A Simulation Of Piston Engines Aircraft Based On Wiebe And Woschni Models

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ABSTRACT

This paper presents a combustion simulation for an aircraft piston engine. The aim of this research is to develop a combustion model using accessible and feasible computational tools, ensuring both high accuracy and practicality as a foundation for more advanced studies. The developed model is a zero-dimensional (0-D) and mean-value engine model (MVEM) for a single cylinder, fundamentally based on the first law of thermodynamics. It is meticulously programmed using Matlab, allowing for a detailed description of the intricate relationships among various internal engine characteristics, including cylinder performance parameters, heat release, heat loss, in-cylinder pressure, and an initial estimation of exhaust gas emissions were conducted for the engine operation. Specifically, the heat release process is modeled using the well-established Wiebe function, while heat loss is calculated using the Woschni model, which accounts for the instantaneous heat transfer coefficient. The Runge-Kutta algorithm is employed to solve the differential equations governing the engine's thermodynamic cycles. The research seeks to overcome the inherent limitations of traditional, costly, and time-consuming trial-and-error experimental approaches by leveraging advancements in computational speed and simulation capabilities.

KEYWORDS: Aircraft piston engine, Heat release; Heat loss, Emissions, Brake power

1. Introduction

The piston engine is still the main choice for small aircraft due to its low operating, maintenance costs and high efficiency at low to medium altitudes (below 12,000 ft.). Improving efficiency and reducing emissions are necessary requirements now. Building a simulation which represents the working process of a piston engine is one of the methods used to study real engines.

The objective of this research is to overcome the limitations of the traditional trial-and-error approach, which is costly and time-consuming, by leveraging advances in computational speed.

In this research, the 0-Dimensional model was applied, due to its role in simulating engine combustion by simplifying complex processes into average cylinder properties. It enables fast, efficient analysis of pressure, temperature, and heat release, supporting engine performance prediction and optimization, especially in early design stages.

There are many simulations which are based on a kinetic, dynamic, heat balancing, mass balance models etc. The mathematical differential equations have been used to simulate the working process of piston engines, these equations were especially solved by a programming language with the computer, the results with graphics could describe the characteristic of parameters varying the crank angle (CA). In this research, Wiebe and Woschni models have been used to simulate.

Wiebe function, named after a Russian engineer Ivan Wiebe, is the best-known technique for solving the burn rate and heat release inside the cylinder . The Wiebe function plays an essential role in modern technical applications by accurately modeling combustion in piston engines, including those used in aircrafts. It aids in performance analysis, fuel efficiency optimization, and reliable simulation of combustion dynamics in aviation engineering [1].

The Woschni model provides a widely used empirical formula to calculate the instantaneous heat transfer coefficient in internal combustion engines. Derived from similarity laws for convective heat transfer, it incorporates terms accounting for piston motion and combustion-induced turbulence. In piston aircraft engines, where thermal management is critical, the model enables more accurate simulation of in-cylinder heat transfer. This supports improved cooling design, engine efficiency, and reliability under varying flight conditions [2].

In the case study of the Rotax 912 engine, Grabowski et al. [3] employed the AVL Boost software to develop a one-dimensional (1-D) engine model primarily for performance calibration, including power, torque, and fuel-flow characteristics. The simulation results showed minor deviations from experimental data, within approximately ±3 kW in brake power and ±2 Nm in engine torque, indicating good agreement with manufacturer specifications. This 1-D approach provides a useful reference for other engine models.

G. Parker and G. Rizzoni, in their study [4], presents a dynamic simulation model of a general aviation piston engine, capturing individual cylinder behavior. It aids in developing advanced control, diagnostics, and emissions strategies.

In their research, Jaeheun Kim et al shows how spark timing and combustion duration affect piston dynamics and efficiency in a free piston engine using Wiebe function [5].

Fotis *et al* estimated heat transfers in internal combustion engines. It incorporates piston speed and gas pressure, making it highly applicable for simulating thermal behavior in piston aircraft engines under dynamic operating conditions [6].

The aim of this simulation is to build a working process simulation code for predicting what happens inside cylinders. The mean value model and 0-D dimensional model have been used in this simulation due to the simplicity and affordable results. Runge Kutta algorithm was used to solve the differential equations. The object of the study is to build a simulation model of the new engine, therefore, the initial parameters (intake pressure, intake temperature) were taken from engine documents and test records provided by the manufacturer. The key outputs of simulation (brake power, peak pressure, and specific fuel consumption) were compared with engine specifications, the gaps were not over 5% between them. The results of the simulation gave us the accurate characteristics of the internal cylinder.

This paper employs the Runge-Kutta algorithm for numerically solving the ordinary differential equations (ODEs) [7] that govern the piston engine's thermodynamic cycles. Specifically, Matlab's built-in ode23 solver would be a suitable choice for this task. ode23, a Runge-Kutta method of orders 2 and 3, is adept at handling initial value problems arising from the 0-D and mean-value engine model. It efficiently integrates the differential equations that describe internal characteristics such as pressure, temperature, heat release (via the Wiebe function), and heat loss (using the Woschni model), providing accurate and practical solutions for engine performance analysis.

2. Model structure

2.1. Engine performance parameters

Figure 1 shows the schematic structure of the piston engine model, illustrating the main subsystems such as the intake, compression, combustion, expansion, and exhaust processes. This diagram provides a general overview of how thermodynamic interactions are modeled and defines the boundary conditions used in the simulation.

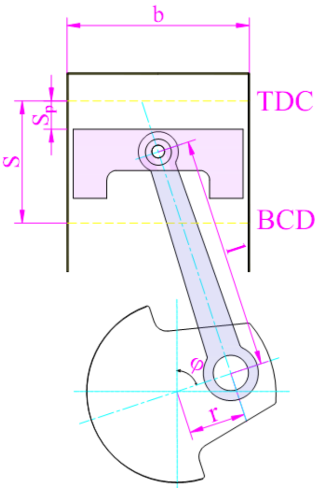

Figure 1: Engine performance

Assuming that crank angle (CA) φ=0, when the piston at top dead center (TDC), the dimensionless instantaneous stroke is [8]:

$$\bar{s}_p = \frac{s_p}{s} = \frac{1}{2}(1-\cos\varphi) + \frac{1}{2e}\left[1-\left(1-e^2\sin^2\varphi\right)^{1/2}\right] \quad (1)$$

where s is the distance between top dead centre (TDC) and bottom dead center (BDC); s_p is the instantaneous stroke, e is the ratio of the crankshaft radius r to the connecting rod length l, e=r/l.

The Instantaneous volume displacement Vd(φ) is:

$$V_d(\varphi)=\pi.b^2.s_p/4 \quad (2)$$

Dimensionless instantaneous cylinder volume:

$$\bar{V}_d = \frac{V_d(\varphi)}{V_a} = \frac{1}{\varepsilon}\left[1+\frac{\varepsilon-1}{2}(1\text{-}\cos\varphi)+\frac{1}{2e}[1\text{-}(1\text{-}e^2\sin^2\varphi)^{1/2}]\right] \quad (3)$$

where Va is cylinder volume when piston at BDC, ε is the compression ratio; e is the ratio of the crankshaft radius r to the connecting rod length l, e=r/l.

2.2. Heat release

Cumulative heat release versus crank angle based on Wiebe's formula [1]

$$x = 1-\exp\left[-a\left(\frac{\varphi-\varphi_0}{\Delta\varphi}\right)^{m+1}\right] \quad (4)$$

where φ is the crank angle, φ_0 is the start of combustion, Δφ is the total combustion duration, and a and m are adjustable parameters. Varying a and m changes the shape of the curve significantly [9].

The heat release rate is calculated as [10]:

$$\frac{\partial Q_{in}}{d\varphi} = Q_{in}\frac{dx}{d\varphi} \quad (5)$$

2.3. Heat loss and heat flux

The heat transfer through the surrounded wall of combustion chamber ∂Q_{loss} is calculated as [2]:

$$\frac{\partial Q_{loss}}{d\varphi} = h_g(\varphi)A_w(\varphi)(T\text{-}T_w)/n_e \quad (6)$$

where T(K) is the instantaneous bulk gas temperature; T_w(K) is the mean temperature of cylinder wall; A_w(m^2) is the area of combustion chamber surface, n_e(rev/min) is the engine speed; and h_g($W/m^2.K$) is the Woschni heat transfer coefficient which is calculated as follows:

$$h_g = 3{,}26p^{0.8}U^{0.8}b^{-0.2}T^{-0.55} \quad (7)$$

where b(m) is the cylinder bore and U(m/s) is the characteristic gas velocity within the cylinder as [2]:

$$U = 2.28\bar{v}_p + 0.00324\frac{V_d T_a}{p_a V_a}p \quad (8)$$

where T_a(K) and p_a(N/m^2) are the temperature and pressure at the end of the charging process, respectively. $\bar{v}_p$ (m/s) is mean piston speed, V_d(m^3) is the displacement volume, V_a(m^3) is cylinder volume when piston at BDC.

A key aspect of the Woschni model is its consideration of the effective gas velocity within the cylinder (U), which is influenced by piston motion, swirl, and particularly the turbulence generated by combustion and fuel injection. This allows it to quite effectively capture the complex heat transfer phenomena occurring in an operating engine [2].

Heat flux is the rate of heat energy transfer per unit area, typically across a surface. It describes how quickly thermal energy flows and is measured in watts per square meter (W/m²).

$$q^{\cdot} = \frac{q}{A} \tag{9}$$

where q is the heat transfer rate (W); A is the surface area (m²).

2.4. Emission Calculation

Calculating emissions from aircraft engines plays a crucial role in assessing their environmental impact and guiding regulatory compliance. Emissions such as NO_x, CO, HC, and particulates are formed during combustion and vary with engine parameters like equivalence ratio, temperature, and pressure. NO_x emissions can be estimated using the extended Zeldovich mechanism [11], which models the temperature-dependent chemical reactions of nitrogen and oxygen species. Following [10], the following expression for the rate of change of nitric oxide concentration, with the brackets denoting molar concentrations in units of mol/cm^3 as below

$$\frac{d}{dt}[NO] = k_1[O][N2] - k_{1r}[NO][N] + k_2[N][O2] - k_{2r}[NO][O] + k_3[N][OH] - k_{3r}[NO][H] \tag{10}$$

The rate constants have units of cm^3/(mol s), the additional subscript r on the rate constants denotes the reverse reaction rate constant, and the temperature T is in Kelvin.

$$\begin{aligned} k_1 &= 1.8 \times 10^{14}\exp(-38{,}370 / T); \\ k_{1r} &= 3.8 \times 10^{13}\exp(425 / T); \\ k_2 &= 1.8 \times 10^{10}\exp(4{,}680 / T); \\ k_{2r} &= 3.8 \times 10^{9}\exp(-20{,}820 / T) \\ k_3 &= 7.1 \times 10^{13}\exp(-450 / T); \\ k_{3r} &= 1.7 \times 10^{14}\exp(-24{,}560 / T) \end{aligned} \tag{11}$$

A high concentration of particulate matter (PM) is manifested as visible smoke or soot
in the exhaust gases. Consider a simple soot formation model with a two-stage reaction path for the
fuel-rich combustion of a hydrocarbon fuel [10]:

Stage 1: $C_xH_\gamma + cO_2 \rightarrow 2cCO + \sim H_2 + (a - 2c)C(s)$

Stage 2: $C(s) + O_2 \rightarrow CO_2$

Where C(s) represents solid carbon (soot). If oxygen is insufficient in Stage 1, soot forms. Stage 2 oxidizes soot if enough oxygen is present.

2.5. The First law of thermodynamics

The First Law of Thermodynamics, the principle of energy conservation, is fundamental to simulating aircraft piston engine combustion. By quantifying the energy balance, it allows for accurate modeling of the Wiebe and the Woschni model, directly impacting predictions of engine performance, efficiency, and specific fuel consumption. The compression, combustion and expansion processes include the processes of heat release, heat transfer as shown in the following heat balance differential equation [12]:

$$\frac{\partial Q_{in}}{d\varphi} - \frac{\partial Q_{loss}}{d\varphi} - p\frac{dV}{d\varphi} = mc_v\frac{dT}{d\varphi} + c_vT\frac{dm}{d\varphi} \tag{12}$$

where c_v is constant volume specific heat (J/kg.K).

From the equation of state pV=mRT, which in differential form is [12]*:*

$$mdT = \frac{1}{R}(pdV + Vdp) \tag{13}$$

To combine equation (13) with equation (14), the differential equation of cylinder pressure varying with the crank angle (CA) is written as follows [12]:

$$\frac{dp}{d\varphi} = -k\frac{p}{V}\frac{dV}{d\varphi} + \frac{k-1}{V}\frac{\partial Q_{in}}{d\varphi} - \frac{k-1}{V}\frac{\partial Q_{loss}}{d\varphi} \tag{14}$$

where k(-) is the gas constant (cp/cv).

3. Case study

The flow diagram used to compute the gas pressure that is built up in a working cylinder chamber at a working cycle to compute the power of the engine is shown in the **Error! Reference source not found.**. The particular submodel and respective model in Simulink is shown in the Table 1, The sample piston engine used in this study is a Rotax 912iS, installed on the Tecnam P2008 light aircraft.

The Tecnam P2008 is a modern light sport aircraft that has gained popularity in Vietnam, particularly for flight training and aviation career orientation. At Bay Việt Flight Training Center in Rạch Giá, the P2008 JC MkII is used to offer hands-on flying experiences for aspiring pilots, including pre-flight inspections, actual flight sessions, and post-flight briefings. This aircraft features a composite fuselage and metal wings, combining lightweight design with structural strength, making it ideal for training and recreational flying. With its Rotax engine, advanced avionics, and low operating costs, the P2008 is well-suited for Vietnam’s growing aviation education sector. Programs using the P2008 help participants explore the profession of commercial piloting and receive certification for familiarization flights.

Figure 2: Rotax 912iS engine on P2008 aircraft

Table 1: Engine parameters

Key parameters	Values
Type of engine	Horizontally opposed pistons
Turbocharger	Aspiration
Number of cylinders	4
Bore x stroke (mmxmm)	84x61
Crank radius x Connecting rod length, (mmxmm)	30.5 x130
Compression ratio (-)	10.5
Fuel consumption at 100% load (L/h)	27.6
Brake Power at 5800 rpm, kW	73.5
Weight (with gearbox), kg	56.6

Calculating flow diagram

This calculating flow diagram plays a vital role in simulation by visually outlining the sequence of calculations, data flow, and dependencies among variables and sub-models.

Figure 3 presents the updated simulation flow diagram, which outlines the computational procedure used to evalua te the engine's performance. Each block represents a sub-m odel airflow, combustion (Wiebe), heat transfer (Woschni), and emission estimation demonstrating the logical interconn ection between thermodynamic and chemical processes.

The structure parameters: bore cylinder, piston stroke, compression ratio… are taken from engine specifications (Table 1), which are fixed.

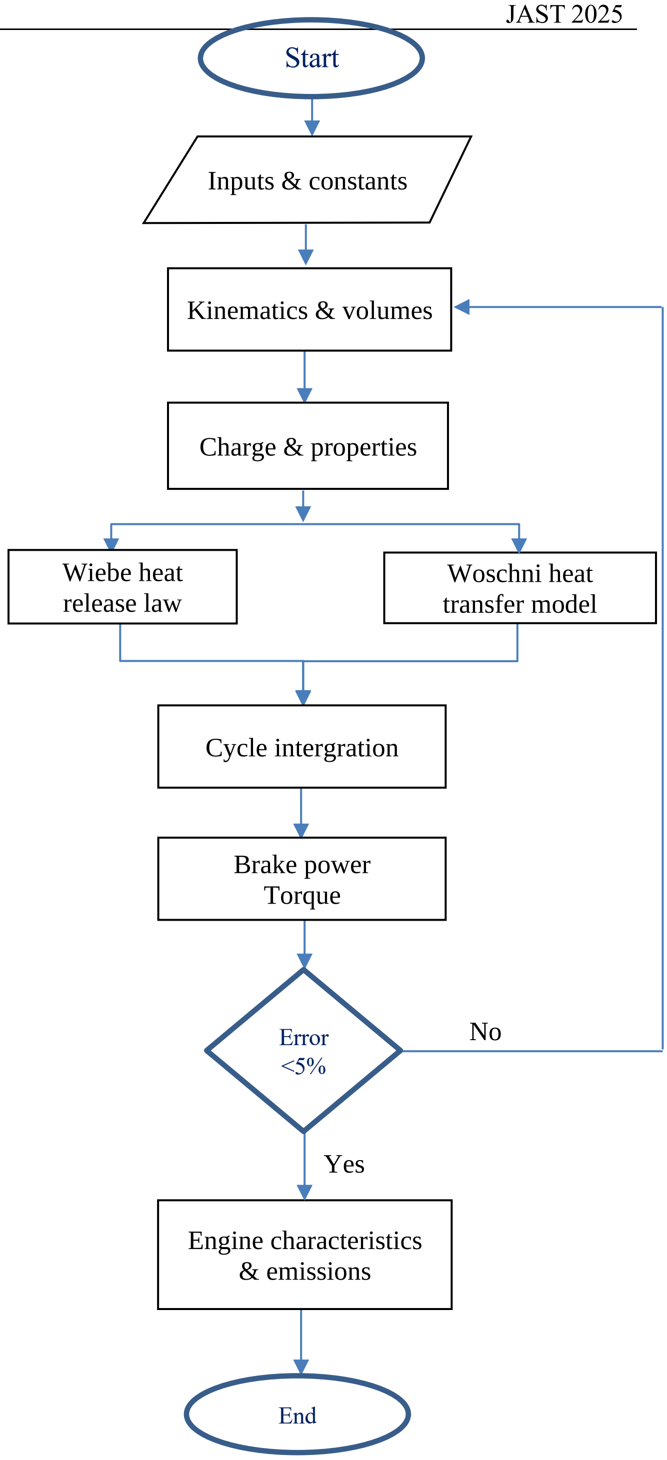

Figure 3: Flow diagram for simulation

The initial parameters: LHV represents the amount of heat released by the complete combustion of a unit of fuel, assuming that the water produced during combustion remains in a vapor state and does not condense into liquid, LHV = 44650 kJ/kg; engine speed can be taken as engine records (n=5800 rpm at nominal regime).

Boundary Conditions: The model accounts for the influence of altitude on air density, which is a critical environmental condition. Two parameters present: pressure and temperature, which calculated by ICAO Standard, Ta=297

K; Pa=90 kPa [13]; In this combustion process simulations, the wall temperature (piston crown, cylinder head, liner) is often used as a thermal boundary condition. This allows the model to account for heat transfer between the hot combustion gases and the solid walls, Tw=450K.

The value of the tuning parameters: the Wiebe's function parameters (a, n , φ_i) , which are tuned by the description of [10] firstly and step by step (automatic) was corrected via feed-back loop in the model.

4. The predictions and discussions

4.1. Variation of cylinder volume and piston velocity

Variation of cylinder volume and piston velocity were calculated as shown in
Figure **4** and
Figure **5**.

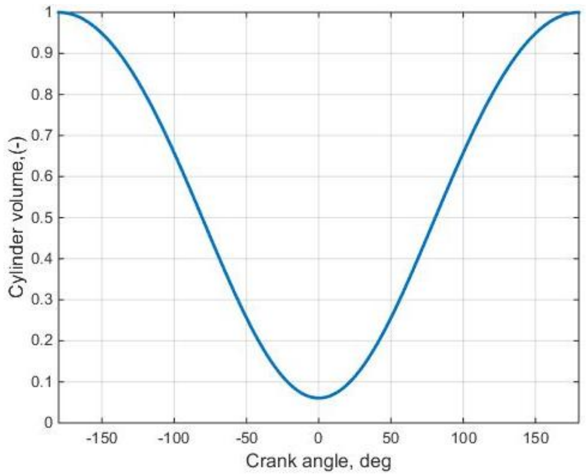

Figure 4: Cylinder volume varies with CA

The volume decreases smoothly during the compressio n stroke, reaches its minimum at top dead center (TDC), and t hen increases symmetrically during the expansion stroke, conf irming the correct geometric relationship between piston moti on and crank rotation.

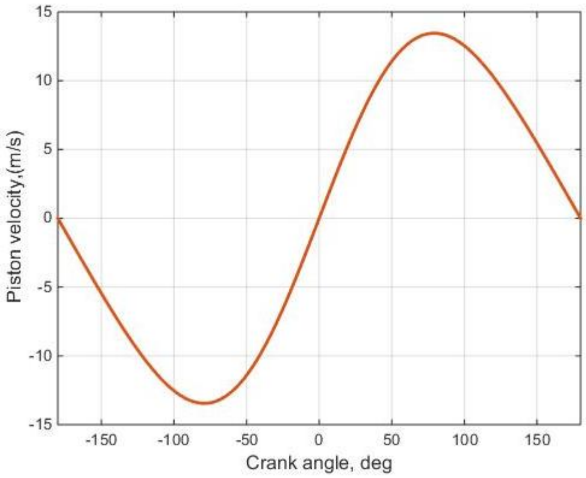

Figure 5: Piston velocity (m/s) varies with CA (deg)

The piston velocity is negative at compression stroke ($-180^0 \div 0^0$ crank angle), equals zero at TDC (0^0CA), positive during the expansion stroke ($0^0 \div 180^0$ CA)

4.2. Variation of heat flux and temperatures

The heat flux was calculated as shown in Figure 6.

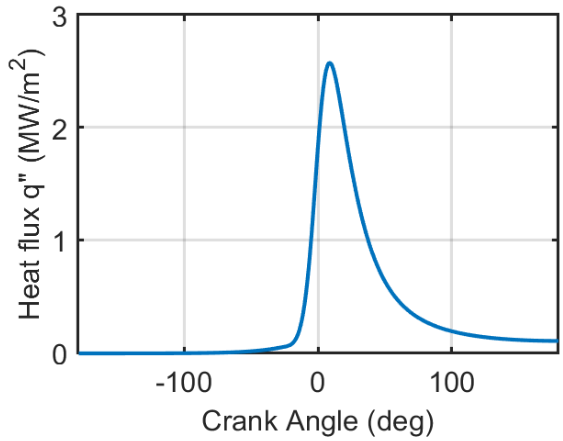

Figure 6: Heat flux with crank angle (deg)

Figure 6 shows that the peak heat flux reaches approximately 2.8 MW/m² near the TDC, which is higher with the typical range of 2–3 MW/m² reported for gasoline spark-ignition engines [10]. This result is suitable for the Rotax 912iS engine with very high speed (5800 rpm).

The simulated profile agrees well with standard experimental data, indicating that the Woschni heat-transfer model accurately represents the transient heat exchange between the gas and cylinder wall.

The temperature of mixed gas is presented as Figure 7.

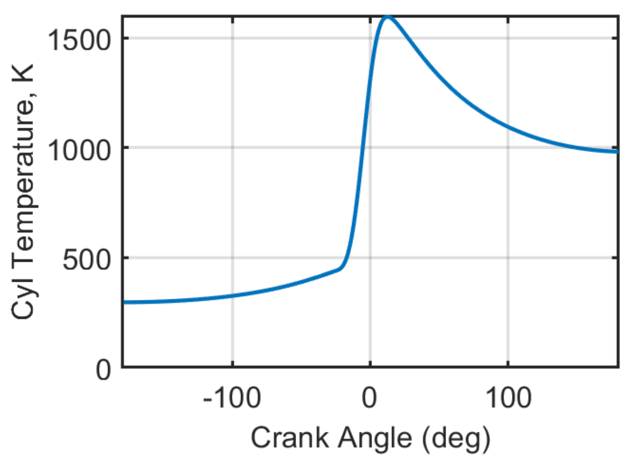

Figure 7: Mixed gas temperature with CA (deg)

Figure 7 the mixed gas temperature rises rapidly during the combustion phase, reaching about 1600 K near the top dead center (TDC) due to the combustion process. This peak temperature is within the typical range for gasoline spark-ignition engines [10]. These trends confirm that the Wiebe combustion and Woschni heat-transfer models accurately represent the thermodynamic behavior of the engine cycle.

4.3. Variation of heat release and pressure

In this simulation, the heat release rate (HRR) is expressed in kilowatts (kW), representing the instantaneous rate of energy release during the combustion process.

The 3D heat release diagram illustrates that combustion begins slightly before top dead center (TDC) and reaches its peak around 5–10° after TDC, corresponding to the main combustion phase. The maximum heat release rate of approximately 0.04 kW indicates rapid energy conversion and efficient flame propagation.

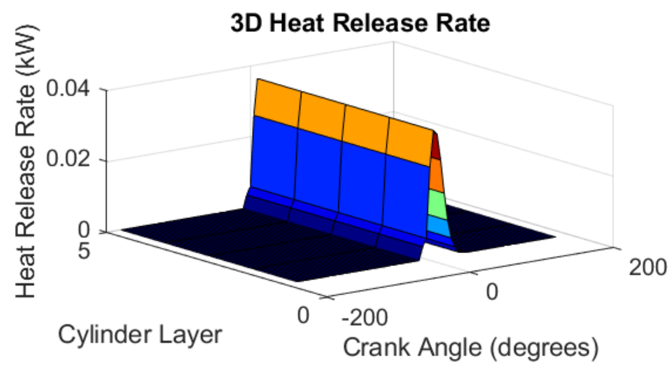

Figure 8: Heat release rate with crank angle

The in-cylinder pressure curve (Figure 9) closely follows the heat release rate profile, where the rapid increase in heat release near top dead center (TDC) causes a sharp rise in pressure, demonstrating the direct thermodynamic coupling between combustion energy release and pressure buildup inside the cylinder.

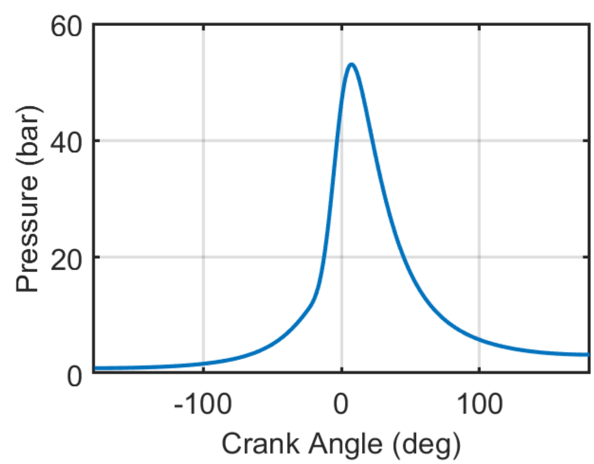

Figure 9: The pressure with crank angle

The in-cylinder pressure curve shows a sharp peak of about 53 bar occurring near the TDC. This profile indicates proper ignition timing and efficient combustion, where most of the heat release occurs shortly after TDC, ensuring effective conversion of thermal energy into mechanical work

4.4. Emission calculation

Emissions were calculated as shown in Figure 10 and Figure 11.

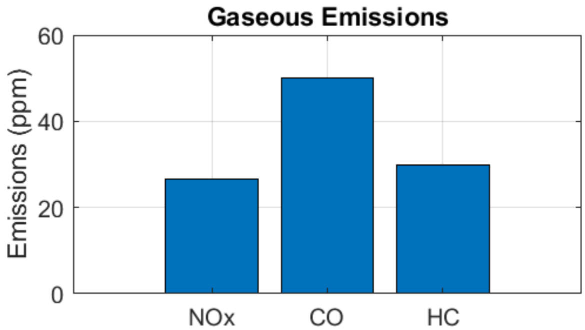

Figure 10: Concentration of a gas NOx, CO and HC

The simulation results indicate low levels of gaseous emissions, with $NO_x \approx 25$ ppm, CO ≈ 50 ppm, and HC ≈ 30 ppm. These values reflect efficient and stable combustion, achieved through effective control of temperature and fuel–air mixture strength. The results validate that the developed model reliably captures the low-emission characteristics of a modern spark-ignition aircraft piston engine.

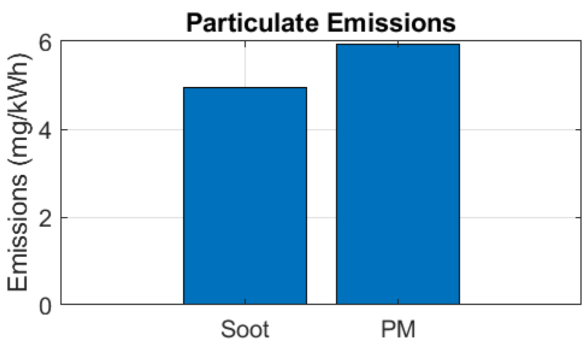

Figure 11: Soot and PM

The graph illustrates that particulate matter (PM) emissions are notably higher than soot emissions, with PM reaching approximately 5.8 mg/kWh compared to 4.5 mg/kWh for soot. These values indicate minimal particulate formation. This outcome is consistent with the expected behavior of a well-optimized gasoline spark-ignition engine operating under clean combustion conditions.

4.5. The cumulative heat loss and cumulative work

Cumulative heat loss represents the total thermal energy transferred from the working gases to engine components (e.g., cylinder walls, piston, head) during a cycle.

Cumulative work refers to the total mechanical work produced by the gas within the cylinder over an engine cycle.

The cumulative work and heat loss are plotted in Figure 12. The cumulative work is initially negative due to the piston moving up (compression stroke), and becomes positive in expansion stroke.

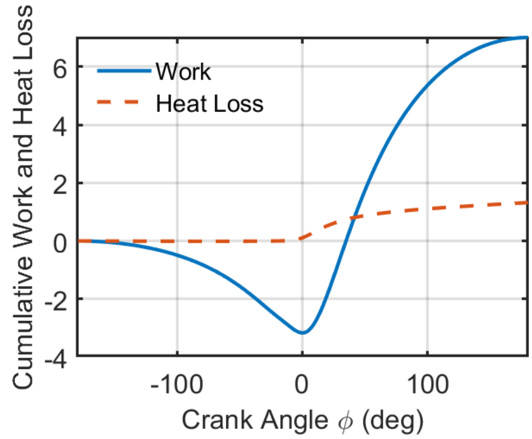

Figure 12: Cumulative Work and Heat loss (kJ)

The present simulation results (cumulative work ≈ 6. 8 kJ and heat loss ≈ 1.8 kJ), with heat loss accounting for approximately 21% of the total fuel energy, fall well within the reported theoretical and experimental ranges. These resu lts were obtained from the newly developed engine simulati on model, confirming that the proposed model accurately re presents the energy balance of the thermodynamic cycle. Fu rthermore, the Rotax 912iS simulation exhibits realistic and reliable behavior consistent with standard spark-ignition eng ine performance.

Key engine output parameters of the model and their comparison with test records are presented in

Table **2**

Table 2: Comparing output parameters of simulation and test records.

Key parameters	Test records	Simulation	Error (%)
Speed at the Nominal Power, rpm	5800	5800	0
Max. combustion pressure, bar	55	53	3.6
Brake power, kW	73.5	77.2	2.5
Specific fuel consumption, g/kWh	264	254	4.9

Output parameters: peak combustion pressure (p_z), brake power (P_w), specific fuel consumption (g_e). The output parameters were taken from the simulation and standard values (from test records). These errors in

Table **2** are not over 5%, therefore the modelling of the engine can be accepted and using it for presentation of internal characteristics inside the cylinder as in the following part.

5. Conclusion

The research has gained some results:

A Matlab model of a piston engine was developed that can describe their characteristics and relations among factorials (heat release, heat loss, work). The inputs of the model are the same with engine technical documents, the outputs are compared with test records (provided by manufacturer) to evaluate its accuracy.

Based on the model, the predictions of internal characteristics were presented: Cumulative energy release curve, rate of energy release curve, temperature, heat flux, pressure, cumulative work, heat transfer, and emissions. The engine power output, efficiency, fuel specific consumption can also be displayed.

This paper focuses on developing an individual cylinder model, where interactions between cylinders are not yet considered. However, this model is helpful for training courses of this engine and the simulation can be used for studying other kinds of engine based on their documents.

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