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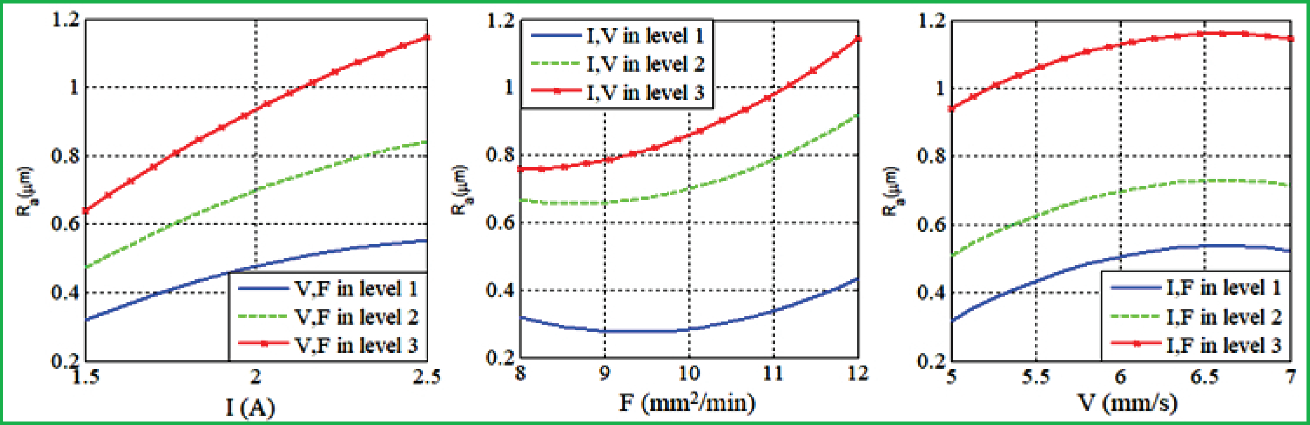

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STATISTICAL MODELING AND OPTIMIZATION OF SURFACE ROUGHNESS IN WIRE EDM OF NONCIRCULAR GEARS MADE OF SKD11 STEEL

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Abstract:

This study presents the statistical modeling and optimization of surface roughness in wire EDM of SKD11 noncircular gears. A Taguchi L9 design was employed to evaluate the effects of discharge current (I), feed rate (F), and wire winding speed (V). ANOVA results indicated that discharge current had the greatest influence (58.68%), followed by feed rate (31.95%) and wire winding speed (9.37%). A quadratic model developed using Response Surface Methodology (RSM) showed excellent accuracy ($R^2 > 0.99$). Surface roughness increased monotonically with higher parameter levels due to thermal effects. The optimized parameter combination significantly reduced surface roughness, confirming the effectiveness of the proposed statistical approach for precision gear manufacturing by WEDM.

Keywords: *Wire electrical discharge machining (WEDM), Surface roughness, Taguchi design, Analysis of variance (ANOVA), Response Surface Methodology (RSM), SKD11 noncircular gears.*

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1. Introduction

Wire Electrical Discharge Machining (Wire EDM) is suitable for machining parts with complex shapes, such as non-circular gears, particularly those made of hard materials such as SKD11 steel [1-3]. Surface quality in Wire EDM is governed by multiple process parameters, including discharge current, pulse duration, wire speed, and feed rate [4,5]. To address the multi-factorial nature of the Wire EDM process, statistical optimization techniques such as the Taguchi method and Response Surface Methodology (RSM) have been extensively used [6,7]. The Taguchi design of experiments (DOE) provides a systematic, economical approach to evaluating parameter effects using orthogonal arrays. RSM, on the other hand, allows the development of predictive models to describe nonlinear interactions among variables and to identify optimal process settings [8,9]. Recent studies have demonstrated that combining Taguchi and RSM methods enhances prediction accuracy and reduces experimental effort [10, 11]. However, very few previous studies have quantified the relationship between process parameters and surface roughness of out-of-round gears fabricated from SKD11 material. Therefore, this study aims to develop a statistical model to predict and optimize surface roughness in the Wire EDM of SKD11 noncircular gears based on Taguchi experimental design and response surface method (RSM). A Taguchi L9 orthogonal array is employed to design the experiments with three control factors: discharge current (I), wire winding speed (V), and cutting speed (F). ANOVA is used to quantify the contribution of each parameter, while RSM is applied to construct a predictive model and identify the optimal combination of factors. The proposed framework not only enhances understanding of parameter interactions but also provides a robust basis for process control in precision gear manufacturing.

2. Summary of Taguchi's method, analysis of variance, and response surface method [15]

2.1. Summary of Taguchi's method

The Taguchi method uses predefined orthogonal arrays to design experiments to evaluate the simultaneous effects of input parameters on target outputs [12]. The trial results are converted into an intermediate signal-to-noise ratio (S/N) index using one of three characteristics, as in Eqs.

(1~3).

Quality characteristics of the larger are better:

$$S/N = -10\log\left(\frac{1}{n}\Sigma\frac{1}{y_i^2}\right) \quad (1)$$

Quality characteristics of the nominal are the best:

$$S/N = -10\log\left(\frac{\overline{y}^2}{D^2}\right) \quad (2)$$

Quality characteristic of the smaller is better:

$$S/N = -10\log\left(\frac{1}{n}\Sigma y_i^2\right) \quad (3)$$

Where: $S/N; y_i; \overline{y}; D; n$ are the signal/noise ratio, the measured value of the i^{th} trial, the mean of all measurements, the variance, and the number of repetitions of the ith trial, respectively.

The signal-to-noise (S/N) ratio serves as a crucial performance index for evaluating a process's robustness, and optimizing the S/N ratio enables the identification of the optimal level for parameters.

2.2. Steps of analysis of variance

Analysis of variance (ANOVA) is a statistical technique employed to evaluate the significance of input factors on a system's response. This technique provides a rigorous basis for identifying the most influential parameters, optimizing process conditions based on the experimental outcomes. The main steps to analyze variance are as follows:

Step 1: Computing the mean S/N ratio at each level for all parameters, as defined in Eq. (4)

$$m_{ji} = \frac{1}{k}\sum_{i=1}^{k}\left((S/N)_j\right)_i \quad (4)$$

Step 2: Determining the total value of the experimental results (T) and the parameter adjustment coefficients (CF) according to Eq. (5)

$$T = \sum_{i=1}^{n_a} y_i \text{ and } CF = T^2/n_a \quad (5)$$

Step 3: Determining the sum of squares of the parameters according to Eq. (6)

$$S_j = \sum_{i=1}^{n_{ji}} (j_i^2 - CF.n_{ji})/n_{ji} \quad (6)$$

Step 4: Determining the sum of squares of variance of the parameters according to Eq. (7)

$$S_j = 3\left(m_{ji} - m\right)^2 + 3\left(m_{ji} - m\right)^2 + 3\left(m_{ji} - m\right)^2 \quad (7)$$

Step 5: Determining the degrees of freedom of the experiment (f_T) and of the parameters (f_j), as Eq. (8)

$$f_T = n_a - 1 \text{ and } f_j = k - 1 \quad (8)$$

Step 6: Determining the variance of the parameters according to Eq. (9)

$$V_j = S_j / f_j \quad (9)$$

Step 7: Determining the sum of squares according to Eq. (10)

$$S_T = \sum_j S_j \quad (10)$$

Step 8: Determining the effect percentage of the parameters on the output target according to Eq. (11)

$$P_j = S_j / S_T \quad (11)$$

Step 9: Predict the output quality with the optimal level of parameters according to Eq. (12)

$$Y_{opt} = T + \sum_{j=1}^{n}(j_i - T) \quad (12)$$

In the above Eqs: n_a is the number of experimental, m_{ji} is the average of the S/N ratios for ith level (i = 1,2 … k) of parameter j, n_{ji} is the number of trials of parameter *j* at the i^{th} level, and j_i is the sum of the results of parameter *j* at the i^{th} level, *m* is the average of the S/N ratios, and *k* is the number of levels of the parameter *j*.

2.3. Summary response surface method (RSM) [16]

RSM is a collection of mathematical and statistical techniques used to model the relationship between a response and multiple input variables. Its goal is to build an empirical model that quantifies how process parameters influence the output. A typical example is a quadratic regression model relating tooth surface roughness (Ra) to discharge current (I), wire winding speed (V), and cutting speed (F) as follows:

$$\tilde{R}_a = a_1 I^2 + a_2 F^2 + a_3 V^2 + a_4 F.V + a_5 I + a_6 F + a_7 V + a_8 \quad (13)$$

The coefficients a_1 to a_8 are the unknowns to be found and are typically estimated using least-squares regression based on the extreme condition of the expression:

$$S(a_1 \to a_8) = \sum_{i=1}^{n}\left[\left(R_a\right)_i - \tilde{R}_a(I_i, F_i, V_i)\right]^2 \to \min \quad (14)$$

By solving the extreme condition, the regression equation for tooth surface roughness is obtained.

3. Experimental design

3.1. Machine, workpiece, and processing materials of the experiment

This study conducts experimental machining of non-circular gears that form an ellipse. The gear material is SKD11 steel. The CNC wire-cutting machine, Excetek V400G, is used for gear machining. The cutting wire used is copper wire with a diameter of 0.25 mm. Tooth surface roughness was measured using a Mitutoyo SJ-410 roughness meter. The workpiece, wire cut, machine in machining of non-circular gears, and tooth surface roughness meter are shown in **Fig. 1.** Machine parameters and machined gear images are given in **Tab.1**. Machined material composition is given in **Tab.2.**

3.2. Selection of study parameters and experimental matrix

In this study, the parameters I, V, and F are selected to determine their impact on the tooth surface roughness during WEDM machining of non-circular gears using the L9 orthogonal array of Taguchi. The nine test conditions and their corresponding parameter values are given in columns 1-4 of **Tab.3**.

4. Result and discussion

The machined gear samples corresponding to the 9 test conditions are shown in **Fig.2**. The

Workpiece

Wire

CNC wire cutting machine
Exetek V400G

Roughness measuring device
Mitutoyo SJ-410

Figure 1. Workpiece, wire, machine in machining of non-circular gears, and tooth surface roughness meter

Table 1. Basic parameters of machined gears

Machined gear parameters	**Value (*unit*)**
Number of teeth of the gear	25
Normal module	2,4
Tooth thickness according to the normal direction	5,04 (*mm*)
Pressure angle in the normal direction	22°
Tooth width	20 (*mm*)
Center distance	89,64 (*mm*)

Table 2. Material composition of SKD11 steel

Chemical composition of SKD 11 steel	Carbon (C)	Manganese (Mn)	Silicon (Si)	Sulfur (S)	Phosphorus (P)
Percentage of main elements of SKD 11 steel	0.9% – 1.5%	<= 0,4%	<= 0,4%	≤ 0.025%	≤ 0.025%

Table 3. Levels of the experimental parameters, trial results, and S/N ratios

Trial	**Discharge current I (A)**	**Wire winding speed V (mm/s)**	**Cutting speed F (mm^2 /s)**	**Surface roughness y_i (µm)**	y_i^2	**S/N**
1	1.5	5	8	0.31	0.0961	10.1728
2	1.5	6	10	0.48	0.2304	6.3752
3	1.5	7	12	0.64	0.4096	3.8764
4	2	5	10	0.51	0.2601	5.8486
5	2	6	12	0.91	0.8281	0.8192
6	2	7	8	0.69	0.4761	3.2230
7	2.5	5	12	0.95	0.9025	0.4455
8	2.5	6	8	0.74	0.5476	2.6154
9	2.5	7	10	0.85	0.7225	1.4116

average tooth surface roughness measurement results are given in column 5 of **Tab. 3**. The S/N ratio of the test results is calculated according to the quality characteristic that smaller is better (Eq. (3)). The results are given in column 7 of **Tab.3**. By using Eq. (4), the level of the parameters (*I, F, V*) can be calculated. The graph of leveling and the percent effect chart of the parameters (*I, F, V*), as presented in **Fig. 3**.

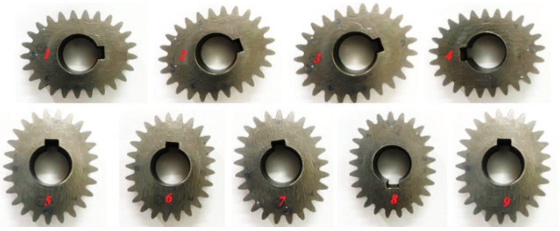


Figure 2. Gear samples corresponding to nine different machining modes according to Taguchi's orthogonal arrays

Based on the results shown in **Fig. 3**, the optimal parameter levels for tooth surface roughness are I_1, F_1, and V_1. The tooth surface roughness value achieved with the combination of parameters at the optimal level can be determined by Eq. (12)

$$R_{aopt} = T + (I_1 - T) + (V_1 - T) + (F_1 - T) = 0.3 \ (\mu m) \quad (15)$$

It can be seen that the optimal conditions coincide with experiment No.1, and the statistically calculated optimal results are very consistent with the experimental results.

Based on the effective distribution chart in Fig. 3, the parameters of tooth surface roughness and discharge current (I) have the most significant influence on Ra (58.68%). Next is the cutting speed (F) with 31.95% influence. The wire-winding speed (V) has a negligible impact on tooth surface roughness (only 9.37%). This means the surface roughness needs to be adjusted; we should prioritize adjusting the discharge current and cutting speed. The above has determined the optimal level of the parameters (I, F, V) for the tooth surface roughness. However, the law governing tooth surface roughness as parameters (I, F, V) change has not yet been clarified. To solve this problem, the response surface methodology with a quadratic regression model based on the least-squares criterion mentioned in Section 2.3 is used. The results are as follows:

$$\tilde{R}_a = -0.167I^2 - 0.086V^2 + 0{,}023F^2 + 0.068IF + \\ +0.353I + 1.133V - 0.54F - 1.347 \quad (16)$$

Calculating the coefficient of determination (R^2) for the regression function yields $R^2 = 0.999$. This very high index indicates that the regression function fits the experimental data set very well and can be used to describe the influence law of the input parameters (*I, F, V*) on the output, the tooth surface roughness.

From the regression function (Eq.(16)), the 2D and 3D responses showing the relationship between tooth surface roughness (R_a) and each input parameter (*I, F,* and *V*) and the combination of input parameter pairs in optimal level (*VF*; *IF*, *VI*) are given in **Fig. 4** and **Fig. 5**.

The three plots in Fig. 4 show clear and consistent trends in how surface roughness (Ra) varies with the machining parameters I, F, and V. First, surface roughness increases monotonically with current I for all V–F levels, and the separation between the curves indicates that higher V–F settings always produce higher Ra. This confirms that discharge energy is the dominant factor

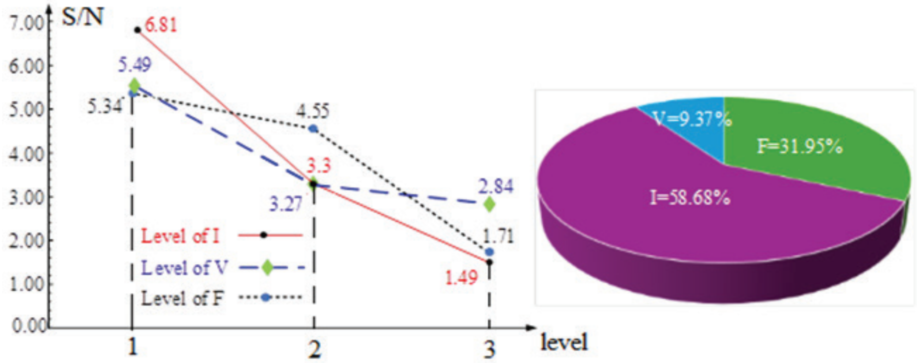


Figure 3. Graph of leveling and percent effect chart of the I, F, V parameters on tooth surface roughness

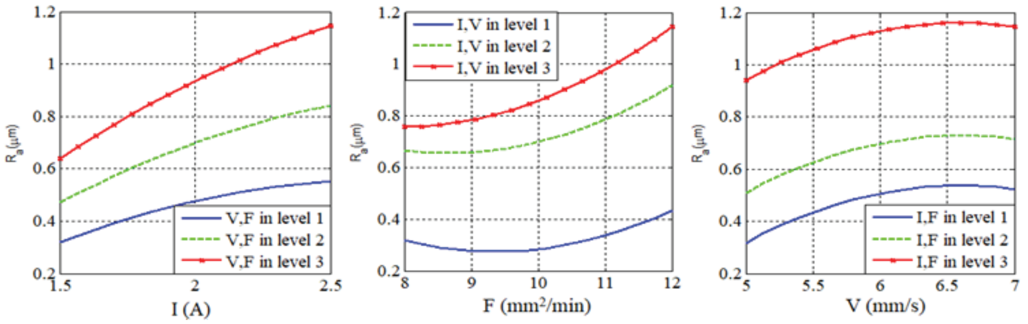
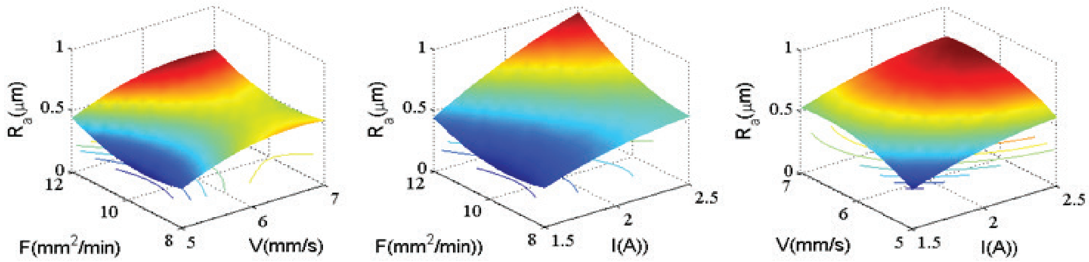


Figure 4. The dependence of tooth surface roughness on parameters I, F, and V in a 2D graph



a) I in level 1 b) V in level 1 c) F in level 1

Figure 5. The dependence of tooth surface roughness on parameters I, F, and V in a 3D graph

governing surface quality. For cutting speed F, Ra exhibits a mild non-linear trend: it decreases slightly at moderate F and rises again at higher F. The effect is most pronounced under high-energy settings, implying that F mainly acts as a secondary parameter that adjusts the balance between material removal and achievable finish. With wire speed V, Ra changes only moderately: low-to-medium V slightly improves surface finish, while excessively high V causes Ra to increase again. Under high-energy combinations, the influence of V becomes notably weaker compared with I.

Overall, current I is the primary factor determining surface roughness, while F and V play supporting roles that fine-tune the resulting quality. The best surface finish is obtained at low I, moderate F, and moderate V. Operators should avoid simultaneously high I, F, and V, which consistently correspond to the highest measured Ra.

The three response surfaces in Fig. 5(a–c) collectively describe how surface roughness varies when one parameter is fixed at its optimal level. In Fig. 5(a), with current held constant, Ra forms a shallow trough across the F-V plane: roughness reaches a minimum in the vicinity of level 1 of the parameters feed and wire speed. When wire speed is fixed, as shown in Fig. 5(b), a similar feed-dependent pattern appears; however, the principal gradient aligns with the current axis, indicating that even at low wire speed, the increase in I dominates the surface response. This trend becomes clearer in Fig. 5(c), where feed is fixed and Ra increases sharply with current while only modestly varying with wire speed. Taken together, these three surfaces show that although feed and wire speed contribute to smoothing the surface, their influence is secondary compared with the strong and consistent effect of current. The lowest roughness is therefore achieved under low-current conditions combined with in the vicinity of level 1 of the parameters feed and wire-speed settings, where the response surface exhibits a stable and well-defined optimum.

5. Conclusions

This study examined the effects of discharge current, cutting speed, and wire winding speed on the surface roughness of SKD11 noncircular gears machined by WEDM using a combined Taguchi–ANOVA–RSM approach. Results showed that discharge current was the dominant factor, accounting for approximately 58.7% of the variation, while cutting speed and wire speed exerted secondary influences. The optimal parameter combination significantly reduced roughness, and the developed quadratic regression model demonstrated excellent agreement with experimental data ($R^2 > 0.99$). The response surfaces revealed that crater morphology, thermal accumulation, and debris evacuation jointly govern roughness evolution. Although the study focused on three process parameters and one gear geometry, the findings provide practical guidance for precision machining of hard tool steels. Future work should expand the parameter space and incorporate additional surface integrity metrics.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

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MÔ HÌNH HÓA THỐNG KÊ VÀ TỐI ƯU HÓA ĐỘ NHÁM BỀ MẶT TRONG GIA CÔNG EDM CÁC BÁNH RĂNG KHÔNG TRÒN LÀM TỪ THÉP SKD11

Tóm tắt:

Nghiên cứu này trình bày mô hình thống kê và tối ưu hóa độ nhám bề mặt bánh răng không tròn chế tạo từ thép SKD11 khi gia công trên máy cắt dây. Thiết kế Taguchi L9 được sử dụng để đánh giá ảnh hưởng của cường độ dòng điện (I), tốc độ cắt (F) và tốc độ cuộn dây (V) tới độ nhám bề mặt răng. Kết quả phân tích phương sai (ANOVA) cho thấy cường độ dòng điện có ảnh hưởng lớn nhất (58,68%), tiếp theo là tốc độ cắt (31,95%) và cuối cùng là tốc độ cuộn dây (9,37%). Mô hình thống kê bậc hai giữa độ nhám bề mặt răng và các tham số đầu vào (I, F, V) được phát triển bằng phương pháp nội suy bình phương tối thiểu cho thấy độ chính xác tuyệt vời ($R^2 > 0,99$). Độ nhám bề mặt tăng dần theo mức độ gia tăng của các tham số do ảnh hưởng nhiệt. Sự kết hợp các tham số được tối ưu hóa đã làm giảm đáng kể độ nhám bề mặt, khẳng định hiệu quả của mô hình được đề xuất trong sản xuất bánh răng chính xác bằng WEDM.

Từ khóa: *Gia công bằng máy cắt dây (WEDM), Độ nhám bề mặt, Thiết kế Taguchi, Phân tích phương sai (ANOVA), Phương pháp bề mặt đáp ứng (RSM), Bánh răng không tròn SKD11.*