DESIGNING AN ADAPTIVE CONTROL SYSTEM BASED ON INDIRECT MRAS FOR TWIN ROTOR MIMO SYSTEM

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ABSTRACT

This paper presents an indirect Model Reference Adaptive System design for a Twin Rotor MIMO system, where the nonlinear system exhibits significant cross-coupling between the two rotors, making the control process challenging. The main objective of the design is to accurately track the process's trajectory in space. Firstly, the nonlinear equations governing the system are provided and then linearized into two distinct linear equations. Based on these linearized equations, two indirect adaptive controllers are developed: one to control the pitch angle and the other to control the yaw angle of the system's beam. By applying Lyapunov's stability theory, the adaptive laws are established, ensuring stability and rapid convergence. The simulation results demonstrate that the system performs effectively, with the output signal successfully tracking the reference signal when the proposed controller is applied. These results demonstrate the effectiveness, reliability, and capability of the adaptive control strategy in accurately tracking the trajectory despite significant nonlinearities and cross-coupling in the system.

THIẾT KẾ HỆ THỐNG ĐIỀU KHIỂN THÍCH NGHI DỰA TRÊN MRAS GIÁN TIẾP CHO HỆ THỐNG TWIN ROTOR MIMO

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TỪ KHÓA

Thích nghi theo mô hình mẫu
Hệ thống Twin Rotor MIMO
Hệ thống phi tuyến
Điều khiển thích nghi
Lý thuyết ổn định Lyapunov

TÓM TẮT

Bài báo này trình bày thiết kế hệ thống thích nghi theo mô hình mẫu gián tiếp cho hệ thống Twin rotor nhiều đầu vào ra, hệ thống này là phi tuyến có sự xen kênh đáng kể giữa hai rotor, khiến quá trình điều khiển trở nên khó khăn. Mục tiêu chính của thiết kế là theo dõi chính xác quỹ đạo quá trình trong không gian. Ban đầu, các phương trình phi tuyến mô tả hệ thống được đưa ra và sau đó được tuyến tính hóa thành hai phương trình tuyến tính riêng biệt. Dựa trên các phương trình tuyến tính này, hai bộ điều khiển thích nghi gián tiếp được phát triển: một bộ điều khiển để điều chỉnh góc chao dọc và bộ còn lại để điều chỉnh góc đảo lái của cánh tay đòn tự do. Bằng cách áp dụng lý thuyết ổn định Lyapunov, các quy luật thích nghi được thiết lập, đảm bảo tính ổn định và hội tụ nhanh chóng. Kết quả mô phỏng cho thấy hệ thống hoạt động hiệu quả, với tín hiệu đầu ra bám tín hiệu đặt ở đầu vào khi áp dụng bộ điều khiển đề xuất. Những kết quả này chứng minh tính hiệu quả, độ tin cậy và khả năng của chiến lược điều khiển thích nghi trong việc theo dõi quỹ đạo chính xác dù có sự phi tuyến và xen kênh mạnh trong hệ thống.

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1. Introduction

The Twin Rotor MIMO System (TRMS) serves as a model for a helicopter, as shown in Figure 1. However, several significant simplifications are made in the model [1]. Controlling the motion trajectory of the TRMS is one of the most effective ways to test control algorithms for nonlinear systems. The key objectives for controlling the TRMS include:

- Tracking: Moving quickly and accurately to follow a given trajectory.
- Stability: Achieving a steady state around a desired point in state space in the shortest time possible.
- Robustness: Ensuring the system is sensitive to unknown parameters and resistant to disturbances.

In recent years, many studies have been conducted on TRMS to find effective ways to control it. Control methods such as PID control, full-state feedback, neural networks, fuzzy control, and genetic algorithms have been explored [2] – [7]. Conventional PID controllers, however, often do not provide robust performance in systems with parametric uncertainties and internal or external disturbances. Therefore, alternative approaches are being sought to improve control effectiveness in such systems.

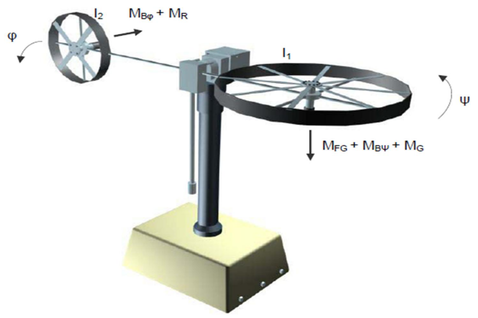

Figure 1. *TRMS*

In this study, the design of an indirect MRAS-based adaptive control system is developed for the TRMS, which acts on the errors to reject system disturbances and cope with parameter changes. In adjustable model reference adaptive systems, the desired closed-loop response is specified through the reaction of the process. The control system aims to make the process output similar to the reference model output [8] – [11]. The proposed controller is expected to improve tracking performance and increase robustness under the effects of disturbances and parameter changes. Two separate adaptive controllers are designed based on Lyapunov's stability theory to control the pitch and yaw angles of the TRMS' beam.

The paper is organized as follows: Section 2 introduces the design of adaptive laws for the adaptive controller based on the indirect MRAS. Section 3 presents the TRMS design and simulation results. Finally, the paper concludes with a summary.

2. Designing adaptive laws indirect MRAS

There are different types of adaptive system, but the general ideal behind Model Reference Adaptive System (MRAS) is to create a closed loop controller with parameters that can be updated to change the response of the system. The output of the system is compared to parameters are update based on this error. The goal is for the parameters to converge to ideal values that cause the process respond to match the response of the reference model [12] – [14]. The structure described in Figure 2 can be used as an indirect adaptive PD controller system. That is combination of the adjustable controller together with adjustable model. A second-order process is controlled with the aid of a PD-controller. The variation parameters of this controller are K_p and K_d. If variations in process parameters b_p and a_p is changed values, they will change b_m and a_m. Therefor, the K_p and K_d is changed values, too. We are going to find the form of the adjustment laws for b_m and a_m. The adaptive laws used in Figure 2 are similar to the ones used for direct adaptive control with the method of Lyapunov [10] – [12]. The adaptive laws used for second-order identification [8] are shown in formulas (1) and (2).

$$a_m = \frac{1}{\alpha_{am}} \int (p_{21}\, e_1 + p_{22} e_2)\ \widehat{\dot{y}_p}\, dt + a_m(0) \tag{1}$$

$$b_m = \frac{1}{\alpha_{bm}} \int (p_{21} e_1 + p_{22} e_2) u dt + b_m(0) \quad (2)$$

Where a_m, b_m, $\widehat{y_p}$, u, e and $\dot{e}$ are defined in Figure 2, p_{21} and p_{22} are elements of P matrix, obtained from the solution of Lyapunov equation indicated in Equation (3).

$$A_p^T P + P A_p = -Q \quad (3)$$

In this Equation (3), Q is a definite positive dependent matrix, and matrix A_p belongs to the process model. The matrix Q has to be chosen to find a positive P. The adaptive system is influenced by the choice Q. The performance of the system on the choice Q, the stability the resulting system can be guaranteed [12].

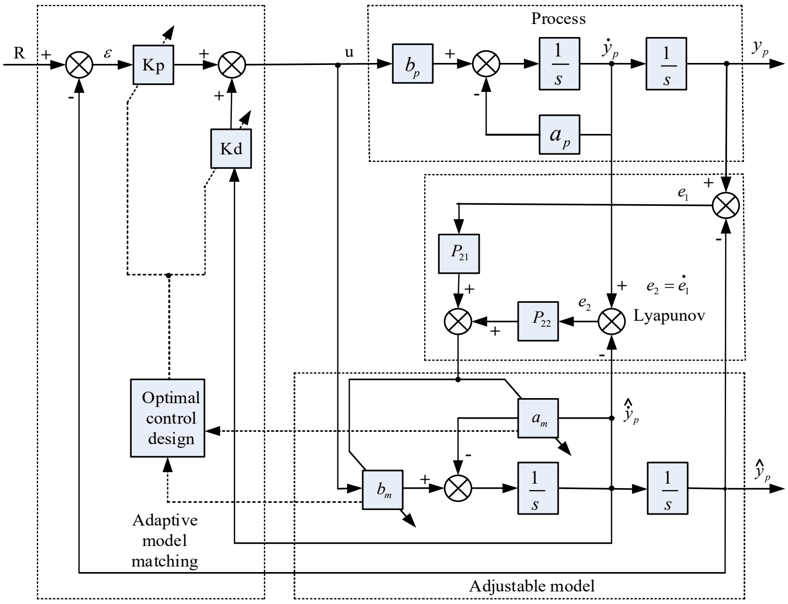

Figure 2. *Indirect adaptive system designed with Lyapunov [8]*

Otherwise, can detemine p_{21}, p_{22} parameters belong to $A_p \approx A_m$ [8], obtained:

$$p_{21} = \frac{1}{a_m}\left(\frac{q_{22}}{2a_m} + q_{21}\right);\ p_{22} = \frac{q_{22}}{2a_m} \quad (4)$$

Because the parameters a_p and b_p are unknown, with adjustable model [8], we have $a_m \approx a_p$; $b_m \approx b_p$. Therefore, the K_p and K_d values are obtained as follows:

$$K_p = \frac{\omega_0^2}{b_m};\ K_d = \frac{2\xi\omega_0}{b_m} \quad (5)$$

3. Simulation results

3.1. The mathematical model of TRMS

As presented in Section 1, the TRMS is a nonlinear model with significant cross-coupling. To control the TRMS, we need its mathematical model. The dynamic model of the TRMS can be represented in state-space form as follows:

$$\frac{d\Psi}{dt} = \dot{\Psi} \quad (6)$$

$$\frac{d\dot{\Psi}}{dt} = \frac{a_1}{I_1}\tau_1^2 + \frac{b_1}{I_1}\tau_1 - \frac{M_g}{I_1} sin\,\Psi - \frac{B_{1\Psi}}{I_1}\dot{\Psi} + \frac{0.0326}{2I_1} sin(2\Psi)\,\dot{\varphi}^2 - \frac{k_{gy}}{I_1} a_1 \, cos(\Psi)\,\dot{\varphi}\tau_1^2 + \frac{k_{gy}}{I_1} b_1 \cos(\Psi)\,\dot{\varphi}\tau_1 \quad (7)$$

$$\frac{d\varphi}{dt} = \dot{\varphi} \quad (8)$$

$$\frac{d\dot{\varphi}}{dt} = \frac{a_2}{I_2}\tau_2^2 + \frac{b_2}{I_2}\tau_2 - \frac{B_{1\varphi}}{I_2}\dot{\varphi} - \frac{1.75 k_c a_1}{I_2}\tau_1^2 - \frac{1.75 k_c b_1}{I_2}\tau_1 \quad (9)$$

$$\frac{d\tau_1}{dt} = -\frac{T_{10}}{T_{11}}\tau_1 + \frac{k_1}{T_{11}}u_1 \quad (10)$$

$$\frac{d\tau_2}{dt} = -\frac{T_{20}}{T_{21}}\tau_1 + \frac{k_2}{T_{21}}u_2 \quad (11)$$

Where Ψ: pitch (vertical) angle, φ: yaw (horizontal) angle, τ_1: main motor momentum, τ_2: tail motor momentum.

From Equations (6), (7), (8) and (9) we perform linearized, the state space representation of system is derived:

$$\begin{bmatrix}\dot{\Psi}\\ \ddot{\Psi}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{B_{1\Psi}}{I_1}\end{bmatrix}\begin{bmatrix}\Psi\\ \dot{\Psi}\end{bmatrix} + \begin{bmatrix}0\\ \frac{b_1}{I_1}\end{bmatrix}\tau_1 \quad (12)$$

$$\begin{bmatrix}\dot{\varphi}\\ \ddot{\varphi}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{B_{1\varphi}}{I_2}\end{bmatrix}\begin{bmatrix}\varphi\\ \dot{\varphi}\end{bmatrix} + \begin{bmatrix}0\\ \frac{b_2}{I_2}\end{bmatrix}\tau_2 \quad (13)$$

The model parameters are more or less chosen experimentally. Table 1 gives their approximate values.

Table 1. *TRMS model parameters*

Parameter	Value
I_1- moment of inertial of vertical rotor	$6.8x10^{-2}$ kgm^2
I_2 - moment of inertial of horizontal rotor	$2x10^{-2}$ kgm^2
a_1 - static characteristic parameter	0.0135
b_1 - static characteristic parameter	0.0924
a_2 - static characteristic parameter	0.02
b_2 - static characteristic parameter	0.09
M_g - Gravity momentum	0.32Nm
$B_{1\Psi}$- Friction momentum function parameter	$6x10^{-3}$ Nms/rad
$B_{1\varphi}$- Friction momentum function parameter	$1x10^{-1}$ Nms/rad
k_{gy} - Gyroscopic momentum parameter	0.05 s/rad
k_1 - Motor 1 gain	0.1
k_2 - Motor 2 gain	0.8
T_{11}- Motor 1 denominator parameter	1.1
T_{10}- Motor 1 denominator parameter	1
T_{21}- Motor 2 denominator parameter	1
T_{20}- Motor 2 denominator parameter	1
T_p - Cross reaction momentum parameter	2

3.2. Indirect Adaptive Control System

An indirect MRAS control system for the TRMS system is designed (Figure 3). This control structure uses two indirect adaptive PID controllers, which adapt only the K_p, K_d components, while the K_i component remains constant.

This adaptive control structure consist of concept block:

- Adjustable model

The reference model, referred to as the ‘adjustable model’ in this case, will track the process response. The ‘adjustable model’ functions as an ‘adaptive observer’. The goal of process identification is to develop a satisfactory model of a real process by observing its input-output behavior. The parameters of the identified model and the process are assumed to be 'identical,' and the states of the model can be considered estimates of the process states.

Follow Equation (1) and Equation (2) the complete adaptive laws in intergral form for adjustable model of pitch angle controller are given by:

$$a_{m\Psi} = -\alpha_{a\Psi}\int(p_{21\Psi}\,e_{1\Psi} + p_{22\Psi}e_{2\Psi})\widehat{\dot{\Psi}_p}dt + a_{m\Psi}(0) \quad (14)$$

$$b_{m\Psi} = \alpha_{b\Psi}\int(p_{21\Psi}\,e_{1\Psi} + p_{22\Psi}e_{2\Psi})u_1 dt + b_{m\Psi}(0) \quad (15)$$

The complate adaptive laws in intergral form for adjustable model of yaw angle controller are given by:

$$a_{m\varphi} = -\alpha_{a\varphi} \int (p_{21\varphi}\, e_{1\varphi} + p_{22\varphi} e_{2\varphi}) \widehat{\dot{\varphi}_p} dt + a_{m\varphi}(0) \quad (16)$$

$$b_{m\varphi} = \alpha_{a\varphi} \int (p_{21\varphi}\, e_{1\varphi} + p_{22\varphi} e_{2\varphi}) u_2 dt \; + b_{m\varphi}(0) \quad (17)$$

In the form of the adjustment laws, $p_{21\Psi}$, $p_{22\Psi}$, $p_{21\varphi}$ and $p_{22\varphi}$ are elements of matrices P_Ψ and P_φ, which are obtained from the solution of the Lyapunov equations indecated in Equation (18) and Equation (19).

$$A_\Psi^T P_\Psi + P_\Psi A_\Psi = -Q_\Psi \quad (18)$$

$$A_\varphi^T P_\varphi \; + P_\varphi A_\varphi \; = -Q_\varphi \quad (19)$$

Where Q_Ψ and Q_φ are positive definited matrices, with A_ψ and A_φ are taken from Equation (12) and Equation (13), respectively. This yields:

$$A_\Psi = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_{1\Psi}}{I_1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -0.088 \end{bmatrix}; \quad A_\varphi = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_{1\varphi}}{I_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -5 \end{bmatrix}$$

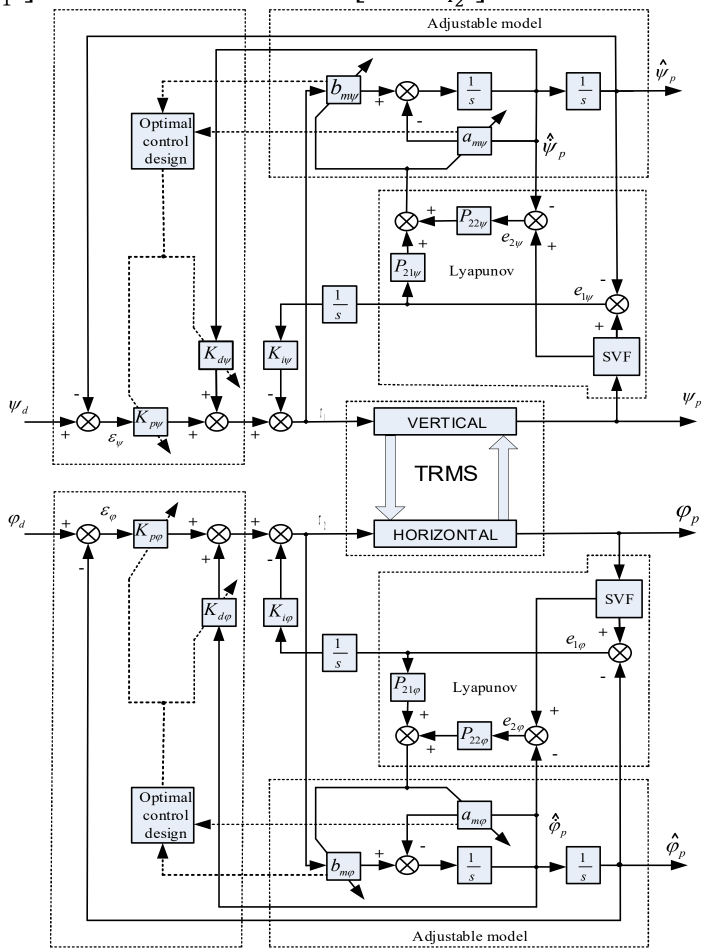

Figure 3. *Indirect adaptive control structure for TRMS*

From Equation (4), and Equation (5), it is obtained as below:

$$p_{21\Psi} = \frac{1}{a_{m\Psi}}\left(\frac{q_{22\Psi}}{2a_{m\Psi}} + q_{21\Psi}\right); \; p_{22\Psi} = \frac{q_{22\Psi}}{2a_{m\Psi}} \qquad p_{21\varphi} = \frac{1}{a_{m\varphi}}\left(\frac{q_{22\varphi}}{2a_{m\varphi}} + q_{21\varphi}\right); \; p_{22\varphi} = \frac{q_{22\varphi}}{2a_{m\varphi}}$$

$$K_{p\Psi} = \frac{2500}{b_{m\Psi}}; \; K_{d\Psi} = \frac{70-a_{m\Psi}}{b_{m\Psi}} \qquad K_{p\varphi} = \frac{2500}{b_{m\varphi}}; \; K_{d\varphi} = \frac{70-a_{m\varphi}}{b_{m\varphi}}$$

- State Variable Filter

The TRMS controller does not contain velocity sensors to determine the angular velocities of TRMS' beam. Therefore, the velocity must be determined from the beam's position. This is achieved using a 2nd-order State Variable Filter (SVF) [12], as shown in Figure 4, selected in such a way that the parameters of the adjustable model can vary without the need to modify the SVF parameters each time. The chosen parameters are: $\omega_m = 50$ (rad/s), $\xi = 0.7$.

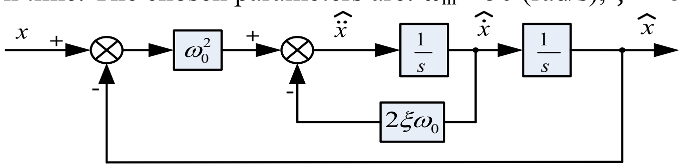

Figure 4. *2nd order State Variable Filter*

3.3. Simulation results

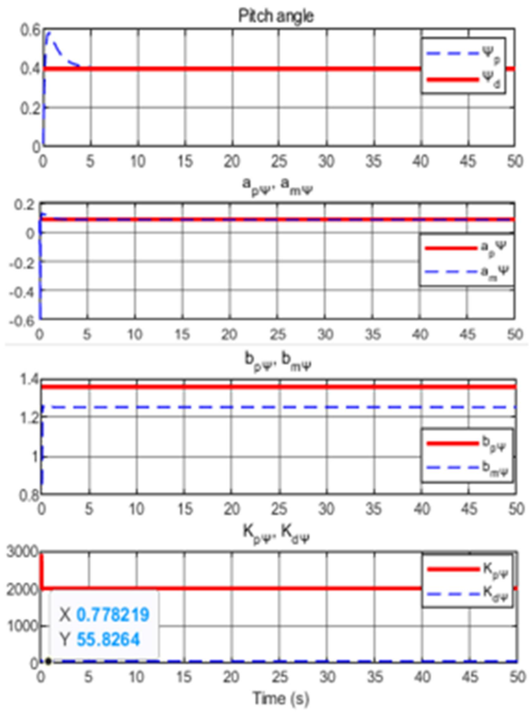

Figure 5. *Responses of* $\Psi_p,\ a_{m\Psi}, b_{m\Psi}, K_{p\Psi}$ *and* $K_{d\Psi}$

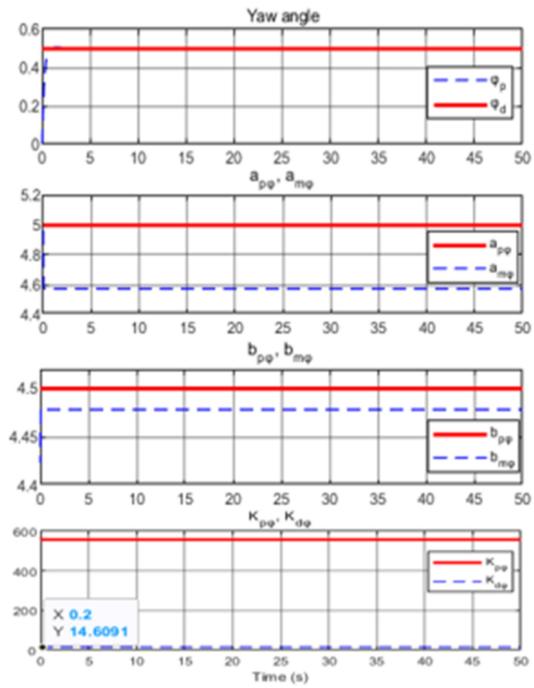

Figure 6. *Responses of* $\varphi_d, a_{m\varphi}, b_{m\varphi}, K_{p\varphi}$ *and* $K_{d\varphi}$

- PID_Ψ adaptive controller for pitch angle

$\Psi_d = 0.4\ [rad]$

$$a_{m\Psi} = -\alpha_{a\Psi}\int(p_{21\Psi}\, e_{1\Psi} + p_{22\Psi}e_{2\Psi})\widehat{\dot{\Psi}_p}dt + a_{m\Psi}(0)$$
$$b_{m\Psi} = \alpha_{b\Psi}\int(p_{21\Psi}\, e_{1\Psi} + p_{22\Psi}e_{2\Psi})u_1 dt\ + b_{m\Psi}(0)$$
$$K_{p\Psi} = \frac{2500}{b_{m\Psi}};\ K_{d\Psi} = \frac{70-a_{m\Psi}}{b_{m\Psi}};\ K_{i\Psi} = 3$$
$$Q_\Psi = \begin{bmatrix}35 & 25\\ 25 & 35\end{bmatrix};\ \omega_{0\Psi} = 50;\ \xi = 0.7;\ a_{m\Psi}(0) = 0; b_{m\Psi}(0) = 0$$

- PID_φ adaptive controller for yaw angle

$\varphi_d = 0.4\ [rad]$

$$a_{m\varphi} = -\alpha_{a\varphi}\int(p_{21\varphi}\, e_{1\varphi} + p_{22\varphi}e_{2\varphi})\widehat{\dot{\varphi}_p}dt + a_{m\varphi}(0)$$
$$b_{m\varphi} = \alpha_{a\varphi}\int(p_{21\varphi}\, e_{1\varphi} + p_{22\varphi}e_{2\varphi})u_2 dt\ + b_{m\varphi}(0)$$

$K_{p\varphi} = \frac{2500}{b_{m\varphi}}$; $K_{d\varphi} = \frac{70-a_{m\varphi}}{b_{m\varphi}}$; $K_{i\varphi} = 8.5$

$Q_{\varphi} = \begin{bmatrix} 40 & 18 \\ 18 & 40 \end{bmatrix}$; $\omega_{0\varphi} = 50$; $\xi = 0.7$; $a_{m\varphi}(0) = 0$; $b_{m\varphi}(0) = 0$

Figure 5, shows that the ψ_p angle tracks ψ_d angle; however, the percent overshoot compared to the reference is approximately 40. Figure 6 demonstrates that the φ_p angle tracks the φ_d angle, the percent overshoot compared to the reference is approximately 5%. The values of $a_{m\psi}$, $b_{m\psi}$, $a_{m\varphi}$, $b_{m\varphi}$ reach constant values. This difference is due to the linearization of the system. The feedback controller gains $K_{p\psi}$, $K_{d\psi}$ or $K_{p\varphi}$, $K_{d\varphi}$, which are calculated based on the adaptive gains $a_{m\psi}$, $b_{m\psi}$ or $a_{m\varphi}$, $b_{m\varphi}$, also reach constant values.

4. Conclusion

Adaptive controllers based on indirect MRAS have been successfully designed to track the preset trajectory of the TRMS. Adaptive control laws have been applied to the TRMS using Lyapunov's stability theory for second-order systems. The system has been validated through simulations in MATLAB/Simulink, demonstrating stability and effectiveness in handling parameter variations and uncertain disturbances. However, this study has only been conducted through simulations and has not yet been implemented experimentally. Future research should focus on the practical implementation of the control system and the optimization of adaptive control parameters to significantly enhance performance in aerodynamic environments affected by disturbances.

REFERENCES

[1] Feedback Instruments Ltd, “Twin Roto MIMO System Control Experiments 33-949S,” U.K., 2006. [Online]. Available: http://www.cpdee.ufmg.br/~palhares/33-942rotor.pdf. [Accessed Feb. 15, 2025]

[2] R. Mok and M. A. Ahmad, “Performance evaluation of smoothed functional algorithm based methods for sigmoid-PID control optimization in MIMO twin-rotor systems,” in *Proceedings of the 7th International Conference on Electrical, Control and Computer Engineering* –vol. 1, in *ECCE 2023. Lecture Notes in Electrical Engineering*, vol. 1212, 2023, Singapore: Springer, doi: 10.1007/978-981-97-3847-2_35.

[3] A. Rahideh and H.M. Shaheed, “Hybrid Fuzzy-PID-based Control of a Twin Rotor MIMO System,” in *IEEE Conference on Industrial Electronics*, *France*, 2006, pp. 49-54.

[4] J.-G. Juang, M.-T. Huang, and W.-K. Liu, “PID control using presearched genetic algorithms for a MIMO System,” *IEEE Transactions on systems, Man and cybernetics*, Part C, vol. 38, no. 5, pp.716-727, 2008.

[5] P. Wen and T.W. Lu, “Decoupling control of a twin rotor MIMO system using robust deadbeat control technique,” *IET Control Theory Applications*, vol. 2, no. I 1, pp. 999-1 007, 2008.

[6] F. A. Shaik and S. Purwar, “A Nonlinear State Observer Design for 2 – DOF Twin Rotor System Using Neural Networks,” *IEEE Computer society 2009, Int. Conf. on Advances in Computing, Control, and Telecommunication Technologies*, 2009, pp.15-19.

[7] C. W. Taoa, J. S. Taurb, and Y. C. Chena, “Design of a parallel distributed fuzzy LQR controller for the twin rotor multi-input multi-output system,” *Fuzzy Sets and Systems,* vol. 161, pp. 2081–2103, 2010.

[8] T. D. Tran, V. H. Dang, B. L. Dam, and D. C. Nguyen, “Design of indirect mras -based adaptive control systems,” *TNU Journal of Science and Technology*, vol. 139, no. 9, pp. 245-251, 2015.

[9] B. L. Dam, T. V. H. Nguyen, and V. N. Nguyen, “Determining the speed of adaptations of MRAS-based adaptive controller using particle swarm optimization,” *TNU Journal of Science and Technology*, vol.155, no.10, pp. 85-91, 2016.

[10] K. J. Astrom and B. Wittenmark, *Computer-Controlled Systems -Theory and Design*, 3rd ed., Prentice Hall Information and System sciences Series, Prentice Hall, Upper Saddle River, 1997.

[11] Y. D. Landau, *Control and Systems Theory - Adaptive Control - The Model Reference Approach*, Marcel Dekker, 1979.

[12] J. V. Amerongen, *Intelligent Control (part I)-MRAS*, Lecture notes, University of Twen-te, The Netherlands, 2004.

[13] D. C. Nguyen, “Advanced Controller for Electromechanical Motion Systems,” PhD. Thesis, University of Twente, Enschede, The Netherlands, 2008.

[14] D. C. Nguyen *et al.,* “Design of MRAS-based Adaptive Control Systems,” *The IEEE 2013 International Conference on Control, Automation and Information Sciences* (ICCASI), 2013, pp.79-84.