EXPONENTIAL STABILITY FOR A CLASS OF NONLINEAR GUZMÁN FRACTIONAL-ORDER SYSTEMS

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ARTICLE INFO	ABSTRACT

Received: 08/5/2025
Revised: 22/5/2025
Published: 22/5/2025

KEYWORDS

Guzmán fractional-order
Nonliear fractional-order systems
Linear matrix inequalities
Exponential stability
Exponential stabilizability

This paper introduces an efficient analytical method for ad- dressing the problem of exponential stability and stabilizability of a class of nonlinear Guzmán fractional systems. The proposed approach combines mathematical transformations with concepts from fractional calculus, providing a robust framework for the analysis of dynamic systems. Initially, sufficient conditions for ensuring the exponential stability of the system are derived using Lyapunov-based techniques and expressed in terms of strict linear matrix inequalities, which are suitable for computational implementation. Subsequently, a state-feedback control law is designed to guarantee the exponential stabilizability of the closed-loop system. Using linear matrix inequalities in both stability analysis and controller design makes the method easier to apply and understand. Finally, a numerical example is presented to illustrate the feasibility and effectiveness of the proposed strategy, confirming that the theoretical results are valid and can be applied in practice to stabilize complex non-linear fractional-order systems.

TÍNH ỔN ĐỊNH MŨ CHO MỘT LỚP HỆ PHƯƠNG TRÌNH VI PHÂN PHÂN THỨ GUZMÁN PHI TUYẾN

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THÔNG TIN BÀI BÁO	TÓM TẮT

Ngày nhận bài: 08/5/2025
Ngày hoàn thiện: 22/5/2025
Ngày đăng: 22/5/2025

TỪ KHÓA

Đạo hàm phân thứ Guzmán
Hệ phương trình phân thứ phi tuyến
Bất đẳng thức ma trận tuyến tính
Ổn định mũ
Ổn định hóa được dạng mũ

Bài báo giới thiệu một phương pháp phân tích hiệu quả nhằm giải quyết bài toán ổn định mũ và bài toán ổn định hóa được dạng mũ cho một lớp hệ phương trình vi phân phân thứ phi tuyến Guzmán. Phương pháp được đề xuất kết hợp các phép biến đổi toán học với các khái niệm của giải tích phân thứ, qua đó xây dựng một cách thức hữu hiệu cho việc phân tích các hệ động lực phân thứ. Trước hết, điều kiện đủ để đảm bảo tính ổn định mũ được thiết lập bằng phương pháp hàm Lyapunov và được biểu diễn dưới dạng các bất đẳng thức ma trận tuyến tính chặt, thuận tiện cho tính toán. Sau đó, một điều khiển phản hồi trạng thái được thiết kế nhằm đảm bảo cho hệ đóng ổn định hóa được dạng mũ. Việc áp dụng bất đẳng thức ma trận tuyến tính trong phân tích và thiết kế nâng cao tính hiệu quả của phương pháp. Cuối cùng, một ví dụ số được đưa ra để minh họa cho phương pháp đã đề xuất.

DOI: https://doi.org/10.34238/tnu-jst.12752

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1. Introduction

In recent years, fractional calculus and fractional-order differential equations have garnered significant research interest owing to their strong modelling capabilities and wide-ranging applications in diverse fields such as fluid mechanics, chemical processes, and engineering systems [1, 2, 3, 4]. In 2018, P.M. Guzmán et al. [5] introduced, for the first time, a new definition of a fractional derivative and conducted a comprehensive analysis of its properties. In recognition of their pioneering contribution, this derivative is referred to as the Guzmán fractional derivative throughout this study.

As is well known, stability and stabilizability are among the most fundamental qualitative properties of dynamical systems. The Lyapunov function method has proven to be a powerful and widely adopted tool for analyzing the stability of both integer-order and fractional-order systems. In this context, Y. Li et al. [6] were the first to introduce the Lyapunov-based approach for investigating the stability of nonlinear fractional-order differential systems involving both Riemann–Liouville and Caputo derivatives. Regarding Khalil-type fractional differential systems, the Lyapunov function method was investigated and first proposed by A. Souahi et al. [7] for the analysis of their stability properties.

In 2024, N. Echi et al. [8] were the first to extend the Lyapunov function method to study the stability of nonlinear fractional-order differential systems involving the Guzmán derivative. By employing the Grönwall inequality in conjunction with appropriate inequality-based estimates, the authors established a criterion for the fractional exponential stability of this class of nonlinear Guzmán-type fractional differential systems.

In this work, we adopt the Lyapunov-based approach proposed by N. Echi et al. [8], in combination with linear matrix inequality (LMI) techniques, to investigate the exponential stability and exponential stabilizability of nonlinear fractional-order Guzmán-type systems. The class of systems considered in our study encompasses more general nonlinear perturbations and includes, as a special case, the subclass addressed in [8]. This generalization significantly broadens the applicability and robustness of the stability analysis framework.

2. Problem formulation and preliminaries

We begin by introducing the fundamental definitions and essential properties of the Guzmán fractional integrals and fractional derivatives. These core concepts form the analytical foundation of the study and serve as key tools for the subsequent developments presented in this paper.

Definition 2.1. [9] Given a positive scalar $\alpha \in (0,1)$ and a function $f : [t_0, +\infty) \to \mathbb{R}$. The fractional Guzmán derivative of order α of the function $f(.)$ is defined by

$$N_1^{(\alpha)} f(t) = \lim_{\varepsilon \to 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}, \quad \text{for all } t > t_0 \geq 0.$$

If $f(.)$ is α-differentiable on an interval $(0, a)$, and $\lim_{t \to 0^+} N_1^{(\alpha)} f(t)$ exists, we define

$$N_1^{(\alpha)} f(0) = \lim_{t \to 0^+} N_1^{(\alpha)} f(t).$$

Definition 2.2. [9] Given a positive scalar $\alpha \in (0,1)$ and a function $f : [t_0, +\infty) \to \mathbb{R}$. The Guzmán integral of order α for the function $f(t)$ is defined by the expression

$${}_N J_{t_0}^{\alpha} f(t) = \int_{t_0}^{t} \frac{f(s)}{e^{s^{-\alpha}}}\, ds.$$

The following properties of fractional-order calculus are essential for establishing the main results.

Property 2.3. [5] Let $\alpha \in (0,1]$ and the functions f and g be α-differentiable at a point $t > 0$. Then

(a) $N_1^{\alpha}(af+bg)(t) = aN_1^{\alpha}(f)(t) + bN_1^{\alpha}(g)(t)$.

(b) $N_1^{\alpha}(t^p) = e^{t^{-\alpha}} p t^{p-1}, \quad p \in \mathbb{R}$.

(c) $N_1^{\alpha}(\lambda) = 0, \quad \lambda \in \mathbb{R}$.

(d) $N_1^{\alpha}(fg)(t) = fN_1^{\alpha}(g)(t) + gN_1^{\alpha}(f)(t)$.

(e) $N_1^{\alpha}\left(\frac{f}{g}\right)(t) = \frac{gN_1^{\alpha}(f)(t) - fN_1^{\alpha}(g)(t)}{g^2(t)}$.

(f) *If f is differentiable then* $N_1^{\alpha}(f) = e^{t^{-\alpha}} f'(t)$.

(g) *If f is differentiable and $\alpha = n$ integer then* $N_1^{n}(f)(t) = e^{t^{-n}} f'(t)$.

Property 2.4. [10] Let f be α -differentiable function in (t_0, ∞) with $\alpha \in (0,1]$. Then for all $t > t_0$ we have

(a) If f is differentiable then ${}_N J_{t_0}^{\alpha}(N_1^{\alpha} f(t)) = f(t) - f(t_0)$.

(b) $N_1^{\alpha}\left({}_N J_{t_0}^{\alpha} f(t)\right) = f(t)$.

Consider the following Guzmán fractional-order systems described by

$$\begin{cases} N_1^{\alpha}\xi(t) = [A + \Delta A(t)]\xi(t) + [B + \Delta B(t)]u(t) + f(t, \xi(t)), t \geq 0, \\ \xi(0) = \xi_0, \end{cases} \tag{1}$$

where $\alpha \in (0,1)$ is the fractional order of the system, $\xi(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, x_0 is the initial condition. $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$ are known constant matrices. Matrices $\Delta A(t), \Delta B(t)$ represent time-varying parameter uncertainties as follows:

$$\Delta A(t) = M_a \mathcal{T}_a(t) N_a,\ \Delta B(t) = M_b \mathcal{T}_b(t) N_b, \tag{2}$$

where M_a, N_a, M_b, N_b are known constant real matrices and matrices $\mathcal{T}_a(t), \mathcal{T}_b(t)$ are unknown real matrices satisfying

$$\mathcal{T}_a^T(t)\mathcal{T}_a(t) \leqslant I, \mathcal{T}_b^T(t)\mathcal{T}_b(t) \leqslant I.$$

The function $f(t, \xi(t))$ satisfying the condition

$$f^T(t, \xi(t)) Q f(t, \xi(t)) \leq \xi^T(t) R \xi(t),$$

where Q, R are positive definite symmetric matrices.

Lemma 2.5. [11] (Cauchy inequality) Given $x, y \in \mathbb{R}^n$ and a symmetric positive definite matrix $S \in \mathbb{R}^{n\times n}$, it holds that $\pm 2x^T y \leq x^T S x + y^T S^{-1} y$.

Consider the Guzmán fractional-order system

$$N_1^{\alpha}\xi(t) = f(t, \xi(t)), \tag{3}$$

where $\xi(t) \in \mathbb{R}^n$, $f : \mathbb{R}_+ \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ and $N_1^{\alpha}\xi(t)$ is the fractional derivative of order $0 < \alpha < 1$. It is assumed that $f(t, 0) = 0$ for all $t > t_0 > 0$.

Lemma 2.6. [8] Assume that there exists an α-differentiable Lyapunov function $V : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}_+$ and positive constants λ_i, $i = 1, 2, 3$ satisfying:

(i) $\lambda_1 \|\xi(t)\|^2 \leq V(t, \xi(t)) \leq \lambda_2 \|\xi(t)\|^2$,

(ii) $N_1^{\alpha} V(t, \xi(t)) \leq -\lambda_3 \|\xi(t)\|^2$.

Then the origin of system (3) is fractionally exponentially stable.

The following result can be easily proved from the Property 2.3

Lemma 2.7. Let $h : [a, \infty) \to \mathbb{R}^n$ such that $N_1^\alpha h(t)$ exists on (a, ∞), and let P be a positive definite symmetric matrix. Then $N_1^\alpha h^T(t)Ph(t))$ exists on (a, ∞) and

$$N_1^\alpha(h^T(t)Ph(t)) = 2h(t)^T P N_1^\alpha h(t), \quad \forall t > a > 0.$$

Proof. From the Property 2.3, we can compute the Guzmán fractional derivative of the differentiable function $f(t)$ as follows

$$N_1^\alpha f(t) = e^{t^{-\alpha}} \frac{d}{dt} f(t).$$

Consider the homogeneous function

$$\varphi(t) = h(t)^T Ph(t).$$

We have

$$\frac{d}{dt}\left(h(t)^T Ph(t)\right) = h'(t)^T Ph(t) + h(t)^T Ph'(t) = 2h(t)^T Ph'(t).$$

Hence,

$$\begin{aligned} N_1^\alpha(h^T Ph)(t) &= e^{t^{-\alpha}} \cdot \frac{d}{dt}\left(h(t)^T Ph(t)\right) \\ &= e^{t^{-\alpha}} \cdot 2h(t)^T Ph'(t) \\ &= 2h(t)^T P N_1^\alpha h(t). \end{aligned}$$

As a result

$$N_1^\alpha(h^T Ph)(t) = 2h(t)^T P N_1^\alpha h(t).$$

□

3. Main results

3.1. Stability analysis

In the absence of control input vector, the system (1) becomes

$$\begin{cases} N_1^\alpha \xi(t) = [A + \Delta A(t)]\xi(t) + f(t, \xi(t)), t \geq 0, \\ \xi(0) = \xi_0, \end{cases} \tag{4}$$

Theorem 3.1. *Consider the nonlinear fractional-order systems* (4). *If there exist scalars $\delta > 0$, a symmetric positive definite matrix $X \in \mathbb{R}^{n\times n}$ such that the following condition hold*

$$\begin{bmatrix} XA + A^T X + \delta N_a^T N_a + R & X & XM_a \\ * & -Q & 0 \\ * & * & -\delta I \end{bmatrix} \tag{5}$$

then the systems (4) *is fractionally exponentially stable.*

Proof. We construct the following Lyapunov function

$$V(t, \xi(t) = \xi^T(t) X \xi(t),$$

We can easily obtain

$$\lambda_1\|\xi(t)\|^2 \leq V(t,\xi(t)) \leq \lambda_2\|\xi(t)\|^2, \tag{6}$$

where $\lambda_1 = \lambda_{\min}(X), \lambda_2 = \lambda_{\max}(X)$.

By taking the fractional-order derivative of $V(t,\xi(t))$ along the trajectory of system (4) and applying Lemma 2.7, we obtain the following bounds.

$$\begin{aligned} N_1^\alpha V(t,\xi(t) &= 2\xi^T(t)XN_1^\alpha\xi(t) \\ &= 2\xi^T(t)X[(A+\Delta A(t))\xi(t) + f(t,\xi(t))s] \\ &= \xi^T(t)(XA + A^TX)\xi(t) + 2\xi^T(t)X\Delta A(t)\xi(t) + 2\xi^T(t)Xf(t,\xi(t)). \end{aligned} \tag{7}$$

For a positive scalar $\delta > 0$, from using Cauchy matrix inequality, we get the following estimation

$$2\xi^T(t)X\Delta A(t)\xi(t) \leq \delta^{-1}\xi^T(t)XM_aM_a^TX\xi(t) + \delta\xi^T(t)N_a^TN_a\xi(t). \tag{8}$$

Adding inequations (8) and $\xi^T(t)R\xi(t) - f^T(t,\xi(t))Qf(t,\xi(t)) \geq 0$ into (7), we obtain

$$\begin{aligned} N_1^\alpha V(t,\xi(t) \leq\ & \xi^T(t)(XA + A^TX + \delta^{-1}XM_aM_a^TX + \delta N_a^TN_a + R)\xi(t) \\ & + 2\xi^T(t)Xf(t,\xi(t)) - f^T(t,\xi(t))Qf(t,\xi(t)). \end{aligned} \tag{9}$$

We can derived the following result

$$N_1^\alpha V(t,\xi(t) \leq x^T(t)\Theta x(t) \tag{10}$$

where

$$\begin{aligned} & x(t)^T = \begin{bmatrix} \xi^T(t) & f^T(t,x(t)) \end{bmatrix}, \Theta = \begin{bmatrix} \Theta_{11} & X \\ * & -Q \end{bmatrix}, \\ & \Theta_{11} = XA + A^TX + \delta^{-1}XM_aM_a^TX + \delta N_a^TN_a + R. \end{aligned}$$

Applying the Schur complement together with inequality (5), the above inequality readily follows.

With $\lambda_3 = -\lambda_{\max}\Theta > 0$, the following result can be easily proved

$$\begin{aligned} N_1^\alpha V(t,\xi(t) &\leq \lambda_{\max}\Theta\|x(t)\| \\ &\leq \lambda_{\max}\Theta\|\xi(t)\| \\ &= -\lambda_3\|\xi(t)\|. \end{aligned} \tag{11}$$

Combining (6) and (11), according to Lemma 2.7, we obtain the sysems (4) is exponentially stable. This completes the proof. □

3.2. State fedback stabilisation

The main object of this subsection is to design the state feedback controller $u(t) = K\xi(t)$ such that the following closed-loop system of the systems (1) is exponentially stable

$$\begin{cases} N_1^\alpha\xi(t) = [A + \Delta A(t) + BK + \Delta B(t)K]\xi(t) + f(t,\xi(t)), t \geq 0, \\ \xi(0) = \xi_0. \end{cases} \tag{12}$$

Theorem 3.2. *Consider the nonlinear fractional-order closed-loop systems* (12)*. If there exist scalars* $\beta > 0, \delta > 0$*, a symmetric positive definite matrix* $Y \in \mathbb{R}^{n\times n}$*, a matrix* $Z \in \mathbb{R}^{m\times n}$ *such* *that the following condition hold*

$$\begin{bmatrix} \Sigma_{11} & I & YN_a^T & Z^TN_b^T & YR \\ * & -Q & 0 & 0 & 0 \\ * & * & -\delta I & 0 & 0 \\ * & * & * & -\beta I & 0 \\ * & * & * & * & -R \end{bmatrix}, \tag{13}$$

where

$$\Sigma_{11} = AY + YA^T + BZ + Z^TB^T + \delta M_aM_a^T + \beta M_bM_b^T.$$

Then the systems (12) *is fractionally exponentially stable and the state feedback controller is given by* $u(t) = ZY^{-1}\xi(t)$.

Proof. We construct the same Lyapunov function as Theorem 3.1

$$V(t, \xi(t) = \xi^T(t)Y^{-1}\xi(t),$$

We can easily obtain

$$\lambda_1\|\xi(t)\|^2 \leq V(t, \xi(t)) \leq \lambda_2\|\xi(t)\|^2, \tag{14}$$

where $\lambda_1 = \lambda_{\min}(Y^{-1}), \lambda_2 = \lambda_{\max}(Y^{-1})$.

Evaluating the fractional derivative of $V(t, \xi(t))$ along the solutions of system (12) and invoking Lemma 2.7, we arrive at the following estimates.

$$\begin{aligned} N_1^\alpha V(t, \xi(t) &= 2\xi^T(t)Y^{-1}N_1^\alpha\xi(t) \\ &= 2\xi^T(t)Y^{-1}\left[(A + \Delta A(t) + BK + \Delta B(t)K)\xi(t) + f(t, \xi(t))\right] \\ &= \xi^T(t)(Y^{-1}A + A^TY^{-1} + Y^{-1}BK + K^TB^TY^{-1})\xi(t) \\ &\quad + 2\xi^T(t)Y^{-1}\Delta A(t)\xi(t) + 2\xi^T(t)Y^{-1}\Delta B(t)K\xi(t) + 2\xi^T(t)Y^{-1}f(t, \xi(t)). \end{aligned} \tag{15}$$

For $\delta > 0, \beta > 0$, using Cauchy matrix inequality, we get the following estimations

$$2\xi^T(t)Y^{-1}\Delta A(t)\xi(t) \leq \delta\xi^T(t)Y^{-1}M_aM_a^TY^{-1}\xi(t) + \delta^{-1}\xi^T(t)N_a^TN_a\xi(t), \tag{16}$$

$$2\xi^T(t)Y^{-1}\Delta B(t)K\xi(t) \leq \beta\xi^T(t)Y^{-1}M_bM_b^TY^{-1}\xi(t) + \beta^{-1}\xi^T(t)K^TN_b^TN_bK\xi(t). \tag{17}$$

Adding inequations (16)-(17) and $\xi^T(t)R\xi(t) - f^T(t, \xi(t))Qf(t, \xi(t)) \geq 0$ into (15), we obtain

$$\begin{aligned} N_1^\alpha V(t, \xi(t) \leq \xi^T(t)\Big(&Y^{-1}A + A^TY^{-1} + Y^{-1}BK + K^TB^TY^{-1} + \delta Y^{-1}M_aM_a^TY^{-1} \\ &+ \delta^{-1}N_a^TN_a + \beta Y^{-1}M_bM_b^TY^{-1} + \beta^{-1}K^TN_b^TN_bK + R\Big)\xi(t) \\ &+ 2\xi^T(t)Y^{-1}f(t, \xi(t)) - f^T(t, \xi(t))Qf(t, \xi(t)). \end{aligned} \tag{18}$$

We can derived the following result

$$N_1^\alpha V(t, \xi(t) \leq x^T(t)\Psi x(t) \tag{19}$$

where

$$x(t)^T = \begin{bmatrix} \xi^T(t) & f^T(t, x(t)) \end{bmatrix}, \Psi = \begin{bmatrix} \Psi_{11} & Y^{-1} \\ * & -Q \end{bmatrix},$$

$$\Psi_{11} = Y^{-1}A + A^TY^{-1} + Y^{-1}BK + K^TB^TY^{-1} + \delta Y^{-1}M_aM_a^TY^{-1} + \delta^{-1}N_a^TN_a \\ + \beta Y^{-1}M_bM_b^TY^{-1} + \beta^{-1}K^TN_b^TN_bK + R.$$

Letting $K = ZY^{-1}$. By pre- and post-multiplying Ψ with $\mathrm{diag}(Y, I)$ and its transpose matrix, and making use of the Schur complement lemma, one can show that $\Psi < 0$ holds if and only if condition (13) is satisfied.

With $\lambda_3 = -\lambda_{\max}\Psi > 0$, the following result can be easily proved

$$\begin{aligned} N_1^\alpha V(t, \xi(t) &\le \lambda_{\max}\Psi\|x(t)\| \\ &\le \lambda_{\max}\Psi\|\xi(t)\| \\ &= -\lambda_3\|\xi(t)\|. \end{aligned} \tag{20}$$

Combining (14) and (20), according to Lemma 2.7, we obtain the sysems (12) is exponentially stable. This completes the proof. □

Remark 3.3. Compared to the results reported in [8], which focus on the exponential stability analysis of a simplified class of linear differential systems, the conditions proposed in this study extend the scope to a more general class of fractional-order systems. Specifically, our results establish new sufficient conditions for both exponential stability and exponential stabilizability in the presence of system uncertainties and nonlinear disturbances. These extensions significantly enhance the theoretical depth and practical applicability of the proposed framework.

Example 3.4. Consider the following control system

$$\begin{cases} N_1^\alpha\xi(t) = [A + M_a\mathcal{T}_a(t)N_a]\xi(t) + [B + M_b\mathcal{T}_b(t)N_b]u(t) + f(t, \xi(t)), t \ge 0, \\ \xi(0) = \xi_0. \end{cases} \tag{21}$$

where $\xi(t) = (\xi_1(t), \xi_2(t))^T \in \mathbb{R}^2, u(t) \in \mathbb{R}^2$.

$$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}, M_a = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, N_a = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \mathcal{T}_a(t) = \cos t,$$
$$B = \begin{bmatrix} -1 & 0 \\ 1 & -3 \end{bmatrix}, M_b = \begin{bmatrix} -0.3 \\ 0.2 \end{bmatrix}, N_b = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, T_b(t) = \cos t,\ f(t, x(t)) = \begin{bmatrix} \sin(\xi_1(t)) \\ \sin(\xi_1(t)) \end{bmatrix}.$$

The closed-loop system with $u(t) = Kx(t)$ of system (21) is described by

$$\begin{cases} N_1^\alpha\xi(t) = [A + M_a\mathcal{T}_a(t)N_a + BK + M_b\mathcal{T}_b(t)N_bK]\xi(t) + f(t, \xi(t)), t \ge 0, \\ \xi(0) = \xi_0. \end{cases} \tag{22}$$

It is not hard to check that $f(t, \xi(t))$ satisfies with $Q = I, R = I$. By using LMI Control Toolbox in MATLAB, the conditions in Theorem 3.2 are feasible with $\beta = 1.8620, \delta = 1.8620$, and

$$Y = \begin{bmatrix} 0.4851 & -0.0686 \\ -0.0686 & 0.5620 \end{bmatrix}, Z = \begin{bmatrix} 1.4698 & 0.0491 \\ 0.8519 & 0.6545 \end{bmatrix}.$$

According to Theorem 3.2, the closed-loop system (22) is exponentially stable under the state feedback controller $u(t) = \begin{bmatrix} 3.0953 & 0.4650 \\ 1.9542 & 1.4031 \end{bmatrix} \xi(t), \forall t \ge 0.$

4. Conclusion

This paper investigates the problem exponential stability and exponential stabilizability of a class of nonlinear fractional systems of Guzmán. By employing fundamental mathematical transformations and tools from fractional calculus, we propose new sufficient conditions for addressing the problem of designing state feedback controllers, ensuring that the resulting closed-loop system is exponentially stabilizable. An illustrative example is presented to verify the usefulness and effectiveness of the obtained results.

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