CUTTER CORRECTION METHOD FOR IMPROVING THE ACCURACY OF MANUFACTURED SCREW ROTOR BY END MILLING CUTTER

Van-Duong Vuong, Minh-Thuan Hoang*
Hung Yen University of Technology and Education
** Corresponding author: Hoangthuan.hut@gmail.com*

Received: 15/10/2025
Revised: 10/11/2025
Accepted for publication: 10/12/2025

Abstract:

The accuracy of the RP (RP) is a significant factor in the vacuum pump performance. However, the manufactured RP deviates from this theoretical form due to cumulative machine-tool errors—such as temperature changes, humidity, and nearby vibrations. Therefore, a cutter-correction method is proposed to improve the accuracy of the manufactured profile by accounting for RP errors during cutter design. A mathematical model of the design end-milling cutter profile, accounting for compensated cutter parameters, is first proposed. The effect of the compensated cutter parameters on the RP error is investigated and summarized in a sensitivity matrix. The correction cutter profile is established using the correction cutter parameters, which are solved by applying the sensitivity matrix and Levenberg-Marquardt algorithm. The numerical example is provided to validate the proposed approach for reducing RP error.

Keywords: *screw rotor, end-milling cutter, compensated parameters, RP error.*

Nomenclature

A_c	center distance of screw rotors
$\mathbf{M}_{ij}$	transformation matrix from S_j to S_i
n	number of points
$\mathbf{n}$	unit normal vector
$\mathbf{N}$	normal vector
$\mathbf{r}$	position vector
r	inner radius of the rotor
r_p	pitch radius of the rotor
r_c	tool radius
S	coordinate system
s_p	helix parameter
t	curve parameter
$\mathbf{t}$	tangential vector
β	pitch helix angle of the rotor
ϕ	longitudinal parameter of the screw surface
φ	rotation angle

Subscripts

c	cutting tool
s	screw surface

Superscripts

T	matrix transpose
t	cutting tool.

1. Introduction

A vacuum pump is the primary device in vacuum-environment generation systems and is widely used across industries such as semiconductors, electronics, and food processing. [1], [2]. The pump operates by meshing two rotors within the housing; therefore, the profile accuracy of these rotors is a critical factor governing the pump's efficiency [3]. Consequently, enhancing the manufacturing precision of vacuum pumps is a pressing challenge for researchers and production engineers.

End milling has proven well-suited to producing screw rotors (SR), especially the vacuum pump rotor, spurring extensive research into cutter design methods. Tang *et al.* [4] used the meshing principle between the cutter and workpiece to design the cutter profile based on the discrete points of the rotor profile, an interpolation method, and innovated with the form-position geometric method. By a different approach, Popa *et al.* [5] presented the design cutter profile method from discrete points of the RPbased on the principle of the "substitute circle family". Kim *et al.* [6] proposed an end-mill design process in which the cutter's solid model is generated, and cutting is simulated via Boolean operations between a specified grinding wheel and a cylindrical blank; key geometric features are validated by interrogating sectional profiles. The study analyzes parameter interdependencies and introduces an iterative scheme to derive the specified wheel geometry and alignment/CL data to meet the design targets, implemented via API programming in a commercial CAD system and currently deployed by an industrial toolmaker. These previous studies can generate the theoretical cutter profile in full. However, no study has yet proposed a design methodology for end-milling to compensate for machining errors.

To enhance the RP accuracy of the SR profile manufactured by the end milling cutter, Shen *et al.* [7] presented the compensated design method of the cutter profile based on the compensated rotor, based on the profile error and the theoretical rotor profile. In the same approach, Tao *et al.* [8] present the design of the cutter profile process, accounting for error measurements of the manufactured RP to enhance machining accuracy. By a differential approach, Hoang and Wu [9] used the sensitivity matrix and the Levenberg-Marquardt algorithm to modify the cutter profile for reducing the RP error. However, the methods mentioned above may render the cutter deformation non-continuous when errors are introduced into the nominal (theoretical) tool or the rotor.

To address the limitations of previous studies, this study proposed a method to reduce machining errors in milled SR by correcting the end-milling profile. A mathematical model for designing an end-milling cutter with compensated parameters is presented. The sensitivity matrix and Levenberg-Marquardt algorithm are used to obtain the compensated parameters based on the measurement error of the milled rotor profile. The numerical example illustrates the effectiveness of the proposed method.

2. Cutter profile generation method

In the theory of surface design [10], the SR surface is generated by sweeping the 2D profile along the z-axis and rotating it about the rotation axis. Therefore, the position vector of the SR surface $\mathbf{r}_s$, normal vector $\mathbf{N}_s$, and the unit normal vector $\mathbf{n}_s$, are described as follows:

$$\mathbf{r}_s(u,\phi)=\left[x_s(u,\phi),y_s(u,\phi),z_s(u,\phi),1\right]^T$$
$$=\begin{bmatrix}\cos\phi & -\sin\phi & 0 & 0\\ \sin\phi & \cos\phi & 0 & 0\\ 0 & 0 & s_p\phi & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}x_a(u)\\ y_a(u)\\ z_a(u)\\ 1\end{bmatrix} \tag{2}$$

$$\mathbf{N}_s(u,\phi)=\frac{\partial\left[x_s(u,\phi),y_s(u,\phi),z_s(u,\phi)\right]}{\partial u}$$
$$\times\frac{\partial\left[x_s(u,\phi),y_s(u,\phi),z_s(u,\phi)\right]}{\partial\phi} \tag{3}$$

$$\mathbf{n}_s(u,\phi)=\frac{\mathbf{N}_s(u,\phi)}{\left|\mathbf{N}_s(u,\phi)\right|} \tag{4}$$

where ϕ is the longitudinal parameter of the SR; s_p is the helix parameter of the SR surface, and it is calculated as $s_p=r_p\cot\beta$; r_p is the pitch radius of the SR; β is the helix angle of the SR.

In practical manufacturing, the SR of the vacuum pump is machined using a high-speed end-milling cutter that rotates about its axis to generate the cutting speed and translates along the screw rotor axis to machine the entire SR surface. At the same time, the rotor rotates about its axis, helping the cutter mill rotor as a whole. Therefore, the general coordinate system for defining the manufacturing SR process by an end milling cutter is illustrated in Figure 1, where in $S_s(O_s-x_sy_sz_s)$ and $S_c(O_c-x_cy_cz_c)$ are the coordinate systems of the SR surface and the cutter surface, respectively, $S_f(O_f-x_fy_fz_f)$ is the fixed coordinate system, $S_1(O_1-x_1y_1z_1)$ is the auxiliary coordinate system.

The position vector of the end milling cutter $\mathbf{r}_c$ and its unit normal vector $\mathbf{n}_c$ are described as follows:

$$\mathbf{r}_c(\varphi,\phi,u)=\left[x_c(\varphi,\phi,u),y_c(\varphi,\phi,u),z_c(\varphi,\phi,u),1\right]^T$$
$$=\mathbf{M}_{cs}(\varphi)\cdot\mathbf{r}_s(\phi,u) \tag{5}$$

$$\mathbf{n}_c(\varphi,\phi,u)=\mathbf{L}_{cs}(\varphi)\cdot\mathbf{n}_s(\phi,u) \tag{6}$$

$$\mathbf{M}_{cs}=\begin{bmatrix}1 & 0 & 0 & -\left(c_x+r_c\sin\beta\right)\\ 0 & 1 & 0 & -\left(c_y+r\right)\\ 0 & 0 & 1 & -\left(s_p\varphi+r_c\cos\beta+c_z\right)\\ 0 & 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}\cos\varphi & \sin\varphi & 0 & 0\\ -\sin\varphi & \cos\varphi & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{7}$$

Where M_{cs} is the transformation matrix from S_s to S_c, $\mathbf{L}_{cs}$ is a sub-matrix of the first upper-left 3×3

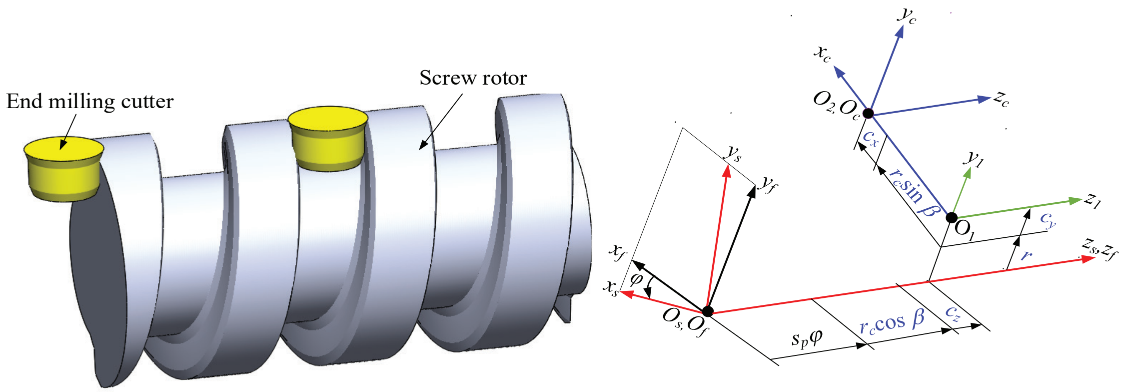

Figure 1. General coordinate system of the designing end milling profile.

elements of $\mathbf{M}_{cs}$, φ is the rotation angle of the rotor, r_c is the outer radius of the side cutting tool, r is the inner radius of the rotor profile, c_i $(i = x, y, z)$ is the compensated parameters along the x, y, and z axes, respectively. In the theoretical cutter profile, the compensated parameters are set equal to zero.

According to the enveloping method, the meshing condition can be written as an equation:

$$f_1(\varphi,\phi,u) = \mathbf{n}_c \cdot \mathbf{t}_c = \mathbf{n}_c \cdot \left(\mathbf{k} \times \frac{\partial\left[x_c(\varphi,\phi,u), y_c(\varphi,\phi,u), z_c(\varphi,\phi,u)\right]}{\partial\varphi} \right) = 0 \tag{8}$$

where $\mathbf{k} = [0,0,1]^T$ is the unit normal vector of the z-axis, symbol T denotes the matrix transposition.

To evaluate the instantaneous contact line between the end mill and the screw-rotor surface, set the rotor's rotation angle to zero, since the tool profile does not influence rotation about the z-axis. Next, obtain the longitudinal parameters of the screw surface ϕ by solving Eq. (8) after substituting the curve parameter u. Finally, substitute these results into Eqs. (5) and (6) to define the position vector and the corresponding unit normal at each contact point.

3. Cutter correction method for reducing the milled RP error

In theory, the SR can be milled without the RP error using the theoretical cutter profile. However, the actual RP may not match the theoretical profile due to machining tool errors, including those arising from humidity, temperature, and vibration. Therefore, this section presents a cutter-correction method to improve the machining accuracy of the milled SR using the end-milling cutter. The proposed method's calculation pipeline, shown in Figure 2, includes the following steps:

1. The theoretical cutter profile is generated from the RP by applying the design end milling cutter profile method presented in Section 2 with the compensated parameters of zero.

2. The RP error of the manufactured rotor δ_i is calculated by comparing the theoretical and actual rotor profiles.

3. The sensitivity matrix $\mathbf{M}_s$ is generated by collecting the RP error as each compensated parameter is incrementally changed by a small value.

4. The compensated parameters c_i are estimated by applying the Levenberg-Marquardt algorithm with the sensitivity matrix $\mathbf{M}_s$ and the profile error of the manufactured rotor in Step 2. The formula can be described as follows:

$$\{c_i\} = \left(\mathbf{M}_s^T \mathbf{M}_s\right) \mathbf{M}_s^T \cdot \{\delta_i\} \tag{9}$$

5. The corrected cutter profile is generated by applying the cutter design method presented in Section 2 with the compensated parameters solved in Step 4.

6. The RP error is calculated by comparing the theoretical RP and the simulated RP manufactured by the corrected cutter generated in Step 5.

7. The process is stopped when the maximum profile error $\delta_{\max}$ is smaller than the allowable error $|\delta|$ or the difference profile error between the two loops is smaller than the tolerance ε.

8. If the condition does not satisfy, the output of RP error at the loop i is set as the input of RP error at the loop i+1.

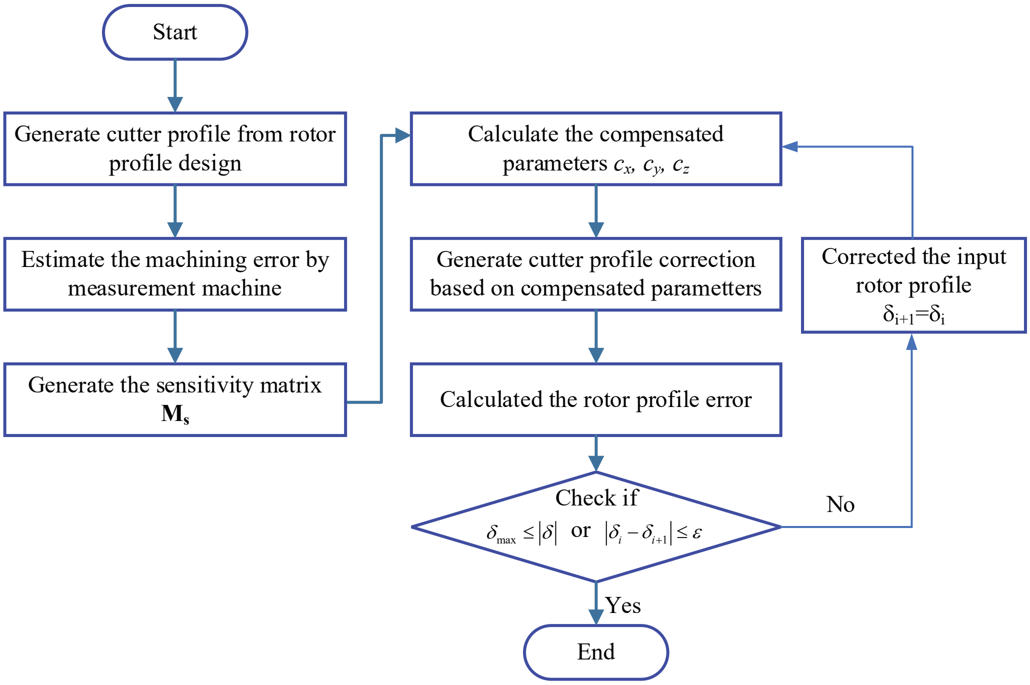

Figure 2. Diagram of the cutter correction process

4. Numerical Example

The RP used in this example is an industry-standard product from Hanbel Co., Ltd., the Kashiyama profile. The outer and inner radii of the RP are 64.87 mm and 40.98 mm, respectively. The center distance between the two rotors is 105.85 mm. The lead of the screw rotor is 72 mm. The cutter diameter in this example is 30 mm. The theoretical cutter profile is first generated based on the discrete RP in the following example, as shown by the dashed line in Figure 3. The profile error of the rotor manufactured by the theoretical cutter is calculated, as indicated by the dashed line in Figure 5.

A sensitivity matrix is first constructed through the profile error between the theoretical and simulated rotor profiles, obtained by perturbing each compensated parameter by 0.01 mm. The form of the sensitivity matrix is described as follows:

$$\begin{array}{cc} & \begin{array}{cccccc} 1 & 2 & 3 & \dots & 89 & 90 \end{array} \\ \mathbf{M}_S = & \left[\begin{array}{cccccc} 1.33 & 1.33 & 1.32 & \dots & 0.10 & 0.46 \\ -1.53 & -1.53 & -1.52 & \dots & -1.15 & -1.14 \\ -1.53 & -1.53 & -1.52 & \dots & -1.15 & -1.54 \end{array}\right] \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array} \qquad (10)$$

To apply the closed loop presented in Section 3, the compensated parameters are obtained as $c_x = 0.0072\ mm$, $c_y = -0.0247\ mm$, and $c_z = -0.0247\ mm$. The corrected cutter profile is generated using the compensated parameters, and the result is shown in Figure 3. Figure 4 illustrates the normal deviation between the theoretical and corrected cutter profiles. The minimum value is unchanged at the top and bottom of the cutter, and the maximum value is at the middle of the cutter.

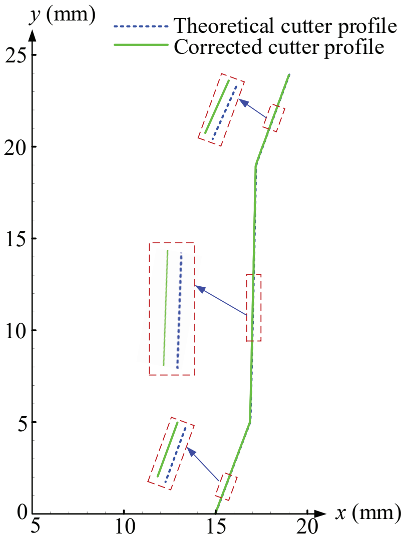

Figure 3. Comparing the theoretical and corrected end milling profile

The profile error of the RP manufactured by the corrected cutter profile is calculated and illustrated in

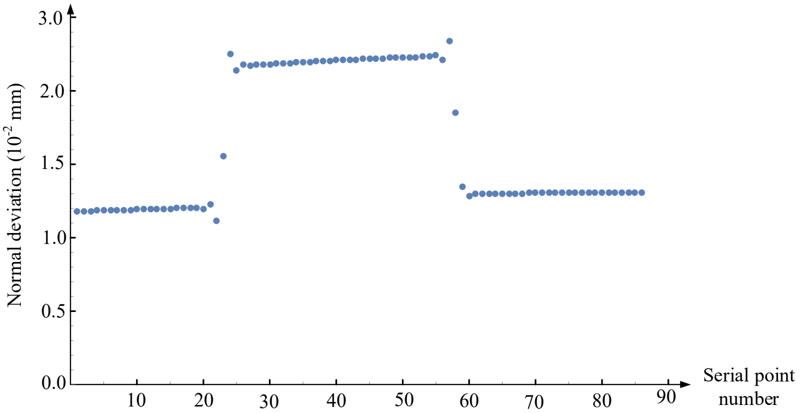

Figure 4. Comparison of the normal deviation between the theoretical and corrected end milling profile

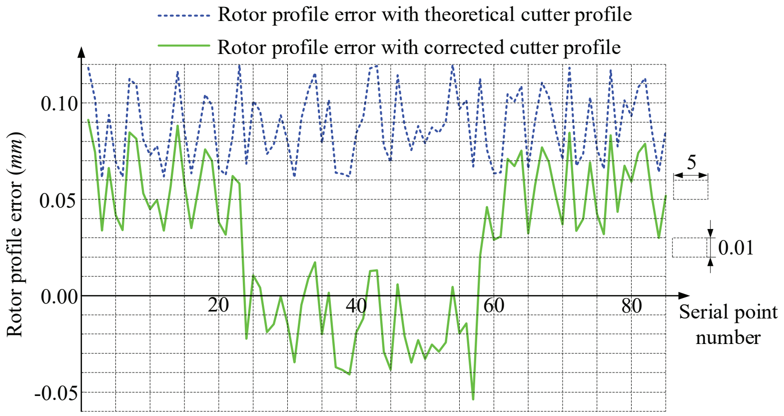

Figure 5. Machining error of rotor profile manufactured by cutter correction

Fig. 5. The mean absolute deviation of the RP reduces from 0.088 mm to 0.0415 mm. The maximum value reduces from 0.12 mm to 0.0912 mm. Besides, the minimum value reduces from 0.0613 mm to -0.054 mm. Therefore, the result shows that machining accuracy is enhanced by modifying the cutter profile using the compensating method.

5. Conclusion

This work proposes a cutter-correction framework to enhance RP accuracy in vacuum pumps by embedding error-compensated parameters directly into an end-milling cutter geometry model. Parameter–profile couplings are quantified through a sensitivity matrix that links minor variations in cutter design to resulting rotor deviations, enabling systematic compensation.

Using these sensitivities, a corrected cutter profile is obtained by solving an inverse identification problem with the Levenberg–Marquardt algorithm, which provides robust, stable convergence. The numerical example showed that the maximum profile error decreased by about 24%, and the mean absolute deviation of the RP decreased by about 52.84%.

Author Contributions

MTH: performed the algorithm design and code writing and was a major contributor to the manuscript. VDV: performed the algorithm design and code writing.

Conflict of Interest

The authors declare no conflict of interest.

Acknowledgments

This work was supported by the Ministry of Education and Training, Vietnam (Grant No. B2025-SKH-03).

References

[1] D. J. Hucknall, *Vacuum technology and applications*, Elsevier, 2013.

[2] M. H. Hablanian, *High-vacuum technology: a practical guide*, Routledge, 2017.

[3] M. T. Hoang and T. Van Tran, "Methodology for generating a general profile of asymmetrical rotors of twin-screw vacuum pumps with point meshing feature by a pre-defined sealing line," *Proc. Inst. Mech. Eng. B J. Eng. Manuf.*, pp. 1–14, 2022, doi: 10.1177/09544054221136383.

[4] Q. Tang, Y. Zhang, Z. Jiang, and D. Yan, "Design method for screw forming cutter based on tooth profile composed of discrete points," *Journal of Mechanical Design*, vol. 137, no. 8, 2015.

[5] V. G. Teodor, I. Popa, and N. Oancea, "The profiling of end mill and planing tools to generate helical surfaces known by sampled points," *The International Journal of Advanced Manufacturing Technology*, vol. 51, pp. 439–452, 2010.

[6] J. H. Kim, J. W. Park, and T. J. Ko, "End mill design and machining via cutting simulation," *Computer-Aided Design*, vol. 40, no. 3, pp. 324–333, 2008, doi: https://doi.org/10.1016/j.cad.2007.11.005.

[7] Z. Shen, B. Yao, B. Chen, W. Feng, and X. Zhang, "A novel rotor profile error tracing and compensation strategy for high precision machining of screw rotor based on trial cutting of limited samples," *Shock and vibration*, vol. 2015, no. 1, p. 978325, 2015.

[8] L. Tao, M. Yuan, and H. Fang, "A pre-compensation method for profile errors of screw rotors under precision form grinding," *The International Journal of Advanced Manufacturing Technology*, vol. 117, no. 11, pp. 3229–3239, 2021.

[9] M.-T. Hoang and Y.-R. Wu, "Error compensation method for milling single-threaded screw rotors with end mill tools," *Mech. Mach. Theory*, vol. 157, p. 104170, 2021.

[10] F. L. Litvin and A. Fuentes, *Gear geometry and applied theory*. Cambridge University Press, 2004.

PHƯƠNG PHÁP HIỆU CHỈNH DỤNG CỤ CẮT ĐỂ NÂNG CAO ĐỘ CHÍNH XÁC GIA CÔNG ROTOR TRỤC VÍT BẰNG DAO PHAY NGÓN

Tóm tắt:

Độ chính xác của biên dạng rotor là yếu tố quyết định đến hiệu suất của bơm. Tuy nhiên, biên dạng gia thực tế của rotor khó có thể đạt được như biên dạng lí thuyết do các sai số gia công như nhiệt độ, sai sót của người thực hiện, dao động... Do đó, phương pháp hiệu chỉnh dụng cắt được trình bày nhằm nâng cao độ chính xác gia công của biên dạng rotor thông qua việc bù sai số trong quá trình thiết kế dụng cụ cắt. Mô hình toán học cho quá trình thiết kế dao phay ngón có xét đến các yếu tố bù sai số được đưa ra. Các yếu tố ảnh hưởng của các thông số bù sai số đến sai số biên dạng rotor được phân tích và tổng hợp thành ma trận độ nhạy. Biên dạng hiệu chỉnh của dụng cụ cắt được tạo ra dựa vào thông số bù sai số được giải dựa vào ma trận độ nhạy và thuật toán Levenberg-Marquardt. Ví dụ số được sử dụng để chứng minh tính đúng đắn của phương pháp đề xuất trong việc giảm sai số của biên dạng rotor.

Từ khóa: *rotor trục vít, dao phay ngón, bù sai số, sai số biên dạng rotor.*