ENHANCING SIMULATION EFFICIENCY THROUGH MODAL TRUNCATION: A STUDY ON CONVECTION REACTION MODEL REDUCTION

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ABSTRACT

The paper explores the application of the Modal Truncation algorithm in Model Order Reduction, focusing on its effectiveness in reducing high-dimensional mathematical model. The algorithm identifies dominant modes governing dynamic responses, discards the high-order model, and reconstructs a new model with reduced dimensions. Much recent scientific literature has demonstrated the algorithm's versatility in systems stability analysis, dynamic analysis of systems, and other diverse applications. This study applies Modal truncation to the Convection Reaction model, an 84th-order system representing a chemical reaction. Results show that 5th and 6th-order reduced-order models effectively replicate the original system's behavior, with the 6th-order model exhibiting superior accuracy. Visualizations of transient and frequency domain responses provide insights into the reduced-order models' applicability. Based on the H∞ norm, error analysis emphasizes the 6th-order model's accuracy, which is crucial for selecting an appropriate reduced-order model based on desired accuracy in various applications. This study underscores the Modal Truncation algorithm's significance in achieving computational efficiency without compromising simulation fidelity. The continued refinement and application of this method play a crucial role in addressing challenges associated with high-dimensional systems.

NÂNG CAO HIỆU QUẢ MÔ PHỎNG THÔNG QUA CẮT NGẮN PHƯƠNG THỨC: NGHIÊN CỨU VỀ GIẢM MÔ HÌNH PHẢN ỨNG ĐỐI LƯU

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TỪ KHÓA

Giảm bậc mô hình
Thuật toán cắt ngắn phương thức
Hệ thống đa chiều
Mô hình phản ứng đối lưu
Phương trình đối lưu-khuếch tán

TÓM TẮT

Bài báo khám phá ứng dụng của thuật toán Cắt ngắn phương thức trong Giảm bậc mô hình, tập trung vào hiệu quả của nó trong việc giảm kích thước của mô hình toán học đa chiều. Thuật toán nhận diện các chế độ chi phối đáp ứng động, loại bỏ các mô hình bậc cao và xây dựng một mô hình mới có bậc thấp hơn. Nhiều tài liệu khoa học gần đây đã chứng minh tính linh hoạt của thuật toán trong phân tích hệ ổn định, phân tích hệ động học, và nhiều ứng dụng đa dạng khác. Nghiên cứu này áp dụng Cắt ngắn phương thức vào mô hình Phản ứng đối lưu, một hệ thống có bậc 84, biểu diễn một phản ứng hóa học. Kết quả cho thấy mô hình giảm về bậc 5 và bậc 6 tái tạo hiệu quả hành vi của hệ thống ban đầu, trong đó mô hình bậc 6 thể hiện độ chính xác vượt trội. Trực quan hoá về đáp ứng tức thời và phản hồi tần số cung cấp góc nhìn sâu sắc về khả năng ứng dụng của các mô hình giảm bậc. Dựa trên chuẩn H∞, phân tích sai số nhấn mạnh độ chính xác của mô hình bậc 6, điều này rất quan trọng để chọn một mô hình giảm bậc phù hợp dựa trên độ chính xác mong muốn trong các ứng dụng khác nhau. Nghiên cứu này nhấn mạnh sự quan trọng của thuật toán Cắt ngắn phương thức trong việc đạt được hiệu suất tính toán mà không làm suy giảm độ chính xác trong mô phỏng. Việc tiếp tục cải tiến và ứng dụng phương pháp này đóng một vai trò quan trọng trong việc giải quyết các thách thức liên quan đến hệ thống giảm kích thước mô hình.

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1. Introduction

The Modal truncation algorithm is a dimension reduction technique in Model Order Reduction that aims to compress the size of high-dimensional mathematical models leading to reduced computational cost and simulation expense, while still ensuring the desired level of accuracy. The key idea of this algorithm is to Identify the dominant modes (characteristic equations) that govern the system's dynamic response. These are usually the low-frequency modes. Discard (truncate) high-frequency modes from the model, as they have little influence on the overall system response. Reconstruct a new model with only a few modes compared to the original [1].

In the realm of model reduction algorithms, Modal Truncation stands out as a versatile and efficient technique, finding widespread application across diverse scientific domains. This method has undergone continual refinement to tailor its effectiveness to specific objects of study. Notably, recent research has demonstrated the prowess of Modal Truncation in various contexts. In the field of power systems stability analysis [1], a robust method was introduced, leveraging dimension reduction and modal truncation to discern critical oscillation modes. The application of diagonal expansion facilitated the derivation of characteristic polynomials, with the resulting model successfully tested on a substantial 547-machine 8647-bus model of the North China system. Addressing challenges in the dynamic analysis of engineering systems [2], another study focused on accurate frequency response analysis of non-classically damped systems. Through the integration of Neumann expansion theory and frequency shifting techniques, modal truncation issues were effectively addressed, demonstrating improved accuracy in comparison to traditional methods. Furthermore, diverse applications were explored, such as model reduction for cable mesh reflector antennas [3], compliant lightweight robots with flexible links [4], and large-scale wind farm models [5]. Article [6] proposes a method for reducing the dimensions of Hurty-Craig-Bampton components in high spatial resolution models by employing multi-fidelity models. The approach involves interface reduction through mesh coarsening, minimizing computational costs. The study demonstrates effectiveness on planar and complex industrial problems. In document [7], the article explores MIMO model reduction challenges in modern power systems with renewables. Traditional approaches face limitations, and the Iterative Rational Krylov Algorithm (IRKA) emerges as a promising alternative. A heuristic-based IRKA is proposed, preserving critical modes, and the method is validated on a 16-machine system with wind farms and a larger Brazilian system. The scientific work [8] investigates the impact of modal truncation in experimental modal analysis using finite modes. The study's dependency on the frequency and load distribution emphasizes potential errors if mode shapes don't efficiently represent load spatial distribution. Each study showcased the adaptability and efficiency of Modal Truncation in capturing essential system dynamics while significantly reducing computational complexity.

The evolution and application of Modal Truncation in recent scientific literature underscore its pivotal role as a potent tool for model reduction across a spectrum of complex systems, contributing to advancements in stability analysis, dynamic modeling, and control design. To validate the efficacy of the method, the author implemented and applied the algorithm to the model [9]. Simulations were conducted using Matlab, followed by a comparative analysis between the original system and the reduced-order systems at orders 5 and 6. This assessment aimed to evaluate the algorithm's applicability to the target object across both time and frequency domains.

2. Materials and Methods

Modal truncation is a mathematical algorithm employed in the field of system reduction and approximation. This technique is particularly relevant in dealing with high-dimensional systems, where the goal is to reduce the complexity of a model while retaining its essential dynamics. The algorithm is designed to identify and retain the most significant modes or components of a system, discarding less influential ones to achieve a lower-order representation. In the context of dynamical systems, such as those described by partial differential equations or other mathematical models, modal truncation proves valuable for simplifying the computational burden associated with simulations and analyses. By preserving the dominant modes that contribute most to the system's behavior, modal truncation enables a more efficient and manageable representation of complex systems. The implementation of modal truncation involves a careful selection of the dominant eigenmodes of a system, often determined through techniques like singular value decomposition (SVD) or other modal analysis methods. Once identified, these dominant modes are retained, while the less significant modes are truncated or neglected [1]. The algorithm is described as follows:

- Step 1: Initialize the environment by clearing all variables, closing figures, and clearing the command window. This ensures a clean workspace for subsequent computations and visualizations. Then, determine the order n of the system.
- Step 2: Load data into the workspace. This step provides the necessary input data for the subsequent analysis and computations.
- Step 3: Compute the eigenvalues and eigenvectors of matrix **A**. This step is fundamental in understanding the dynamic behavior of the system described by matrix **A**.
- Step 4: Sort the eigenvalues in descending order along with their corresponding eigenvectors. Sorting provides a clearer representation of the dominant modes of the system.
- Step 5: Prompt the user to input a value for variable r. This user input defines the dimensionality of the reduced system and influences subsequent matrix manipulations.
- Step 6: Extract the first r columns of the sorted eigenvector matrix and its inverse. This step forms the basis for the reduced system representation.
- Step 7: Form a projection matrix by combining the real and imaginary parts of the selected eigenvectors. The projection matrix is crucial for transforming the original system into a reduced form.
- Step 8: Perform transformations on matrices **A**, **B**, and **C** to obtain reduced matrices **Ar**, **Br**, and **Cr**.
- Step 9: Generate Bode plots, impulse response plots, and enable grid display for enhanced visualization. This step involves creating frequency response plots (Bode), impulse response plots, and improving overall plot readability by adding a grid. These visualizations provide valuable insights into the dynamic behavior and characteristics of both the full and reduced state-space models.
- Step 10: Calculate the absolute and relative errors between the full and reduced state-space models. Error analysis provides insights into the accuracy of the reduced-order representation.

3. Results and Discussion

The presented benchmark simulates a chemical reaction through the utilization of a convection-reaction partial differential equation on a unit square [9]. The model incorporates Dirichlet boundary conditions and is discretized using a centered difference approximation. The convection-diffusion equation amalgamates the diffusion and convection (advection) equations, delineating physical phenomena wherein particles, energy, or other physical quantities undergo transfer within a physical system, influenced by two primary processes: diffusion and convection. Depending on the specific application, the same equation may be referred to as the advection-diffusion equation, drift-diffusion equation, or the generic scalar transport equation.

The dynamical equation of the Convection Reaction model [9] is described by the state space matrix **A**, **B**, **C**, and **D** (where **D** = 0). Applying the Modal truncation algorithm to reduce the order of this 84th-order system to models of order 5 and 6. Implementation and programming were carried out in Matlab, and simulations were conducted to obtain the transient responses between the original system and the reduced-order systems over the time domain, as depicted in Figure 1, and the frequency domain responses, illustrated in the Bode plot in Figure 2.

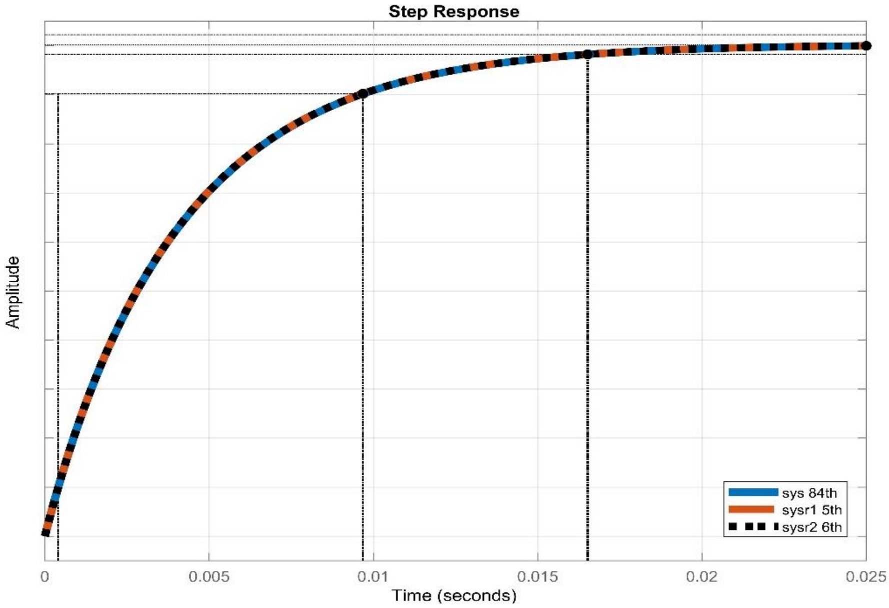

Figure 1. *Impulse response Comparison between Reduced-Order Systems and the Original System*

From Figure 1, it is observed that the 5th and 6th-order reduced-order systems exhibit transient responses matching those of the original system. Hence, either the 5th or 6th-order reduced-order system can be employed as a viable replacement for the original system in applications across the time domain.

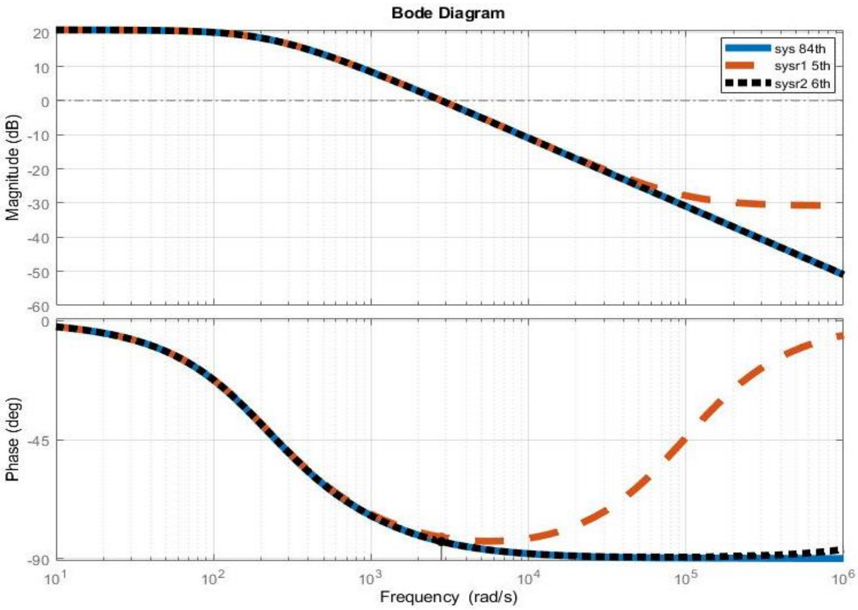

Figure 2. *Frequency response Comparison between Reduced-Order Systems and the Original System*

Figure 2 reveals that the 6th-order reduced-order system demonstrates a frequency domain response that aligns with that of the original system. On the other hand, the 5th-order reduced-order system exhibits a Magnitude response matching the original system at frequencies below $3x10^4$ (rad/s) (at 20 dB) and a Phase response matching the original system at frequencies below $2x10^3$ (rad/s) (at 80 deg). However, significant discrepancies are evident at higher frequency ranges. Consequently:

- The 6th-order reduced-order system can be effectively employed as a substitute for the original system across the entire frequency spectrum.

- The 5th-order reduced-order system can serve as a replacement for the original system in applications involving Magnitude response at frequencies below $3x10^4$ (rad/s) and Phase response at frequencies below $2x10^3$ (rad/s).

The order reduction system has the following state space model:

- The system reduces to order 5^{th}:

$$\mathbf{A}_{sys1} = \begin{bmatrix} -323.2 & 217 & -15.49 & 40.73 & -5.681 \\ 215 & -840.3 & 27.95 & -18.25 & -165.9 \\ 2.325 & 32.77 & -250.2 & 96.57 & -40.49 \\ 42.63 & -23.93 & -94.26 & -299.9 & -238.6 \\ -3.246 & -149.9 & 37.57 & -209.7 & -1193 \end{bmatrix}, \mathbf{B}_{sys1} = \begin{bmatrix} 53.13 \\ 0.1013 \\ 0.003456 \\ 0.1089 \\ -0.06563 \end{bmatrix}$$

$$\mathbf{C}_{sys1} = \begin{bmatrix} 53.13 & 0.1039 & 0.008405 & 0.1131 & -0.0706 \end{bmatrix}, \mathbf{D}_{sys1} = [-8.42e-06]$$

- The system reduces to order 6^{th}:

$$\mathbf{A}_{sys2} = \begin{bmatrix} -322 & 164.5 & 145.9 & -2.406 & -38.34 & -6.59 \\ 169.6 & -479.5 & -266.7 & -94.94 & 72.39 & -84.37 \\ 141.9 & -267 & -631.8 & 65.76 & -10.68 & -95.37 \\ 2.382 & 99.53 & -60.4 & -244.9 & -7.087 & 39.93 \\ 25.67 & -39.1 & 35.14 & -1.491 & -296.1 & -165.2 \\ -2.326 & -74.52 & -85.88 & -27.27 & 185.1 & -557.8 \end{bmatrix}, \mathbf{B}_{sys2} = \begin{bmatrix} 53.13 \\ -0.02177 \\ 0.03364 \\ -0.003578 \\ 0.01048 \\ 0.02778 \end{bmatrix}$$

$$\mathbf{C}_{sys2} = \begin{bmatrix} 53.13 & -0.0224 & 0.03342 & 0.004399 & -0.01651 & 0.02849 \end{bmatrix}, \mathbf{D}_{sys2} = \begin{bmatrix} -3.467 \times 10^{-7} \end{bmatrix}$$

The absolute and relative errors based on the H∞ norm between the original system and the 5th and 6th-order reduced-order systems are presented in Table 1.

From Table 1, it is evident that the absolute error between the original system and the 5th-order reduced-order system consistently exceeds that of the 6th-order reduced-order system. The absolute error, as per the H∞ norm, indicates the maximum deviation between the original system and the reduced-order system across the entire time domain, while the relative error, as per the H∞ norm, signifies the greatest discrepancy between the original system and the reduced-order system across the entire frequency spectrum.

Therefore, when accuracy is paramount, the BPRU algorithm appears to be the most effective choice among these three algorithms for model reduction in this context.

Table 1. *Comparison of Model Reduction Algorithms*

Reduced Order (r)	Absolute Error	Relative Error
5	0.032286063250813	0.002979566832949
6	0.012067214855912	0.001113640671253

These results suggest that the 6th-order reduced-order system provides a more accurate approximation of the original system, with significantly lower errors in both absolute and relative terms. This information is crucial for understanding the trade-offs and selecting an appropriate reduced-order model based on the desired accuracy for various applications across both time and frequency domains.

4. Conclusion

This study applied the Modal Truncation algorithm to the Convection Reaction model, demonstrating its effectiveness in reducing an 84th-order system to 5th and 6th-order models.

The analysis of transient and frequency domain responses revealed that both the 5th and 6th-order reduced-order models can serve as viable replacements for the original system in specific applications. However, error analysis, considering absolute and relative errors, highlighted the superior accuracy of the 6th-order reduced-order system.

These findings underscore the significance of the Modal Truncation algorithm in achieving a balance between computational efficiency and simulation fidelity. As technology advances and the need for efficient modeling and simulation grows, the continued refinement and application of Modal Truncation are poised to play a crucial role in addressing challenges associated with high-dimensional systems.

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