the Lyapunov stability theory, the stability of the close- loop quadrotor system is guaranteed. Finally, a simulation built in MATLAB is given to prove the effectiveness of the proposed control law.
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Input Time Delay		

## ĐIỀU KHIỂN TRƯỢT TÁC ĐỘNG NHANH CHO HỆ QUADROTOR THIỂU CƠ CẤU CHẤP HÀNH DƯỚI ẢNH HƯỞNG CỦA NHIỄU VÀ TRỄ TÍN HIỆU ĐẦU VÀO

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THÔNG TIN BÀI BÁO		TÓM TẮT
<b>Ngày nhận bài:</b>	<b>04/6/2024</b>	Bài viết là công trình nghiên cứu về vấn đề bám quỹ đạo của Quadrotor dưới sự xem xét ảnh hưởng của nhiễu loạn bên ngoài và trễ tín hiệu đầu vào. Mỗi Quadrotor được xem xét gồm 6 bậc tự do, bằng sử dụng định luật Euler-Newton, một mô hình động học của Quadrotor được đưa ra gồm 2 hệ thống con: hệ tịnh tiến và hệ chuyển động quay. Theo xấp xỉ Pade, một nghiên cứu về giảm thiểu ảnh hưởng của trễ tín hiệu đầu vào được đề cập trong bài báo. Tiếp đó, một bộ điều khiển trượt tác động nhanh bền vững được thiết kế cho cả 2 hệ thống con. Với luật điều khiển được đề xuất sẽ đảm bảo: i. Thời gian hội tụ sai lệch nhanh, ii. Giảm hiện tượng chattering, iii. Xử lý vấn đề nhiễu và trễ tín hiệu đầu vào bằng tính bền vững trong luật điều khiển. Thêm vào đó, bằng sử dụng Lý thuyết ổn định Lyapunov, sự ổn định của hệ thống sẽ được chứng minh và đảm bảo. Cuối cùng, một mô phỏng được xây dựng trên phần mềm Matlab được đưa ra để kiểm chứng sự hiệu quả của phương án đề xuất.
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## 1. Introduction

In recent years, unmanned aerial vehicles (UAVs) have garnered significant attention within the research community due to their vast potential applications in numerous tasks and fields where human presence may be challenging to attain. These applications include disaster assessment, wildfire detection, agricultural applications, and more. Quadrotor of the typical types of unmanned aerial vehicles which has been increasingly popular due to its capability of taking off and landing vertically, maintaining position stability, and flexible trajectory. Consequently, one of the common and significant challenges posed is the problem of control for quadrotors. This is a non-trivial problem due to the quadrotor's 6 degrees of freedom with 4 control inputs. Furthermore, in practical problems and applications, accurately knowing the object model is often difficult to achieve, coupled with frequent model uncertainties. Therefore, addressing model uncertainties is a crucial requirement in designing control systems for quadrotors.

In numerous previous studies, various control structures have been proposed to achieve good control quality, including PID controllers [1], [2], LQR controllers [3], backstepping controllers [4], [5] and sliding mode controllers [6]-[8]. In recent works [6]-[12], the nonlinear quadrotor is studied; by using Euler- Lagrange principle, a math mode of quadrotor is given with two subsystems (translational subsystem and rotational subsystem). In [8], an adaptive backstepping fast terminal sliding mode control (ABFTSMC) is developed to track a desired trajectory of a 6DOF quadrotor attitude and position in the finite- time. Additionally, many observer-based methods [9]- [11] are employed to compensate for dynamic model inaccuracies of the ideal quadrotor model, followed by the design of a nonlinear feedback controller to address the tracking control problem.

In practical engineering applications, input time delay usually exists and is a result of the low of performance. To eliminate the influence of input time delay, in this work in [13] the adaptive control approach was employed for the SISO high-order nonlinear systems. Based on Lyapunov–Krasinski theory, a robust compensator was proposed to address the unknown fixed time delay [9]. In the work [11], the input time delay is studied for quadrotor system, where, by using Pade approximation, the small-time delay can be eliminated.

In this work, a robust controller for a quadrotor UAV for both position and attitude is developed. The main goal of the paper is to track the position of quadrotor UAV to the desired reference trajectory under the external disturbance, wind gust and input time delay. The stability of closed loop of the quadrotor system and the global convergence of the position and attitude tracking errors are proof by the Lyapunov stability theory. Inspired by the work in [11], by using Pade approximation and Laplace transform, the time input delay can be eliminated. Then, a new robust control based on Integral Fast Terminal Sliding Mode Control (IFTSM) algorithm is proposed for both translational and rotational systems of quadrotor. The main contribution of the paper.

i) By using the new IFTSMC algorithm, the position and attitude tracking errors of quadrotor convergence to zero asymptotically in the fast finite- time.

ii) By the strong robustness of the proposed control law, the external disturbance and input time delay of quadrotor system can be eliminated.

iii) By using integral component in the sliding surface and  $\tanh(x)$  function in the switching control, the chattering is reduction.

This paper is organized as follows. Section 2 presents the dynamic model of quadrotor and the control problem. Then, the proposed position and attitude control schemes based IFTSM algorithm, and the stability analysis of the closed- loop system is given to achieve path tracking. In Section 3, a simulation result is given to verify the proposed control law. Section 4 concludes the paper.

## 2. Problem formulation and control design

### 2.1. Problem formulation

#### 2.1.1. Dynamic model of quadrotor

The configuration of quadrotor UAV is described by Figure 1 in [8]. This system is a rigid body with four rotors and six degrees of freedoms following three directions  $(x, y, z)$  and three motion  $(\phi, \theta, \psi)$ . Because of four inputs while six output, the system is underactuated and unstable [2]. Besides, in this paper, the quadrotor is considered under some assumptions as following as

i. The quadrotor is rigid body, so the dynamic model of quadrotor can be analyzed by the Euler-Newton formulation [14].

ii. The desired forces and moments to control the attitude motion of quadrotors can be generated by the four rotors [10], [15].

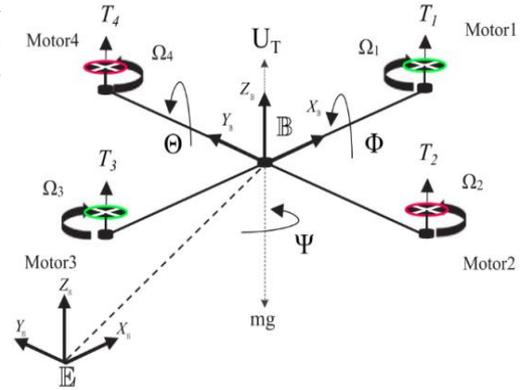


Figure 1. The quadrotor UAV [8]

Let  $B = [x_B, y_B, z_B]$  donates the body fixed frame,  $E = [x_E, y_E, z_E]$  donates the earth fixed inertial frame. Let  $p = [x, y, z]$  represent the position of quadrotor in the earth fixed inertial frame,  $\varepsilon = [\phi, \theta, \psi]$  represent the Euler angle (roll angle  $\Phi$ , pitch angle  $\theta$ , and yaw angles  $\psi$ ), satisfy that:  $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$  and  $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ . Although the behavior of quadrotor is described in the body fixed frame, the position of quadrotor is described in the earth fixed inertial frame. So, it needs a rotation matrix [8] which transfers the coordinates from the body fixed frame to the earth fixed inertial frame.

$$\mathbf{R}_{B2E} = \begin{bmatrix} c_\psi c_\theta & s_\theta s_\phi s_\psi - s_\psi c_\phi & c_\phi c_\psi c_\theta + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\psi s_\theta + c_\phi s_\psi & c_\phi s_\psi s_\theta + s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \quad (1)$$

With  $S_i = \sin(i)$  and  $C_i = \cos(i)$ . Based on the Euler-Lagrange formulation, following the work in [8], the translational dynamic and rotational dynamic of quadrotor is given:

$$\begin{aligned} \ddot{x} &= -\frac{K_x}{m} \dot{x} + \frac{1}{m} (c_\phi c_\psi s_\theta + s_\phi s_\psi) U_T + d_x \\ \ddot{y} &= -\frac{K_y}{m} \dot{y} + \frac{1}{m} (c_\phi s_\psi s_\theta + s_\phi c_\psi) U_T + d_y \\ \ddot{z} &= -\frac{K_z}{m} \dot{z} + \frac{1}{m} (c_\phi c_\theta) U_T - g + d_z \\ \ddot{\phi} &= \frac{1}{J_\phi} (\dot{\theta} \dot{\psi} (J_\theta - J_\psi) - J_r \dot{\theta} w - K_\phi \dot{\phi}^2 + U_\phi) + d_\phi \\ \ddot{\theta} &= \frac{1}{J_\theta} (\dot{\phi} \dot{\psi} (J_\psi - J_\phi) - J_r \dot{\phi} w - K_\theta \dot{\theta}^2 + U_\theta) + d_\theta \\ \ddot{\psi} &= \frac{1}{J_\psi} (\dot{\phi} \dot{\theta} (J_\phi - J_\theta) - K_\psi \dot{\psi}^2 + U_\psi) + d_\psi \end{aligned} \quad (2)$$

Where  $w = \Omega_4 + \Omega_3 - \Omega_2 - \Omega_1$  ( $\Omega_i$  ( $i = 1, 2, 3, 4$ ) are the speed of rotors),  $J_r$  is the rotor inertia,  $d_i$  ( $i = x, y, z, \phi, \theta, \psi$ ) donate the external disturbance,  $J_\phi, J_\theta, J_\psi$  represent the rotary inertia respect

to x-axes, y-axes, and z-axes, respectively. The  $U_T$  and  $[U_\phi, U_\theta, U_\psi]$  are the desired control forces following as:

$$\begin{aligned} U_T &= k_t (\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_\phi &= k_t l_t (\Omega_4^2 - \Omega_2^2) \\ U_\theta &= k_t l_t (\Omega_3^2 - \Omega_1^2) \\ U_\psi &= k_w (\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{aligned} \tag{3}$$

With  $k_p, k_w$  are quadrotor parameter,  $l_t$  is the distance from the center mass of quadrotor to rotor.

*Remark 1.* The behavior of quadrotor UAV is considered under influence of atmosphere, in where  $K_i (i=x, y, z)$  are the translation drag coefficient, and  $K_i (i=\phi, \theta, \psi)$  are aerodynamic friction coefficients.

*Remark 2.* The external disturbance  $d_i (i=x, y, z, \phi, \theta, \psi)$  are unknown, but they are assumed that  $d_i, \dot{d}_i$  are bounded.

So that the convenience of control design, the dynamic model of quadrotor is rewritten following as:

$$\begin{aligned} \ddot{p} &= -\frac{K_p}{m} \dot{p} + u_p + d_p \\ \ddot{\epsilon} &= f_\epsilon(\dot{\epsilon}) + J^{-1} \tau_\epsilon + d_\epsilon \end{aligned} \tag{4}$$

Where  $K_p = \text{diag}([K_x, K_y, K_z])$  is drag matrix,  $J = \text{diag}([J_\phi, J_\theta, J_\psi])$  donates the inertia matrix,  $f_\epsilon(\epsilon) \in R^3$  is nonlinear function vector,  $d_p = [d_x, d_y, d_z]^T$  and  $d_\epsilon = [d_\phi, d_\theta, d_\psi]^T$  are external disturbance vectors.

$$\begin{aligned} u_p &= \begin{bmatrix} u_{px} \\ u_{py} \\ u_{pz} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} c_\phi c_\psi s_\theta + s_\phi s_\psi \\ c_\phi c_\psi s_\theta - s_\phi c_\psi \\ c_\phi c_\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \\ \tau_\epsilon &= \begin{bmatrix} u_\phi \\ u_\theta \\ u_\psi \end{bmatrix} \end{aligned} \tag{5}$$

### 2.1.2. Input time delay

In practical engineering applications, input time delay usually exists and is a result of the low of performance. That is to say, the input time delay can be written as  $u(t-\tau)$  in which  $\tau$  is time delay. To track the work of [16], based on Laplace transform and Pade approximation, with small time delay, the input time delay can be solved by:

$$\begin{aligned} u(t-\tau) &= \xi - u(t) \\ \dot{\xi} &= \frac{4}{\tau} u(t) - \frac{2}{\tau} \xi \end{aligned} \tag{6}$$

Where  $\xi$  is intermediate variable.

*Remark 3.* Although the intermediate variable  $\xi$  is not real state of system, it can be viewed as an error variable that needs to be addressed. This means that the problem of input delay will be eliminated.

*Remark 4.* To apply this work to quadrotor, in the study [11], the input time delay can be eliminated by a robustness control scheme.

2.1.3. Integral fast terminal sliding mode control

The traditional fast terminal sliding mode (FTSM) control [17] is followed by.

$$s = \dot{x} + \alpha x + \beta |x|^\lambda \text{sign}(x) \tag{7}$$

where  $s$  is sliding surface,  $0 < \lambda < 1$ ,  $\alpha$  and  $\beta$  are positive constant. When  $s = 0$ , in result in  $\dot{x} = -\alpha x - \beta |x|^\lambda \text{sign}(x)$ , which will reach  $x \rightarrow 0$  in the fast finite time. But, using  $\text{sign}$  in the sliding surface which is result of high chattering. To address this problem, in this paper, a new integral fast terminal sliding mode (IFTSM) control is proposed which is following by.

$$s = \dot{x} + \alpha x + \beta \int_0^t |x(\tau)|^\lambda \text{sign}(x(\tau)) d\tau \tag{8}$$

With the new integral fast terminal sliding mode (IFTSM) control proposed, the chattering problem is reduced.

2.2. Control design

This section is devoted to designing a controller for the quadrotor system. The control objective of the paper is to track the position of quadrotor UAV to the desired reference trajectory under the external disturbance, wind gust and input time delay. To achieve this goal, a structure with two loops (Position loop and Attitude loop) shown in Figure 2 is used and a new IFTSMC scheme is proposed for both two loops. The position loop with the new IFTSMC generates a virtual position control signal which tracks the position of quadrotor UAV with the desired reference trajectory. Then, a function transfer [8] is used to computing the desired reference attitude signal for the rotational subsystem and total thrust  $U_T$  for the translation subsystem of quadrotor under the assumption that the yaw angle  $\psi$  is constant (usually, the yaw angle set equal zero). Finally, the attitude loop with the new IFTSMC generates the desired control signal  $[U_\phi, U_\theta, U_\psi]$  for quadrotor. With the proposed control, the input time delay and the external disturbance is eliminated by its strong robustness, and the tracking error convergence in the fast finite-time. Besides, with integral component in the sliding surface and  $\tanh(x)$  function in the switching control, the chattering is reduction, this means that the desired reference attitude signal is smoother, and the desired control input is less chattering. Based on the Lyapunov stability theory, the stability of the close- loop system is proved.

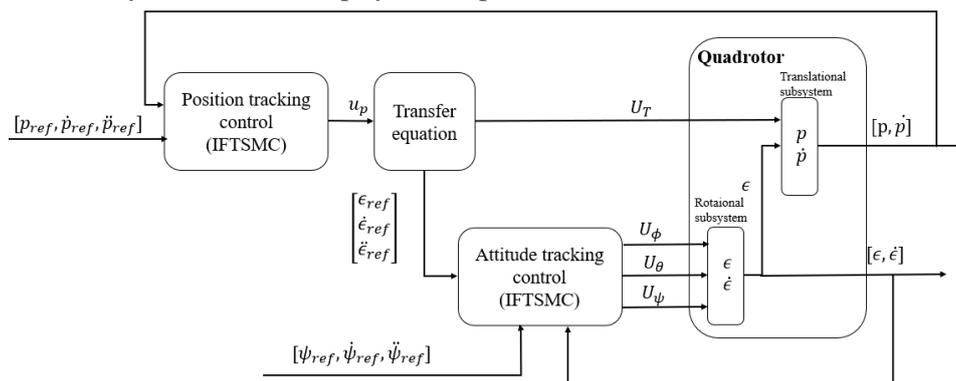


Figure 2. Control structure for quadrotor

2.2.1. Position tracking control based on IFTSMC method

This objective control is to ensure the position tracking error convergence to zero in the fast finite-time. Following the IFTSMC in Section 2, the sliding surface is chosen following as:

$$s_p = \dot{e}_p(t) + k_{p1}e_p(t) + k_{p2} \int_0^t |e_p(\tau)|^{\alpha_p} \text{sign}(e_p(\tau)) d\tau \tag{9}$$

Where  $e_p = p - p_{ref}$  donates the position tracking error,  $p_{ref} = [x_{ref}, y_{ref}, z_{ref}]^T$  is desired position reference. The control parameter  $\alpha_p \in (0,1)$ ,  $k_{p1} = \text{diag}([k_{p11}, k_{p12}, k_{p13}]) \in R^{3 \times 3}$ , and  $k_{p2} = \text{diag}([k_{p21}, k_{p22}, k_{p23}]) \in R^{3 \times 3}$  ( $k_{pij}, (i=1,2; j=1,2,3)$  are positive constants). Take time derivative of the sliding surface.

$$\dot{s}_p = -\frac{K_p}{m} \dot{p} + u_p + d_p - \ddot{p}_{ref} + k_{p1}\dot{e}_p + k_{p2}|e_p(\tau)|^{\alpha_p} \text{sign}(e_p(\tau)) \tag{10}$$

By setting the sliding surface equal zero, the equivalent position control laws can be obtained by:

$$u_{p\_eq} = \frac{K_p}{m} \dot{p} + \ddot{p}_{ref} - k_{p1}\dot{e}_p - k_{p2}|e_p(\tau)|^{\alpha_p} \text{sign}(e_p(\tau)) \tag{11}$$

To reject the input time delay and external disturbance, a robust switching control law is proposed and  $u_{p\_sw} = -k_{p3}s_p - k_{p4} \tanh(s_p)$  is add in the equivalent position control laws where  $k_{p3} = \text{diag}([k_{p31}, k_{p32}, k_{p33}]) \in R^{3 \times 3}$ , and  $k_{p4} = \text{diag}([k_{p41}, k_{p42}, k_{p43}]) \in R^{3 \times 3}$  satisfy  $k_{p4i} \geq \max(\|d_p(i)\|)$  ( $k_{pij}, (i=1,2; j=1,2,3)$  are positive constants). Hence, the virtual position control input can be obtained  $u_p = u_{p\_eq} + u_{p\_sw}$ .

Choosing the Lyapunov function  $V_p = 0.5s_p^T s_p$ , the time derivative of  $V_p$  along translational subsystem

$$\dot{V}_q = s_p^T \dot{s}_p \leq -k_{p2} s_p^T s_p \leq 0 \tag{12}$$

The stability of translational subsystem has been proved.

### 2.2.2. Attitude tracking control based on IFTSMC method

Using the virtual position control input in the previous subsection, the desired attitude  $\epsilon_{ref} = [\phi_{ref}, \theta_{ref}, \psi_{ref}]$  and total thrust  $U_T$  can be obtained by using the equation and setting the desired yaw angle equal zero  $\psi_{ref} = 0$ .

$$U_T = m \sqrt{u_{px}^2 + u_{py}^2 + (u_{pz} + g)^2}$$

$$\phi_{ref} = \arctan \left( \cos(\theta_{ref}) \left( \frac{u_{px} \sin(\psi_{ref}) - u_{py} \cos(\psi_{ref})}{u_{pz} + g} \right) \right) \tag{13}$$

$$\theta_{ref} = \arctan \left( \frac{u_{px} \cos(\psi_{ref}) + u_{py} \sin(\psi_{ref})}{u_{pz} + g} \right)$$

Following the IFTSMC scheme in Section 2, a robust control law  $u_\epsilon = u_{\epsilon\_eq} + u_{\epsilon\_sw}$  is designed to similar Subsection 3.1 for the rotation subsystem following as:

$$u_{\epsilon\_eq} = J \left( -f_\epsilon(\dot{\epsilon}) + \ddot{\epsilon}_{ref} - k_{e1}\dot{e}_\epsilon - k_{e2}|e_\epsilon(\tau)|^{\alpha_\epsilon} \text{sign}(e_\epsilon(\tau)) \right) \tag{14}$$

$$u_{\epsilon\_sw} = J \left( -k_{e3}s_\epsilon - k_{e4} \tanh(s_\epsilon) \right)$$

Where  $e_\epsilon = \epsilon - \epsilon_{ref}$  donates the attitude tracking error,  $s_\epsilon$  is sliding surface following by:

$$s_\epsilon = \dot{e}_\epsilon(t) + k_{e1}e_\epsilon(t) + k_{e2} \int_0^t |e_\epsilon(\tau)|^{\alpha_\epsilon} \text{sign}(e_\epsilon(\tau)) d\tau \tag{15}$$

Where  $\alpha_p \in (0,1)$ ,  $k_{e1} = \text{diag}([k_{e11}, k_{e12}, k_{e13}]) \in R^{3 \times 3}$ ,  $k_{e2} = \text{diag}([k_{e21}, k_{e22}, k_{e23}]) \in R^{3 \times 3}$ ,  $k_{e3} = \text{diag}([k_{e31}, k_{e32}, k_{e33}]) \in R^{3 \times 3}$ , and  $k_{e4} = \text{diag}([k_{e41}, k_{e42}, k_{e43}]) \in R^{3 \times 3}$  satisfy  $k_{e4i} \geq \max(\|d_{\epsilon}(i)\|)$  ( $k_{eij}, (i = 1, 2, 3, 4; j = 1, 2, 3)$  are positive constants).

Choosing the Lyapunov function  $V_{\epsilon} = 0.5s_{\epsilon}^T s_{\epsilon}$ , the time derivative of  $V_{\epsilon}$  along rotational subsystem

$$\dot{V}_{\epsilon} = s_{\epsilon}^T \dot{s}_{\epsilon} \leq -k_{e2} s_{\epsilon}^T s_{\epsilon} \leq 0 \tag{16}$$

The stability of rotational subsystem has been proved.

### 3. Simulation result

In this section, the performance of the proposed controller is simulated in the MATLAB software. The quadrotor is used in this simulation with the parameter  $m = 1.7$  (kg),  $k_w = 1$ ,  $k_t = 1$ ,  $l_t = 0.2$  (m),  $J = 10^{-3} \text{diag}([4.8, 4.9, 9.8])$  (Nm),  $K_x = K_y = K_z = 5.67e - 4$  (Ns/m),  $K_{\phi} = K_{\theta} = K_{\psi} = 5.67e - 4$  (Ns/rad). This system is considered under external disturbance noise  $d_p = 0.1 \times \sum_{i=1}^{10} \text{rand}(0,1); d_e = 0.02 \times \sum_{i=1}^{10} \text{rand}(0,1)$  and input delay 20 microseconds. The desired position reference is chosen as  $p_{ref} = [\sin(0.5t), 2.0\cos(0.5t), 2.0 + \sin(t)]$ . The parameters of the proposed control law are  $\alpha_p = \alpha_{\epsilon} = 0.5$ ,  $k_{p1} = k_{e1} = \text{diag}([4.0, 4.0, 4.0])$ ,  $k_{p2} = k_{e2} = \text{diag}([0.2, 0.2, 0.2])$ ,  $k_{p3} = k_{e3} = \text{diag}([5.0, 5.0, 5.0])$ ,  $k_{p4} = k_{e4} = \text{diag}([0.1, 0.1, 0.1])$ .

To verify the effectiveness of the proposed controller, a nominal sliding model control is given:

$$\begin{aligned} u_{p_{smc}} &= \ddot{p}_{ref} + \frac{K_p}{m} \dot{p} - c_{p1} \dot{e}_p - c_{p2} s_{p_{smc}} - c_{p3} \text{sign}(s_{p_{smc}}) \\ u_{\epsilon_{smc}} &= \ddot{\epsilon}_{ref} - f_{\epsilon}(\dot{\epsilon}) - c_{e1} \dot{e}_{\epsilon} - c_{e2} s_{\epsilon_{smc}} - c_{e3} \text{sign}(s_{\epsilon_{smc}}) \end{aligned} \tag{17}$$

Where  $s_{p_{smc}} = c_{p1} e_p + \dot{e}_p$  and  $s_{\epsilon_{smc}} = c_{e1} e_{\epsilon} + \dot{e}_{\epsilon}$  are sliding surfaces,  $c_{p1} = c_{e1} = \text{diag}([1.0, 1.0, 1.0])$ ,  $c_{p2} = c_{e2} = \text{diag}([3.0, 3.0, 3.0])$ , and  $c_{p3} = c_{e3} = \text{diag}([0.1, 0.1, 0.1])$  are control parameters.

Figure 3 shows the position tracking between the desired position reference with the real position of quadrotor. Figure 4 shows the attitude error tracking of quadrotor. The result shows that the proposed method achieves high tracking performance in the fast finite time. Besides, under the influence of external disturbance and input time delay, the proposed control law robust with negative factor. Figure 5 shows the desired control inputs of quadrotor. To view the action of quadrotor in real time, Figure 6 shows the trajectory of quadrotor in 3-dimensional space.

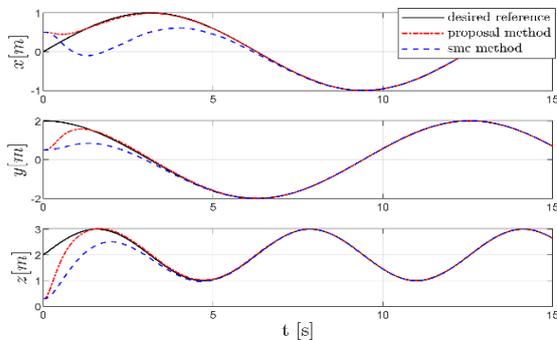


Figure 3. The position responses of quadrotor

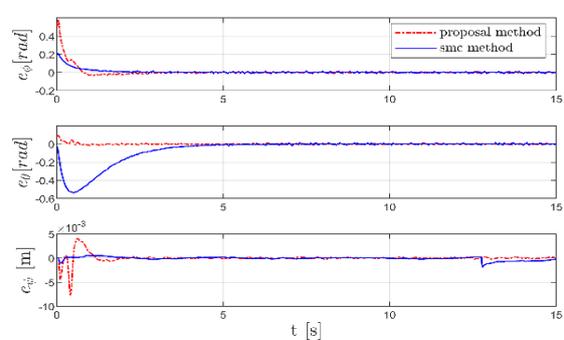


Figure 4. The attitude tracking errors of quadrotor

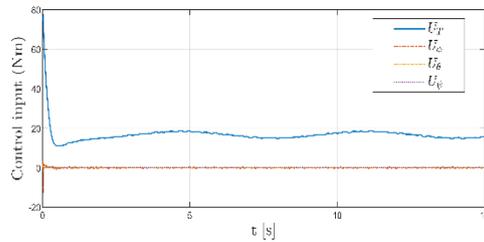


Figure 5. The control inputs of quadrotor

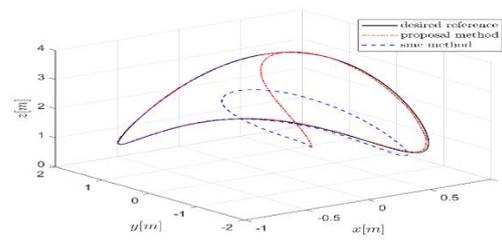


Figure 6. 3-D trajectory of quadrotor

#### 4. Conclusion

This paper focuses on investigating the trajectory tracking problem for a UAV in the presence of external disturbances and input time delays. To mitigate the effects of input time delays, Pade approximation is employed. Subsequently, a novel robust control strategy based on the Integral Fast Terminal Sliding Mode Control (IFTSM) algorithm is introduced for both translational and rotational dynamics of the quadrotor. The proposed IFTSM control law offers fast error convergence, reduced chattering, and robustness against external disturbances and input time delays.

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