COMPARISON AND EVALUATION OF THE MODAL TRUNCATION AND BALANCED TRUNCATION TECHNIQUES IN MODEL ORDER REDUCTION FOR IIR FILTERS

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ABSTRACT

This paper investigates model order reduction for a 30th-order IIR filter in telecommunication systems with the objectives of simplifying the structure, preserving dynamic characteristics, and reducing computational costs. Two algorithms, Modal Truncation and Balanced Truncation, are compared at reduced orders r = 13 and r = 15. At order 13, the H_∞ and H_2 errors for Balanced Truncation are 1.109873×10^{-2} and 5.661614×10^{-3}, respectively, which are lower than those for Modal Truncation, recorded at 2.097335×10^{-2} and 1.805762×10^{-2}. Response analysis shows that with Balanced Truncation, after 30 seconds the time response of the reduced-order system closely follows that of the original system, and the frequency response above 10 rad/s also approximates that of the original system, whereas Modal Truncation consistently produces significant discrepancies. At order 15, the errors for Balanced Truncation further decrease (H_∞: 1.233049×10^{-3}, H_2: 8.219160×10^{-4}) with the response nearly matching the original system, while Modal Truncation continues to exhibit substantial deviations. The results confirm that Balanced Truncation is a superior choice over Modal Truncation for model order reduction of IIR filters in telecommunication and signal processing applications.

SO SÁNH, ĐÁNH GIÁ KỸ THUẬT CẮT NGẮN PHƯƠNG THỨC VÀ CẮT NGẮN CÂN BẰNG TRONG GIẢM BẬC MÔ HÌNH CHO BỘ LỌC IIR

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TỪ KHÓA

Xử lý tín hiệu viễn thông
Bộ lọc IIR
Giảm bậc mô hình
Cắt ngắn phương thức
Cắt ngắn cân bằng

TÓM TẮT

Bài báo nghiên cứu giảm bậc mô hình cho bộ lọc IIR bậc 30 trong hệ thống viễn thông nhằm đơn giản hóa cấu trúc, bảo toàn đặc tính động lực học và giảm chi phí tính toán. Hai thuật toán Cắt ngắn phương thức và Cắt ngắn cân bằng được so sánh ở các bậc r = 13 và r = 15. Ở bậc 13, sai số H_∞ và H_2 của Cắt ngắn phương thức lần lượt là $1,109873\times10^{-2}$ và $5,661614\times10^{-3}$, thấp hơn so với Cắt ngắn phương thức là $2,097335\times10^{-2}$ và $1,805762\times10^{-2}$. Phân tích đáp ứng cho thấy, với Cắt ngắn cân bằng, sau 30 giây đáp ứng thời gian của hệ giảm bậc bám sát với hệ gốc, và đáp ứng tần số trên 10 rad/s cũng xấp xỉ với hệ gốc, trong khi Cắt ngắn phương thức luôn cho sai khác đáng kể. Ở bậc 15, sai số của Cắt ngắn cân bằng giảm nhiều hơn (H_∞: $1,233049\times10^{-3}$, H_2: $8,219160\times10^{-4}$) với đáp ứng gần như trùng khớp với hệ gốc, còn Cắt ngắn phương thức vẫn sai khác đáng kể. Kết quả khẳng định Cắt ngắn cân bằng là lựa chọn ưu việt trong việc giảm bậc bộ lọc IIR cho các ứng dụng viễn thông và xử lý tín hiệu.

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1. Introduction

In telecommunications networks, filters play a crucial role in ensuring signal quality and optimizing transmission performance. They operate by allowing certain frequencies to pass while blocking unwanted ones, thereby cleaning the signal and minimizing interference. Filters can be categorized into various types, including low-pass, high-pass, band-pass, and band-stop filters, each serving specific purposes in signal conditioning and processing. Their significance lies in improving signal quality as well as optimizing bandwidth, reducing latency, and enhancing the overall performance of telecommunication systems. In the context of the ever-advancing information and communication technology, the effective use of filters enhances data transmission capabilities, ensuring that information is conveyed accurately and reliably. This is particularly important in applications such as television, mobile communications, and the Internet, where signal quality directly affects the user experience. Owing to their ability to condition and process signals, filters have become an indispensable component in the design of modern telecommunication systems [1], [2].

High-order filters in telecommunication systems and signal processing typically deliver superior performance; however, they are also computationally complex. The sophisticated algorithms required for their analysis and design demand significant computational resources, which increases processing time and complicates implementation—especially when stringent reliability and compatibility with existing systems are required. The heavy computational load can restrict data processing speeds, thereby affecting system responsiveness, particularly in real-time applications. To address these challenges, the use of auxiliary tools becomes essential. Techniques such as simulation and modeling can help alleviate the complexity of high-order filters by allowing researchers to test and optimize designs in a virtual environment prior to actual deployment. Moreover, object recognition technology can further streamline the development process by automating the analysis and optimization of filter parameters [3].

In this context, model order reduction has emerged as an important method under active research and development, with wide-ranging applications in various fields. This approach allows for the simplification of complex models while maintaining the requisite accuracy, thereby improving performance and reducing computational costs. Model order reduction has been applied to optimize filter designs for large-scale data transmission systems, enhancing signal processing capabilities and minimizing latency [4].

Among model order reduction techniques, two algorithms—Modal Truncation (MT) and Balanced Truncation (BT)—stand out as predominant due to their ability to preserve critical properties of the original system. These two algorithms not only form the foundation for many other model reduction methods but also open new avenues for applications in fields such as signal processing, telecommunications, robotics, and control systems. The flexibility and adaptability of MT and BT have led to their widespread adoption in both research and practical applications, contributing to improved performance and reduced computational costs in the development of complex models.

Modal Truncation (MT) focuses on retaining the dynamic modes that most significantly influence the behavior of the system, thereby ensuring the stability of the reduced model. Additionally, MT preserves crucial information such as pole locations and zero dynamics, which is essential for maintaining system performance [5] - [7].

Balanced Truncation (BT) provides an approach based on balancing the states of the system. This algorithm not only preserves stability but also retains the eigenvalues and Hankel singular values, thereby ensuring that the important dynamic characteristics of the original system are maintained. BT is commonly applied in fields such as automatic control and signal processing, where preserving system quality and accuracy is of paramount importance [8] – [10].

Recognizing the importance and complexity of high-order filters in signal processing, the authors have investigated and applied the MT and BT algorithms for the model order reduction of these filters, with the goal of simplifying the models while preserving essential characteristics such as stability and performance. An analysis and comparison of the capabilities of these two techniques for high-order filters was carried out in [11], thereby evaluating their effectiveness in preserving the essential information of the original system. Comparing these two methods facilitates the selection of specific filter types, thereby opening new research directions in the design and improvement of high-order systems. The significance of this study lies not only in providing insights into the effectiveness of model order reduction algorithms but also in laying the foundation for further developments in the fields of telecommunication signal processing and control, while simultaneously reducing computational costs and deployment time.

2. Materials and Methods

Modeling filters in telecommunication electronic systems as Linear Time-Invariant (LTI) systems not only provides deep insights into system behavior but also supports the design and optimization of filter performance. By employing differential equations, state-space matrices, and transfer functions, engineers can develop effective solutions for complex problems in telecommunications. In telecommunication electronic systems, filters are modeled as LTI systems to facilitate efficient analysis and design. This model is typically represented by differential equations, state-space matrices, and transfer functions, which describe the interaction between the system's input and output, as shown in (1).

$$\mathbf{H}(s):\begin{cases}\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t)=\mathbf{C}\mathbf{x}(t)+\mathbf{D}\mathbf{u}(t)\end{cases}\Leftrightarrow \mathbf{H}(s):=\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}\Leftrightarrow \mathbf{H}(s):=\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D} \quad (1)$$

where $\mathbf{A}\in\mathbb{R}^{n\times m};\mathbf{B}\in\mathbb{R}^{n\times m};\mathbf{C}\in\mathbb{R}^{n\times m};\mathbf{D}\in\mathbb{R}^{n\times m}$, *n* represents the number of state variables (the system's order), while *m* represents the number of inputs and, in a square system, the number of outputs.

The system of differential equations describes the time evolution of state variables, enabling the analysis of the system's dynamic behavior. The state matrix A represents the relationships among state variables, while the matrix B defines how the input influences these states. The matrix C converts the state variables into the output yy, and the matrix D captures the direct effect of the input on the output. The state vector x(t) contains information about the system's state, and the input vector u(t) provides the necessary signals to control the system, affecting the output y(t) through matrices B and D. The transfer function H(s) facilitates the analysis of the filter's frequency response, thereby enhancing the understanding of the system's filtering characteristics.

Two model order reduction algorithms—BT and MT—preserve the stability of the original system because:

- MT: In a stable original system, the eigenvalues (or poles) have negative real parts. By eliminating modes (i.e., eigenvalues or poles with negligible contribution) and retaining only the stable modes, the stability of the system is preserved.
- BT: For a stable original system, the controllability and observability Gramians are positive definite, which allows the system to be transformed into a balanced realization. Sorting according to the Hankel singular values (HSV) ensures that when states with small (less influential) values are discarded, the dominant HSVs—carrying the stability information—are retained, thereby maintaining the stability of the reduced-order system.

2.1. Modal Truncation Algorithm

Modal truncation (MT) is a model order reduction technique that optimizes system accuracy by retaining only the most significant dynamic modes. The principle of this algorithm is based on analyzing the eigenvalues and eigenvectors of the state matrix. By identifying the dominant modes, the algorithm permits the elimination of less influential modes, thereby yielding a simpler model that still preserves the essential characteristics of the system's behavior. Total Cost for the MT Algorithm: $O(2n^3 + n \log n + n^2r + nr^2 + nrm + mnr + m^2)$. The steps for implementing the MT algorithm are as follows [5] – [7]:

Input: A minimal, stable LTI system modeled as in (1).

- Step 1: Compute the eigenvalues of the matrix **A** as in (2).

$$\mathbf{A}v_i = \lambda_i v_i, \quad i = 1, 2, \ldots, n. \tag{2}$$

- Step 2: Construct the modal transformation matrix and its inverse as in (3) and (4), where Λ is the diagonal matrix containing the eigenvalues of **A**.

$$\mathbf{V} = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \tag{3}$$

$$\mathbf{A} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^{-1} \tag{4}$$

- Step 3: Choose the reduction order r such that $r < n$.

- Step 4: Determine the matrices **J** and **K** by selecting the first r columns of **V** and the first r rows of $\mathbf{V}^{-1}$, respectively, and then taking the transpose, as given in (5) and (6).

$$\mathbf{J} = [v_1 \; v_2 \; \ldots v_r] \tag{5}$$

$$\mathbf{K} = (\mathbf{V}^{-1})^T_{1:r} \tag{6}$$

- Step 5: Construct the reduced-order system using MT as in (7).

$$\mathbf{A}_{r\_MT} = \mathbf{K}^T\mathbf{A}\mathbf{J}; \mathbf{B}_{r\_MT} = \mathbf{K}^T\mathbf{B}; \mathbf{C}_{r\_MT} = \mathbf{C}\mathbf{J}; \mathbf{D}_{r\_MT} = \mathbf{D} \tag{7}$$

Output: The reduced-order system ($\mathbf{A}_{r\_MT}$, $\mathbf{B}_{r\_MT}$, $\mathbf{C}_{r\_MT}$, $\mathbf{D}_{r\_MT}$)

2.2. Balanced Truncation Algorithm

Balanced truncation (BT) is a model order reduction technique based on the principle of optimizing the balance between the system states. The operating principle of this algorithm begins with determining the eigenvalues and Hankel singular values of the state matrix, which allows for assessing the relative importance of each state in the model. This process enables the identification of states that contribute less to the overall system behavior, so they can be removed without sacrificing critical dynamic characteristics. Total Cost for the BT Algorithm: $O(6n^3 + rn^2 + r^2n + rnm + mnr + m^2)$. The steps for implementing the BT algorithm are detailed as follows [8] – [10]:

Input: A minimal, stable LTI system modeled as in (1).

- Step 1: Solve the two Lyapunov equations (8) and (9) to obtain the two Gramian matrices P and Q.

$$\mathbf{P}\mathbf{A}^T = -\mathbf{B}\mathbf{B}^T - \mathbf{A}\mathbf{P} \tag{8}$$

$$\mathbf{Q}\mathbf{A} = -\mathbf{A}^T\mathbf{Q} - \mathbf{C}^T\mathbf{C} \tag{9}$$

- Step 2: Perform the Cholesky decompositions of P and Q as in (10).

$$\mathbf{P} = \mathbf{X}\mathbf{X}^T; \mathbf{Q} = \mathbf{Y}\mathbf{Y}^T \tag{10}$$

- Step 3: Compute the singular value decomposition (SVD) as in (11).

$$\mathbf{Y}^T\mathbf{X} = \mathbf{Z}\mathbf{S}\mathbf{V}^T \tag{11}$$

- Step 4: Calculate the balancing transformation matrix as in (12).

$$\mathbf{T} = \mathbf{X}\mathbf{V}\mathbf{S}^{-1/2}; \mathbf{T}^{-1} = \mathbf{S}^{-1/2}\mathbf{Z}^T\mathbf{Y}^T \tag{12}$$

- Step 5: Choose the reduction order r such that $r < n$.

- Step 6: Construct the reduced-order system using BT as in (13).

$$\mathbf{A}_{r\_BT} = \left[\mathbf{T}^{-1}\right]_{1:r,:}\mathbf{A}\left[\mathbf{T}\right]_{:,1:r}; \mathbf{B}_{r\_BT} = \left[\mathbf{T}^{-1}\right]_{1:r,:}\mathbf{B}; \mathbf{C}_{r\_BT} = \mathbf{C}\left[\mathbf{T}\right]_{:,1:r}; \mathbf{D}_{r\_BT} = \mathbf{D} \tag{13}$$

Output: The reduced-order system ($\mathbf{A}_{r\_BT}$, $\mathbf{B}_{r\_BT}$, $\mathbf{C}_{r\_BT}$, $\mathbf{D}_{r\_BT}$)

3. Results and Discussion

IIR filters are one of the key tools in the fields of digital signal processing, communications, and automatic control. A notable characteristic of IIR filters is their ability to maintain an infinite impulse response; that is, the output depends not only on the current input value but also on previous values of both the input and the output. This creates a complex yet efficient structure, enabling IIR filters to perform filtering tasks with high accuracy and fast speed. The 30th-order IIR filter described in [11] is characterized by its frequency-domain transfer function, which is typically expressed as **H**(z)=num(z)/den(z)

Table 1. *Error Norms for BT and MT Model Reductions*

Order	H_∞ Error of BT	H_2 Error of BT	H_∞ Error of MT	H_2 Error of MT
1	2.154578×10^{-1}	4.303870×10^{-2}	2.097335×10^{-2}	1.805762×10^{-2}
2	1.666771×10^{-1}	2.574064×10^{-2}	2.097335×10^{-2}	1.805762×10^{-2}
3	2.562261×10^{-1}	5.123302×10^{-2}	2.097335×10^{-2}	1.805762×10^{-2}
4	7.670426×10^{-2}	1.638993×10^{-2}	2.097335×10^{-2}	1.805762×10^{-2}
5	1.058352×10^{-1}	1.975598×10^{-2}	2.097335×10^{-2}	1.805762×10^{-2}
6	8.626133×10^{-2}	1.843855×10^{-2}	2.097335×10^{-2}	1.805762×10^{-2}
7	4.931243×10^{-2}	1.515404×10^{-2}	2.097335×10^{-2}	1.805762×10^{-2}
8	4.334173×10^{-2}	1.383072×10^{-2}	2.097335×10^{-2}	1.805762×10^{-2}
9	1.292261×10^{-1}	1.916838×10^{-2}	2.097335×10^{-2}	1.805762×10^{-2}
10	2.377069×10^{-2}	9.306082×10^{-3}	2.097335×10^{-2}	1.805762×10^{-2}
11	1.767061×10^{-2}	7.082775×10^{-3}	2.097335×10^{-2}	1.805762×10^{-2}
12	1.945926×10^{-2}	9.185476×10^{-3}	2.097335×10^{-2}	1.805762×10^{-2}
13	1.109873×10^{-2}	5.661614×10^{-3}	2.097335×10^{-2}	1.805762×10^{-2}
14	1.736619×10^{-2}	5.590331×10^{-3}	2.097335×10^{-2}	1.805762×10^{-2}
15	1.233049×10^{-3}	8.219160×10^{-4}	2.097335×10^{-2}	1.805762×10^{-2}
16	1.607460×10^{-3}	7.703399×10^{-4}	2.097335×10^{-2}	1.805762×10^{-2}
17	9.684134×10^{-4}	7.790724×10^{-4}	2.097335×10^{-2}	1.805762×10^{-2}
18	2.673510×10^{-3}	1.308143×10^{-3}	2.097335×10^{-2}	1.805762×10^{-2}
19	7.822645×10^{-4}	3.677773×10^{-4}	2.097335×10^{-2}	1.805762×10^{-2}
20	1.728265×10^{-3}	7.239217×10^{-4}	2.097335×10^{-2}	1.805762×10^{-2}
21	1.115209×10^{-3}	5.241464×10^{-4}	2.097335×10^{-2}	1.805762×10^{-2}
22	7.701682×10^{-5}	3.978911×10^{-5}	2.097335×10^{-2}	1.805762×10^{-2}
23	5.314077×10^{-5}	3.235979×10^{-5}	2.097335×10^{-2}	1.805762×10^{-2}
24	5.314077×10^{-5}	3.235979×10^{-5}	2.097335×10^{-2}	1.805762×10^{-2}
25	5.314077×10^{-5}	3.235979×10^{-5}	2.097335×10^{-2}	1.805762×10^{-2}
26	5.314077×10^{-5}	3.235979×10^{-5}	2.097335×10^{-2}	1.805762×10^{-2}
27	5.314077×10^{-5}	3.235979×10^{-5}	2.097335×10^{-2}	1.805762×10^{-2}
28	5.314077×10^{-5}	3.235979×10^{-5}	2.097335×10^{-2}	1.805762×10^{-2}
29	5.314077×10^{-5}	3.235979×10^{-5}	2.097335×10^{-2}	1.805762×10^{-2}

By applying the Tustin transformation, $z = s + 1$, we obtain the transfer function: $H(s)=num(s)/den(s)$, with

$$\begin{aligned} num(s) = {} & 0.0187s^{30}+0.5044s^{29}+6.574s^{28}+55.13s^{27}+334.2s^{26}+1559s^{25}+5824s^{24}+1.788\times10^{4}s^{23}+4.597\times10^{4}s^{22}+1.003\times10^{5}s^{21} \\ & +1.878\times10^{5}s^{20}+3.035\times10^{5}s^{19}+4.261\times10^{5}s^{18}+5.216\times10^{5}s^{17}+5.578\times10^{5}s^{16}+5.219\times10^{5}s^{15}+4.269\times10^{5}s^{14}+3.05\times10^{5}s^{13} \\ & +1.897\times10^{5}s^{12}+1.023\times10^{5}s^{11}+4.753\times10^{4}s^{10}+1.888\times10^{4}s^{9}+6343s^{8}+1777s^{7}+407.4s^{6}+74.54s^{5}+10.52s^{4}+1.088s^{3} \\ & +0.076s^{2}+0.0031s+7.806\times10^{-18} \end{aligned}$$

$$\begin{aligned} den(s) = {} & s^{30}+25.96s^{29}+325.5s^{28}+2624s^{27}+1.527\times10^{4}s^{26}+6.837\times10^{4}s^{25}+2.448\times10^{5}s^{24}+7.197\times10^{5}s^{23}+1.77\times10^{6}s^{22}+3.69\times10^{6}s^{21} \\ & +6.588\times10^{6}s^{20}+1.015\times10^{7}s^{19}+1.355\times10^{7}s^{18}+1.575\times10^{7}s^{17}+1.597\times10^{7}s^{16}+1.413\times10^{7}s^{15}+1.091\times10^{7}s^{14}+7.338\times10^{6}s^{13} \\ & +4.284\times10^{6}s^{12}+2.161\times10^{6}s^{11}+9.356\times10^{5}s^{10}+3.447\times10^{5}s^{9}+1.068\times10^{5}s^{8}+2.743\times10^{4}s^{7}+5719s^{6}+941.3s^{5}+117.6s^{4} \\ & +10.49s^{3}+0.6116s^{2}+0.0222s+0.0003 \end{aligned}$$

By applying the MT and BT algorithms to this IIR filter, we perform model order reduction on the system, reducing its original order of 30 down to order 1. By calculating the reduction errors in terms of the H_∞ and H_2 norms, we obtained the corresponding errors as shown in Table 1.

From Table 1, we observe differences between the two reduction methods: BT and MT, based on the H_∞ and H_2 error norms. For BT, both the H_∞ and H_2 errors gradually decrease as the reduced order *r* increases toward the original order. This indicates that BT is capable of preserving and reproducing the main dynamic characteristics of the original system as more states are retained, thereby minimizing the overall deviation in the response of the reduced model. In contrast, the MT method produces rather invariant results regardless of the value of r. This invariance may suggest that, in its current implementation, the MT algorithm yields a reduced model with a fixed response structure, thereby failing to leverage the advantage of increasing the number of modes to enhance accuracy.

When reducing the model order for the IIR filter [11], it is observed that besides choosing an order that minimizes the error, other factors must also be considered, such as: a lower reduced order is preferable, and the time-domain as well as the frequency-domain responses of the original and reduced systems should match as closely as possible in terms of shape, characteristic curves, and data patterns. Therefore, after progressively reducing the original 30th-order system down to a 1st-order system while balancing these factors, the authors chose to reduce the original system to the 13th and 15th orders.

Choosing reduced orders of 13 and 15, we obtain the time-domain and frequency-domain responses of the original system versus the reduced-order systems obtained using BT and MT, as shown in Figures 1, 2, 3, and 4.

For a reduced order r = 13:

- From the impulse response plot in Figure 1, we observe that:

+ Over the entire simulated time domain, the order-13 reduced system obtained using MT exhibits a response that deviates significantly from that of the original system.

+ When BT is used to reduce the original system to order 13, the reduced system’s response shows slight discrepancies with the original system for time intervals shorter than 30 seconds; however, for time intervals longer than 30 seconds, the data curve of the reduced system adheres more closely to that of the original system.

+ Consequently, the order-13 reduced system obtained using BT can be considered as a substitute for the original system in time-domain applications, especially for time intervals exceeding 30 seconds.

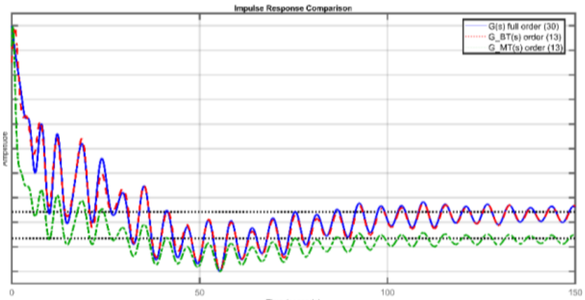

Figure 1. *Impulse response plots of the original system and the reduced 13th-order systems*

- From the Bode plot in Figure 2, we observe that:

+ In the frequency range below 10 rad/s, both the order-13 reduced systems obtained using BT and MT exhibit magnitude and phase responses that differ from those of the original system; however, at frequencies above 10 rad/s, the response of the order-13 reduced system gradually approximates that of the original system, with the BT reduced system’s data curve aligning most closely with the original system.

+ Over the entire simulated frequency range, the frequency response of the order-13 reduced system obtained using BT approximates that of the original system more closely than does the response of the order-13 reduced system obtained using MT.

+ Therefore, the order-13 reduced system obtained using BT can be considered as a substitute for the original system in frequency-domain applications for frequencies above 10 rad/s.

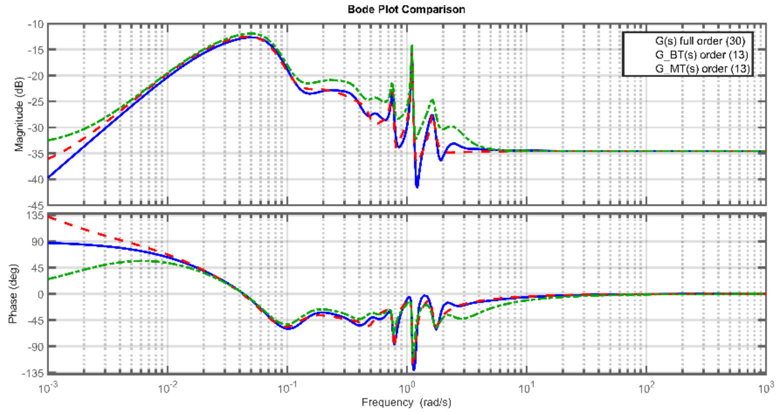

Figure 2. *Bode plots of the original system and the reduced 11th-order systems*

For a reduced order r = 15:

- From the impulse response plot in Figure 3, we observe that:

+ Over the entire simulated time domain, the order-15 reduced system obtained using MT still exhibits a response that deviates significantly from that of the original system, whereas the order-15 reduced system obtained using BT coincides with the original system.

+ Consequently, the order-15 reduced system obtained using BT can be considered as a substitute for the original system in time-domain applications, thereby reducing complexity compared to the 30th-order original system.

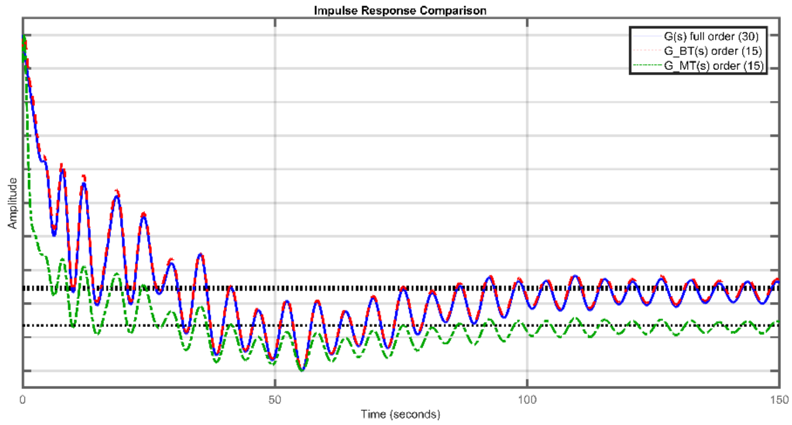

Figure 3. *Impulse response plots of the original system and the reduced 15th-order systems*

- From the Bode plot in Figure 4, we observe that:

+ In the frequency range below 10 rad/s, the order-15 reduced system obtained using MT exhibits magnitude and phase responses that differ from those of the original system; however, at frequencies above 10 rad/s, the response of the reduced system gradually approximates that of the original system.

+ Over the entire simulated frequency range, the frequency response of the order-15 reduced system obtained using BT nearly coincides with that of the original system.

+ Therefore, the order-15 reduced system obtained using BT can be considered as a substitute for the original system in frequency-domain applications, thereby reducing complexity compared to the 30th-order original system.

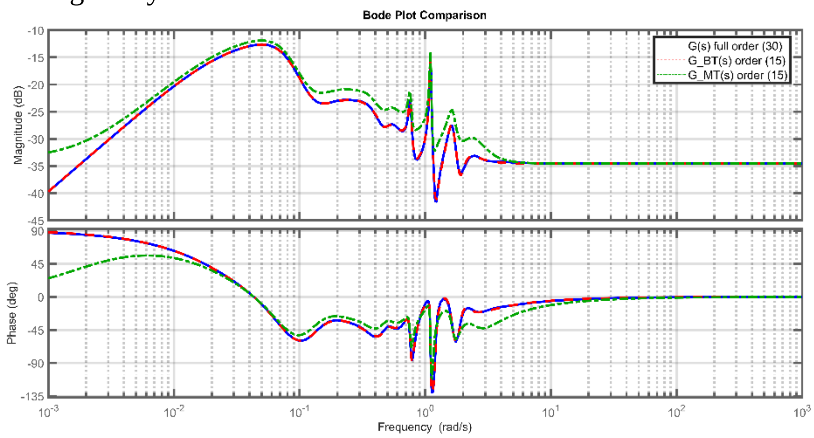

Figure 4. *Bode plots of the original system and the reduced 15th-order systems*

General Evaluation: Based on the reduction error table and the responses of the reduced-order system compared to the original system in both the time and frequency domains, it is evident that for reducing the order of the IIR filter [11], the BT algorithm provides superior reduction quality over the MT algorithm at every reduced order.

4. Conclusion

In this study, we applied and compared two model order reduction algorithms—Modal Truncation (MT) and Balanced Truncation (BT)—on a 30th-order IIR filter. By calculating the reduction errors in the H_∞ and H_2 norms, as well as evaluating the time-domain and frequency-domain responses, BT demonstrated excellent performance in reducing the model order. Specifically, the H_∞ error decreased from approximately 2.15×10^{-1} to 1.23×10^{-3} at order 15 and stabilized at 5.31×10^{-5} when the reduced order reached or exceeded 22, while the H_2 error decreased correspondingly, reaching about 9.31×10^{-3} at order 15 and stabilizing around 3.24×10^{-5} for higher orders. Additionally, BT effectively preserved the system's dynamic characteristics, as evidenced by the time-domain and frequency-domain responses; in particular, for frequencies above 10 rad/s, the BT-reduced models at orders 13 and 15 maintained almost all the features of the original system. In contrast, MT exhibited a constant reduction error—regardless of the number of retained states, the H_∞ error remained approximately 2.10×10^{-2} and the H_2 error about 1.81×10^{-2}. This indicates that the accuracy of the MT-based model, and the response of the reduced-order system, did not improve with increasing order, especially in the time domain and in the low-frequency range (<10 rad/s). Consequently, it can be concluded that for the purpose of reducing the order of IIR filters, BT is a superior method compared to MT, particularly in applications requiring high accuracy and the preservation of the original system's dynamic characteristics.

These results pave the way for further development in the application of model order reduction algorithms in signal processing, telecommunications, and complex control systems, helping to minimize processing time and computational costs while ensuring signal quality. In the future, additional research should focus on further optimizing these algorithms and applying them to various types of filters and systems, thereby broadening their scope of application and enhancing the overall performance of modern telecommunication systems.

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