ANALYSIS OF BEHAVIOUR IN THE SINGLE-STOREY BUILDING FRAME STRUCTURE WITH CRACKS SUBJECTED TO RANDOM LOADS

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ABSTRACT

The paper presents the calculation and behavioral analysis of a single-storey building frame structure with cracks under random loads. The frame structure is assumed to have a crack with a determined depth and a given position. The problem of the paper is to analyze the oscillator of a system of one-degree freedom subject to white noise excitation by using analytical method and Monte Carlo simulation. The results obtained in this paper are the second order moments of horizontal displacements and velocities of the one-storey frame structure. In order to feel the behavior in the structure, this paper has conducted thorough considerations to clarify the effect of crack depth, crack position, viscous damping coefficient and white noise intensity to the quantities considered.

Keywords: *Single-storey frame; crack; random, Monte Carlo; analytical;*

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PHÂN TÍCH ỨNG XỬ KẾT CẤU KHUNG NHÀ MỘT TẦNG CÓ VẾT NỨT DƯỚI TÁC DỤNG CỦA TẢI TRỌNG NGẪU NHIÊN

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TÓM TẮT

Bài báo trình bày cách tính toán và phân tích ứng xử của kết cấu khung nhà một tầng có vết nứt chịu tải trọng ngẫu nhiên. Kết cấu khung được giả thiết tồn tại vết nứt có độ sâu xác định tại vị trí cho trước. Nội dung bài báo là phân tích dao động của hệ một bậc tự do chịu kích động ồn trắng bằng phương pháp giải tích và tính toán mô phỏng theo phương pháp Monte Carlo. Các kết quả nhận được trong bài báo này là mô men bậc hai của chuyển vị ngang và vận tốc của kết cấu khung nhà một tầng. Để cảm nhận được ứng xử trong kết cấu, bài báo đã tiến hành khảo sát làm rõ ảnh hưởng của độ sâu vết nứt, vị trí vết nứt, hệ số cản và cường độ ồn trắng đến các đại lượng được xem xét.

Từ khóa: *khung một tầng; vết nứt; ngẫu nhiên; monte carlo; giải tích*.

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INTRODUCTION

In the process of using houses and structures it is easy to see that they are often changed due to the occurrence of defects such as leaks, corrosiveness, cracks... When studying the behavior of such structures, it usually focuses on two issues. Firstly, studying the behavior of structures under the acting of random loads. Problems of oscillation research under acting of random loads are very interesting not only in computational theory, but also in a model closer than in reality that shown in [3], [4], [7]. Secondly, the frame structures themself are no longer the same as the original ones. The behavioral analysis of structures with defects (such as cracks) is one of the many issues of concern [2], [5], [6], [9].

The studying problem in this paper is that the continuation in the previous paper [1] studied the behavior of structures according to the determined model. The problem here is a combination of two research issues - the structure is subject to the random loads and the existence of cracks. The crack is assumed to have a defined depth at a given position, and is converted into an elastic spring of equivalent stiffness [2], [7]. This paper has conducted random oscillation analysis of a single-degree freedom system in which two methods to be used are analytical [3], [8] and Monte Carlo simulation [3], [4]. And so, the results obtained are the second order moments of horizontal displacements and velocities of the one-storey frame structure. Then, this paper has conducted thorough consideration to clarify the effect of crack depth, crack position, viscous damping coefficient and white noise intensity to the quantities considered.

The contents of this article consist of 6 sections: Introduction; Monte Carlo simulation method of random vibration research; Model of a one-storey building frame structure and problems to solve; Results of solution by analytical methods; Results of Monte Carlo simulation; Conclusion.

MONTE CARLO SIMULATION METHOD OF RANDOM VIBRATION RESEARCH [3], [4]

Let's us considering a random process *f(t)* with autospectral density function *S(ω)*. The second order moment is σ^2, and we have:

$$\sigma^2 = \int_{-\infty}^{\infty} S(\omega)d\omega \qquad (1)$$

Discrete the autospectral density function as shown in Figure 1. Now, we can represent artificial random process *f(t)* as the sum of trigonometric functions with different frequencies:

$$f(t) = \sqrt{2}\sum_{k=1}^{N} A_k \cos(\omega_k + \phi_k) \qquad (2)$$

$$A_k = \sqrt{S_1^0(\omega_k)\Delta\omega} \qquad (3)$$

$$\omega_k = \left(k - \frac{1}{2}\right)\Delta\omega; \qquad \omega_u = N\Delta\omega \qquad (4)$$

where ω_u is the largest frequency of the spectral domain, S_1^0 is the oneside spectral density function $S_1^0 = 2S$, $\Delta\omega$ is the frequency division interval, φ_k is the random phase, taking a random value between 1 and 2π. For each set of random values of φ_k, we get a sample of random functions. However, dividing the spectral domain into regions with equal frequency ranges is usually not optimal. Here we will choose how to divide the spectral domain into intervals so that the area of each rectangle (see Figure 1) is equal.

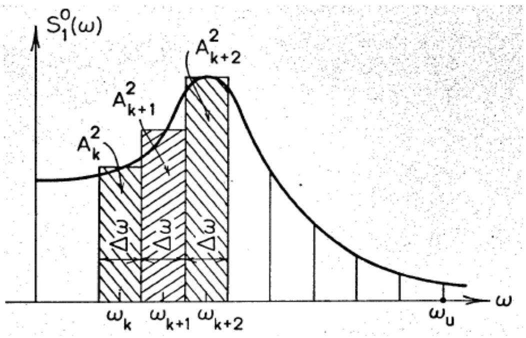

Figure 1. *Oneside spectral density function [4]*

And so the frequencies ω_k can be formulate when they are satisfied the condition

$$\int_0^{\omega_k} S_1^0(\omega)d\omega = \frac{k}{N}\int_0^{\omega_u} S_1^0(\omega)d\omega \quad (5)$$

Additional, the amplitudes A_k (the square root of the rectangular area) will be constant and calculated according to the formula:

$$A_k = \sqrt{\frac{1}{N}\int_0^{\omega_u} S_1^0(\omega)d\omega} \quad (6)$$

We will use Eq.(5), (6) and samples of random functions following Eq.(2) in order to simulate a random process that is implemented by Monte Carlo method below.

MODEL OF A ONE-STOREY BUILDING FRAME WITH CRACKS AND PROBLEMS TO SOLVE

In the document [1] we have a single-storey building frame is modeled as shown in Figure 2a. The floor has mass m. The frame has the height H. The floor is considered to have an infinite stiffness, and two columns have the bending stiffness are EI_1, EI_2, respectively. The damping force has a viscous coefficient of c.

$$k_{cr} = \frac{bh^2E}{36\pi\mu^2\left(0.5033-0.9022\mu+3.412\mu^2-3.18\mu^3+5.793\mu^4\right)};\quad \mu = \frac{2a}{h} \quad (7)$$

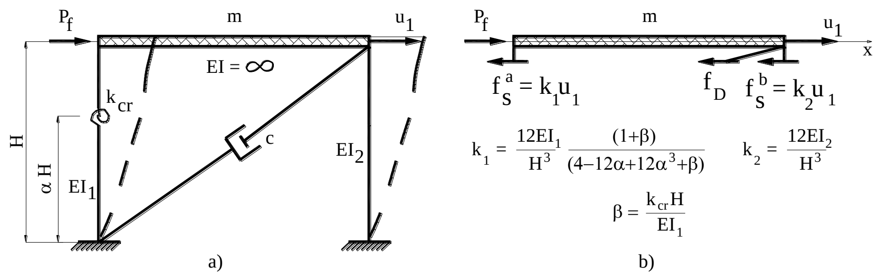

Figure 2. *Model of single-storey building frame with a crack in the column*

The crack (at the position αH (0≤α≤1)) is modeled as a elastic spring with the equivalent stiffness k_{cr}. If the column has cross-sectional area to be rectangular $b\times h$ and a is the depth of the crack, then k_{cr} is shown in Eq.(7) [2]. Unlike in the paper [1], here we assume that the frame is to be subjected to the environmental random loads $P_f=F(t)$, and obtained the equation of motion in the system of single degree of freedom as

$$m\ddot{u}_1 + c\dot{u}_1 + \left[\frac{12EI_1}{H^3}\cdot\frac{(1+\beta)}{\left(4-12\alpha+12\alpha^2+\beta\right)} + \frac{12EI_2}{H^3}\right]u_1 = F(t) \quad (8)$$

where $\beta = (k_{cr}H)/EI_1$. After changing variable $x=u_1$ the Eq.(8) will be rewritten as following:

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = f(t) \quad (9)$$

where

$$\omega_0^2 = \frac{1}{m}\left[\frac{12EI_1}{H^3}\cdot\frac{(1+\beta)}{\left(4-12\alpha+12\alpha^2+\beta\right)} + \frac{12EI_2}{H^3}\right] \quad (10)$$

$$\zeta = \frac{c}{2m\omega_0}; \quad f(t) = \frac{F(t)}{m} \qquad (11)$$

Suppose that with assumption the oscillator in the equation (9) is subjected to white noise excitation, and the spectral density function of *f(t)* is S_0. Then, from Eqs.(5) and (6) received

$$\omega_k = \frac{k}{N}\omega_u; \quad A_k = \sqrt{\frac{2S_0\omega_u}{N}} \qquad (12)$$

Conducting a survey on the effect of changing the depth of crack *a*, its position *α*, receiving the changing results of the variables ω_0 and ζ as shown in the figures from 3 to 6. In the figures we have assigned their values in the range *a=[0.01,0.08]m* and *α=[0.1,0.9]*.

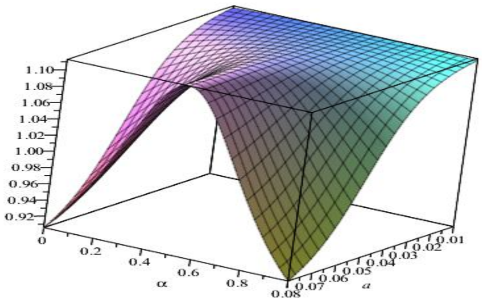

Figure 3. *Values of ω_0 when changing a=[0.01,0.08] and α=[0.1,0.9]*

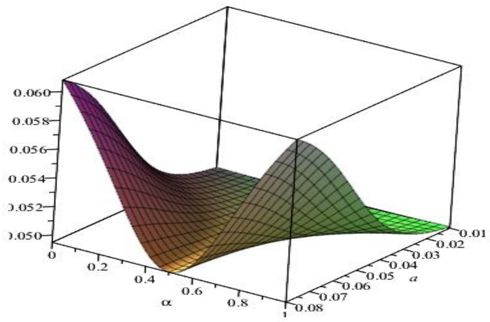

Figure 4. *Values of ζ when changing a=[0.01,0.08] and α=[0.1,0.9]*

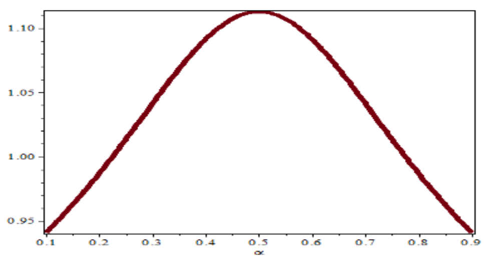

Figure 5. *Values of ω_0 when changing α=[0.1,0.9]*

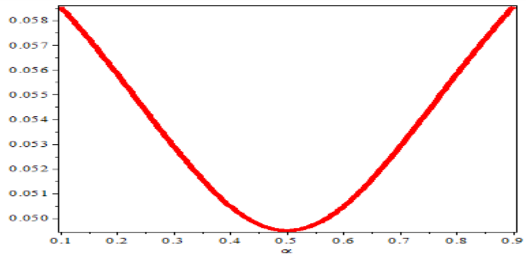

Figure 6. *Values of ζ when changing α=[0.1,0.9]*

The goal of us is that to solve Eq.(9) by using the analytical method and by Monte Carlo simulation method. The results here are values of the second order moments of displacements and their velocities will be implemented as below.

RESULTS OF SOLUTION BY USING ANALYTICAL METHOD

Eq. (9) is written in the form of differential equation of first order [3,8]:

$$\dot{z} = Az + Gf(t) \qquad (13)$$

where

$$z = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}; A = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\zeta\omega_0 \end{bmatrix}; G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (14)$$

Considering the response case is white noise process when considering the system in a steady state (assuming time calculation >= 300s), we have the equation to determine the second order moments of Lyapunov steady state responses as follows [3], [8]:

$$AR + RA^T + GVG^T = 0 \qquad (15)$$

where R is the second order moment of the response, determined by the matrix formula:

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \qquad (16)$$

and V is the steady state white noise intensity of the random process, calculated by the correlation function

$$R_{ff} = V\delta(t) \qquad (17)$$

We have relationship between the spectral density function and correlation function

$$S_{ff} = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega t} R_{ff}\, dt \qquad (18)$$

When $S_{ff}=S_0$ and from Eq.(17) we can substitute them into Eq.(18) will get

$$S_0 = \frac{V}{2\pi} \tag{19}$$

Let's solve Eq. (15) to get the second order moments of displacements R_{11} and their velocities R_{22} as follow

$$R_{11} = \frac{\pi S_0}{2\zeta\omega_0^3}; R_{22} = \frac{\pi S_0}{2\zeta\omega_0} \tag{20}$$

This is the analytical result, and this result is compared with Monte Carlo simulation results below.

From Eq.(20), by changing the crack depth, crack position, viscous damping coefficient and spectral density intensity, we could implement to plot the relationship between the variables and the second order displacements and velocities as shown in figures from 7 to 13. In Figure 7, values of R_{11} change rapidly when the position of crack is at α>= 0.7 and the crack depth a>=0.05m. In Figure 8, we see that R_{22} is in a plane, meaning that its value is constant, regardless of the depth and position.

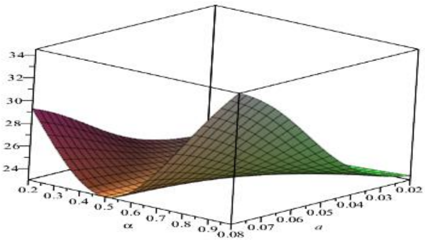

Figure 7. *Values of R_{11} when changing a=[0.01,0.08] and α=[0.1,0.9]*

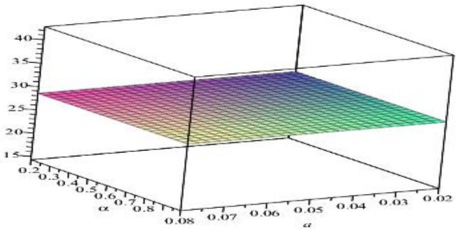

Figure 8. *Values of R_{22} when changing a=[0.01,0.08] and α=[0.1,0.9]*

In Figure 9, with the fixed value of crack depth a=0.08m, values of R_{11} depends on the crack position which almost reaches the minimum at the middle of the column and reaches the max at the top and bottom positions.

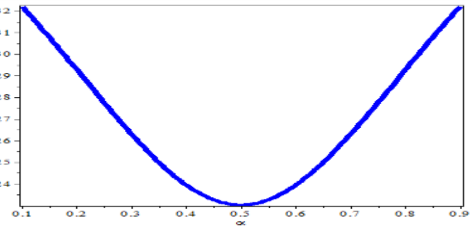

Figure 9. *Values of R_{11} when changing α=[0.1,0.9]*

In Figure 10 shows values of R_{11} when changing crack depth and intensity of spectral density. The values of R_{11} vary greatly and linearly according to S_0. In terms of absolute values, values of R_{11} change according to the intensity of the spectral density is quitely large noise. It is also shown in Figure 11 that values of R_{11} change according to crack position and S_0 spectral density intensity.

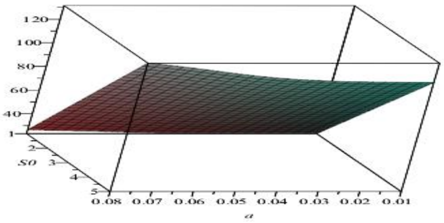

Figure 10. *Values of R_{11} when changing a=[0.01,0.08] and S_0=[1,5]*

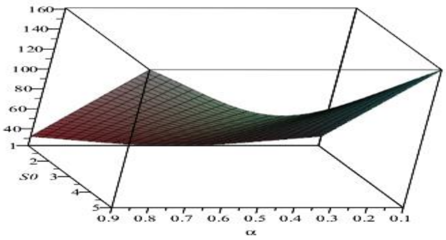

Figure 11. *Values of R_{11} when changing α=[0.1,0.9] and S_0=[1,5]*

Figures 12 and 13 show the second-order moments of displacements and velocities change when the viscous damping coefficient and the intensity of the spectral density vary. Comparing the variation of R_{11} and R_{22}, the change in the value of viscous damping coefficient and the intensity of the spectral density is the greatest influence on their value, meaning that they make the noise being largest. Since then, the concern to reduce noise (making R_{11} and R_{22} smaller) must increase the viscous damping coefficient.

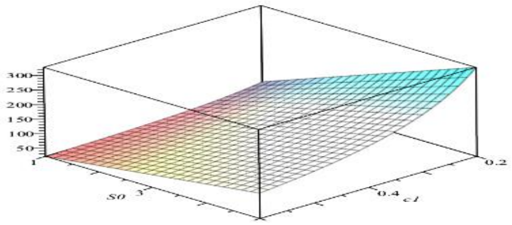

Figure 12. *Values of R_{11} when changing $c=[0.2,0.6]$ and $S_0=[1,5]$*

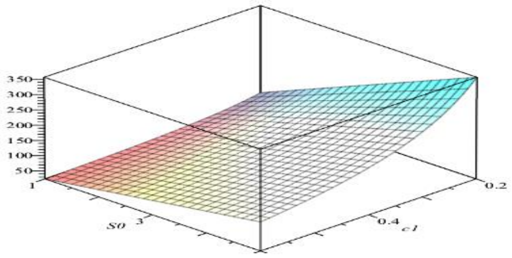

Figure 13. *Values of R_{22} when changing $c=[0.2,0.6]$ and $S_0=[1,5]$*

RESULTS OF SOLUTION BY USING MONTE CARLO SIMULATION

Monte Carlo simulation is performed to get results in the time about 300s, then the response of the system is considered to be steady state. The results in the paper were run with the number of samples being 400. With this number of samples enough for values of R_{11} to converge to analytical results than R_{22}. If the number of samples ≥800, the values of R_{22} can be considered convergence.

In Figure 14 shows the value of R_{11} when changing crack depth and comparing between calculation of analytical theory and Monte Carlo simulation. From the results obtained, the Monte Carlo simulation was found to have a deviation from the theory of about 10%.

Figure 15 shows the value of R11 when changing the crack position. Notice that the R_{11} value according to Monte Carlo simulation is close to the analytical value, with a value of about 5%.

In Figure 16, the case of white noise intensity S_0 changes, getting the result R_{11} is calculated according to analytical and Monte Carlo when the number of samples equals 400 asymptotic very close together (error <0.5%).

On Figure 17 is the value of R_{22} when changing the white noise intensity S_0=[2.5.5], found the value calculated by Monte Carlo simulation with the number of 400 samples with a difference of about 13%. With Monte Carlo calculation, it is found that when changing according to S_0, the degree of convergence results quickly when the value of S_0 is large.

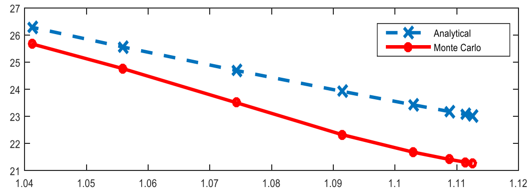

Figure 14. *Values of R_{11} when changing $a=[0.01,0.08]$*

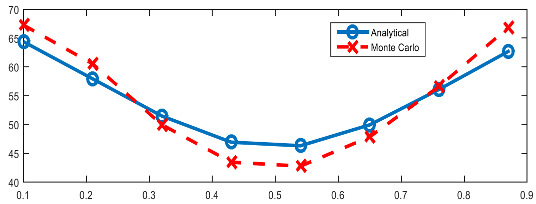

Figure 15. *Values of R_{11} when changing $\alpha=[0.1,0.9]$*

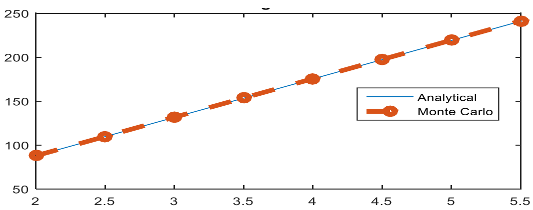

Figure 16. *Values of R_{11} when changing $S_0=[2,5.5]$*

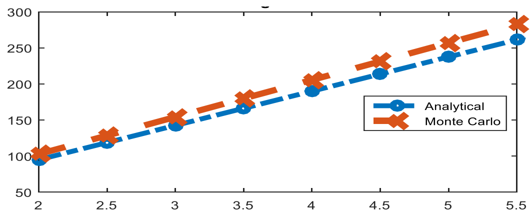

Figure 17. *Values of R_{22} when changing $S_0=[2,5.5]$*

Figures 18 and 19 show values of R_{11} and R_{22} when changing viscous damping coefficient c. With this result, it is shown that with the number of samples equal to 400, the difference in Monte Carlo calculation with analytical calculation is reliable (about 10% error).

After many times running Monte Carlo simulation, it is found that Monte Carlo simulation will converge as quickly as S_0 (calculated in relative value as % compared to analytical calculation). The explanation for this is because the dependence of R_{11} and R_{22} is largest to S_0. The second rapid result convergence after S_0 is calculated for the viscous damping coefficient *c*.

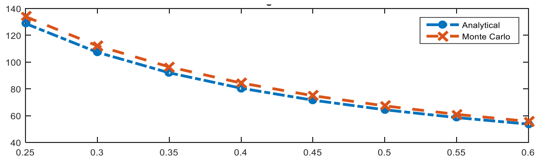

Figure 18. *Values of R_{11} when changing c=[0.25,0.6]*

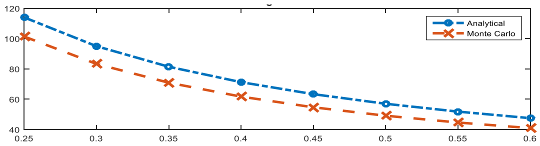

Figure 19. *Values of R_{22} when changing c=[0.25,0.6]*

CONCLUSION

The paper has analyzed and calculated a one-storey frame structure subjected to random loads of white noise by analytical method and simulated by Monte Carlo method. Results obtained are the second order moments of horizontal displacements and their velocities. Compare the results between the two calculation methods shown when Monte Carlo simulation with the number of samples equal to 400 receiving close results with a difference of about 10%. Especially when calculating to get R_{11} changing according to white noise intensity S_0, the difference is very small.

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