FIXED POINT THEOREMS FOR *A*-CONTRACTIVE MAPPINGS IN BOUNDEDLY COMPACT AND *T*-ORBITALLY COMPACT METRIC SPACES

Nguyen Huu Hoc
Faculty of Natural Sciences, Hong Duc University, Thanh Hoa, Vietnam

ARTICLE INFORMATION

Journal: *Vinh University Journal of Science Natural Science, Engineering and Technology*
p-ISSN*: 3030-4563*
e-ISSN: *3030-4180*

Volume: 54
Issue: 1A
****Correspondence:***
huuhocmth@gmail.com

Received: *22 October 2024*
Accepted: *27 December 2024*
Published: *20 March 2025*

Citation:
Nguyen Huu Hoc (2025). Fixed point theorems for A-contractive mappings in boundedly compact and T-orbitally compact metric spaces. ***Vinh Uni. J. Sci.*** *Vol. 54 (1A), pp. 23-32*
doi: 10.56824/vujs.2024a103a

ABSTRACT

In this paper we prove some fixed point theorems for map-pings satisfying an implicit contractive conditions in metric spaces. Our re-sults improve and extend some results in the literature [6]. Some examplesare also given to illustrate our findings.

Keywords: Boundedly compact spaces; Fixed points; Metric spaces; *T*-orbitally compact spaces.

1. Introduction

Fixed point theory plays a fundamental role in nonlinear analysis and its applications. One of its most important results is the Banach contraction principle, which states that if T is a contraction on a complete metric space (X, d), i.e., there exists $k \in [0, 1)$ such that

$$d\,(Tx, Ty) \le kd(x, y)\ \forall x, y\ \in X \qquad (1.1)$$

Then T has a unique fixed point in X. The Banach contraction principle has been extended, generalized, and improved in various ways.

In 1962, Edelstein ([3]) proved the following fixed point theorem.

Theorem 1.1 ([3]). *Let (X, d) be a compact metric space and $T : X \to X$ be selfmapping. If*

$$d\,(Tx, Ty) < d(x, y)\ \ \forall x, y\ \in X, x\ \neq y \qquad (1.2)$$

then T has a unique fixed point.

In Theorem 1.1, the compactness of the metric space (X, d) is essential and it cannot be replace

d by the completeness. Note that both (1.1) and (1.2) imply the continuity of the mapping T. An open question for researchers was whether there exists any contractive-type condition that does not require the mapping to be continuous and still ensures the existence of a fixed point. In 1968, Kannan ([8]) was the first to answer this question and presented the following fixed point result.

Theorem 1.2 ([8]). *Let (X, d) be a complete metric space and T be a self-mapping on X satisfying*

$$d(Tx, Ty) \leq \beta\{d(x, Tx) + d(y, Ty)\} \tag{1.3}$$

for all $x, y \in X$ and $\beta \in \left[0, \frac{1}{2}\right)$. Then, T has a unique fixed point $z \in X$ and for any $x \in X$, the sequence of iterates $(T^n x)$ converges to z.

Note that a mapping T satisfies (1.3) is not neccesarily continuous. Moreover, if a mapping T satisfies (1.3) with $\beta = \dfrac{1}{2}$ in a complete metric space then T may not have a fixed point in X. However, if we further assume that the map T is continuous and the metric space is compact, then the existence of a unique fixed point is guaranteed, as stated in the theorem below.

Theorem 1.3 ([7, 9]). *Let (X, d) be a compact metric space and let $T : X \to X$ be a continuous mapping that satisfies the condition*

$$d(Tx, Ty) < \frac{1}{2}[d(x, Tx) + d(y, Ty)]$$

for all $x, y \in X$ with $x \neq y$. Then T has a unique fixed point in X.

In [6], Garai et al. proved the existence of fixed points in metric spaces where the compactness of the space and the continuity of the mapping are also relaxed.

In 2020, Garai et al. introduced the concepts of $\mathcal{A}$-contractive mappings and proved several related results for these mappings in compact and complete metric spaces. They also showed that these mappings are not necessarily continuous at their fixed points ([5]). We now recall the notions of $\mathcal{A}$-contractive introduced by Garai et al. in [5].

Denote by $\mathcal{A}$ the collection of all mappings $f : \mathbb{R}_+^3 \mapsto \mathbb{R}_+$which satisfy the following conditions:

($\mathcal{A}_1$) f is continuous.

($\mathcal{A}_2$) If $v > 0$ and $u < f(u, v, v)$ or $u < f(v, u, v)$ or $u < f(v, v, u)$, then $u < v$.

($\mathcal{A}_3$) $f(u, v, w) \leq u + v + w$, for all $u, v, w \in \mathbb{R}_+$.

Definition 1.4 ([5]). *Let (X, d) be a metric space and T be a self-mapping of X. Then the mapping T is said to be an $\mathcal{A}$-contractive mapping if there exists an $f \in \mathcal{A}$ such that*

$$d(Tx, Ty) < f\left[(d(x, y), d(x, Tx), d(y, Ty)\right]$$

for all $x, y \in X$ with $x \neq y$.

In [5], Garai et al. proved fixed point existence results for maps satisfying the $\mathcal{A}$-contractive condition in compact metric spaces. We restate the theorem below.

Theorem 1.5 ([5]). *Let (X, d) be a compact metric space and $T : X \to X$ be an $\mathcal{A}$-contractive mapping such that T is orbitally continuous. Then we have the following:*

(1) *T has a unique fixed point.*

(2) *Moreover, if the mapping $f \in \mathcal{A}$, arising in the $\mathcal{A}$-contractiveness of T, satisfies the condition that, if $u > f(u, 0, 0)$ for all $u > 0$, then the sequence $(T^n x)$ of iterates converges to that fixed point for each $x \in X$.*

(3) *Further, if $f(0, 0, u) = 0$ implies $u = 0$, then T is continuous at the fixed point z if and only if $\lim_{x\to z} m(z, x) = 0$, where*

$$m(z, x) = f(d(z, x), d(z, Tz), d(x, Tx)).$$

In this paper, we modify the class $\mathcal{A}$ and use it to obtain new results on the existence of fixed points in boundedly compact and orbitally compact metric spaces which are weaker than compactness. For our aim, we recall some related definitions.

Next, we recall some definitions that will be used in this article.

Definition 1.6 ([4]). *A metric space (X, d) is said to be boundedly compact if every bounded sequence in X has a convergent subsequence.*

It is easy to see that every compact metric space is boundedly compact, but the reverse is not true. A boundedly compact metric space need not to be compact. For example, the set of real numbers $\mathbb{R}$, $X = [0, \infty)$ with the usual metric is not compact but boundedly compact.

Definition 1.7 ([2]). *Let (X, d) be a metric space and T be a self-mapping on X. The orbit of T at $x \in X$ is defined as*

$$O_x(T) = \{x, Tx, T^2x, T^3x, \ldots\}$$

Definition 1.8 ([6]). *Let (X, d) be a metric space and T be a self-mapping on X. Then, X is said to be T-orbitally compact if every sequence in $O_x(T)$ has a convergent subsequence for all $x \in X$.*

From the above definitions, we observe that every compact metric space is T-orbitally compact for any self-mapping T, but the converse does not hold. Moreover, boundedly compactness and T-orbitally compactness are totally independent. Furthermore, T-orbitally compactness of X does not imply that X is a complete metric space. Garai et al. ([5]) illustrates these observations with the following examples.

Example 1.9 ([5]). *Let $X = [0, \infty)$ be a metric space with respect to the usual metric on $\mathbb{R}$. We define two self-mappings T and S on X by*

$$Tx = \frac{x}{n+1}, \text{ if } n-1 \le x < n$$

and

$$Sx = 2x$$

for all $x \in X$ and $n \in \mathbb{N}^*$. Then clearly, X is T-orbitally compact but not S-orbitally compact.

Example 1.10 ([5]). *Let $X = [0, 1)$ be endowed with the usual metric. Define*

$$T : X \to X \text{ by } Tx = \frac{x}{2}.$$

Then it is easy to see that X is T-orbitally compact but it is not complete.

Example 1.11 ([5]). *Let (X, d) be a usual metric space with $X = [0, \infty)$. We define*

$$T : X \to X \text{ by } Tx = 2x.$$

Then, it is trivial to check that X is boundedly compact but not T-orbitally compact.

Definition 1.12 ([1]). *Let (X, d) be a metric space and $T : X \to X$ be a self mapping. T is said to be orbitally continuous at a point z in X if for any sequence $\{x_n\} \subseteq O_x(T)$ for some $x \in X, x_n \to z$ as $n \to \infty$ implies $Tx_n \to Tz$ as $n \to \infty$.*

2 Main results

Now, we are in a position to state our main results.

Theorem 2.1. *Let (X, d) be a boundedly compact metric space and $T : X \to X$ be a orbitally continuous mapping. Assume that there exists a function $f : \mathbb{R}_+^3 \longrightarrow \mathbb{R}_+$ satisfying*

(i) *if $t_1 > 0$ and $t_1 < f(t_2, t_2, t_1)$, then $t_1 < t_2$,*

(ii) *there are $\alpha, \beta, \gamma \geq 0, \alpha < 1$ such that: If $t_1, t_2, t_3 \in \mathbb{R}_+$ then $f(t_1, t_2, t_3) \leq \alpha t_1 + \beta t_2 + \gamma t_3$;*

such that:

$$d(Tx, Ty) < f\left[d(x, y), d(x, Tx), d(y, Ty)\right] \tag{2.1}$$

for all $x, y \in X$ with $x \neq y$. Then, T has a unique fixed point z and for any $x \in X$, the sequence of iterates $(T^n x)$ converges to z.

Proof. Let $x_0 \in X$ be arbitrary and consider the iterated sequence (x_n), where $x_n = T^n x_0$ for each $n \in \mathbb{N}$. We denote $s_n = d(x_n, x_{n+1})$ for all natural numbers $n \geq 0$. If the sequence (x_n) has two equal consecutive terms, that is, $x_n = x_{n+1}$ for some natural number n, then $s_n = s_{n+1} = \cdots = 0$ and so (s_n) converges to 0. Without loss of generality, we may assume that no two consecutive terms of (x_n) are equal. Then we have

$$\begin{aligned} s_n = d\left(x_n, x_{n+1}\right) &= d\left(Tx_{n-1}, Tx_n\right) \\ &< f\left[d(x_{n-1}, x_n), d(x_{n-1}, Tx_{n-1}), d(x_n, Tx_n)\right] \\ &= f\left(s_{n-1}, s_{n-1}, s_n\right). \end{aligned}$$

Therefore, by condition (i), we have

$$s_n < s_{n-1}.$$

This shows that (s_n) is a strictly decreasing sequence of positive real numbers. Hence, it converges to some $b \geq 0$. For each $n \in \mathbb{N}$, we have

$$s_n < s_{n-1} < \ldots < s_1 = K \text{ (say)}.$$

Thus, for all n, m, one has

$$d(x_m, x_n) = d(Tx_{m-1}, Tx_{n-1}) < f\left[d(x_{m-1}, x_{n-1}), d(x_{m-1}, Tx_{m-1}), d(x_{n-1}, Tx_{n-1})\right].$$

By condition (ii), there are $\alpha, \beta, \gamma \geq 0, \alpha < 1$ such that

$$\begin{aligned} d(x_m, x_n) &< \alpha d(x_{m-1}, x_{n-1}) + \beta d(x_{m-1}, x_m) + \gamma d(x_{n-1}, x_n) \\ &\leq \alpha\left[d(x_{m-1}, x_m) + d(x_m, x_n) + d(x_{n-1}, x_n)\right] + \beta d(x_{m-1}, x_m) + \gamma d(x_{n-1}, x_n) \\ &= \alpha d(x_m, x_n) + (\alpha + \beta)s_{m-1} + (\alpha + \gamma)s_{n-1}. \end{aligned}$$

So, we have

$$\begin{aligned} (1-\alpha)\, d(x_m, x_n) &< (\alpha + \beta)s_{m-1} + (\alpha + \gamma)s_{n-1} \\ &< (\alpha + \beta)K + (\alpha + \gamma)K \\ &< (2\alpha + \beta + \gamma)K. \end{aligned} \tag{2.2}$$

This implies that

$$d(x_m, x_n) < \frac{2\alpha + \beta + \gamma}{1 - \alpha}K.$$

Therefore, (x_n) is a bounded sequence in X. By the bounded compactness property of X, (x_n) must have a convergent subsequence, say (x_{n_k}), which converges to some $z \in X$. By the orbital continuity of T, (Tx_{n_k}) converges to Tz. We have

$$\begin{aligned} s_{n_k} &= d(x_{n_k}, Tx_{n_k}) \\ s_{n_k+1} &= d(Tx_{n_k}, T^2x_{n_k}). \end{aligned}$$

Taking the limit as $k \to \infty$, we obtain

$$b = d(z, Tz) = d(Tz, T^2z).$$

We are going to show that $b = 0$. Assume that $b > 0$. Then, $z \neq Tz$, by (2.1) we have

$$d(Tz, T^2z) < f\left[d(z, Tz), d(z, Tz), d(Tz, T^2z)\right].$$

Using condition (i), one has $d(Tz, T^2z) < d(z, Tz)$, which is a contracdition. Hence, we must have $b = 0$ i.e., $d(z, Tz) = 0$, and z is a fixed point of T.

We next prove the uniqueness of the fixed point. Let z' be another fixed point of T and $z \neq z'$. Then, $d(z, z') > 0$, and we have

$$\begin{aligned} d(z, z') = d(Tz, Tz') &< f\left[d(z, z'), d(z, Tz), d(z', Tz')\right] \\ &= f\left[d(z, z'), 0, 0\right]. \end{aligned} \tag{2.3}$$

By condition (ii), we get

$$f\left[d(z, z'), 0, 0\right] \leq \alpha d(z, z') < d(z, z'),$$

this contradicts (2.3). Hence, we have $z = z'$.

Again, in (2.2) letting $n, m \to \infty$, we obtain

$$d(x_n, x_m) \to 0 \quad \text{as } m, n \to \infty.$$

Thus, (x_n) is a Cauchy sequence. As the subsequence (x_{n_k}) of (x_n) converges to z, the limit of (x_n) must be z. That is, the sequence $(T^n x)$ of iterates converges to that fixed point z for each $x \in X$. □

Example 2.2. *Let $X = [1, \infty)$ with the usual metric $d(x, y) = |x - y|$ for all $x, y \in X$. Then, (X, d) is a boundedly compact metric space. We consider the mapping $T : X \to X$ defined by*

$$Tx = \begin{cases} \frac{1}{2}x + 2 & \text{if } 1 \leq x < 2, \\ 2 & \text{if } x \geq 2. \end{cases}$$

Let $f : \mathbb{R}_+^3 \to \mathbb{R}_+$ be defined by $f(t_1, t_2, t_3) = \max\{t_2, t_3\}$ for all $t_1, t_2, t_3 \in \mathbb{R}_+$. Let $x, y \in X$ be arbitrary with $x \neq y$. We have following cases:

Case I: Let $1 \leq x < 2$, $1 \leq y < 2$. Then, $|x - y| < 1$. We have $Tx = \frac{1}{2}x + 2$ and $Ty = \frac{1}{2}y + 2$. Therefore,

$$|Tx - Ty| = \frac{1}{2}|x - y| < \frac{1}{2},$$

and

$$|x - Tx| = \left|\frac{1}{2}x - 2\right| = 2 - \frac{1}{2}x > 1.$$

Thus $d(Tx, Ty) < \max\{d(x, Tx), d(y, Ty)\}$, *i.e.,*

$$d(Tx, Ty) < f(d(x, y), d(x, Tx), d(y, Ty)).$$

Case II: Let $x, y \geq 2$. *Then,* $|Tx - Ty| = 0$ *and*

$$d(x, Tx) = |x - Tx| = |x - 2|,$$

$$d(y, Ty) = |y - Ty| = |y - 2|.$$

Since $x \neq y$, *it follows that* $\max\{d(x, Tx), d(y, Ty)\} > 0$. *Therefore,*

$$d(Tx, Ty) < f\left[d(x, y), d(x, Tx), d(y, Ty)\right].$$

Case III: Let $1 \leq x < 2, y \geq 2$. *Then,* $Tx = \frac{1}{2}x + 2$ *and* $Ty = 2$. *Therefore,*

$$d(Tx, Ty) = \frac{1}{2}|x| = \frac{1}{2}x.$$

Also, $d(x, Tx) = \left|\frac{1}{2}x - 2\right| = 2 - \frac{1}{2}x > \frac{1}{2}x$ *for all* $x < 2$, *i.e.,* $d(Tx, Ty) < d(x, Tx)$. *So we have*

$$d(Tx, Ty) < f(d(x, y), d(x, Tx), d(y, Ty)).$$

Therefore, T *satisfies condition* (2.1). *It is easy to verify that* f *satisfies conditions (i), (ii) in Theorem 2.1 and* T *is orbitally continuous. By Theorem 2.1,* T *has a unique fixed point. Indeed, it is easy to see that* 2 *is the unique fixed point of* T *and one can easily verify that* T *is not continuous at the fixed point* 2.

In the case where $\alpha = 1$, the conditions of Theorem 2.1 do not guarantee the existence of a fixed point. Indeed, consider the following example:

Example 2.3. *Let* $X = [1, \infty)$ *with the usual metric* $d(x, y) = |x - y|$ *for all* $x, y \in X$. *Then,* (X, d) *is a boundedly compact metric space. We consider the mapping* $T : X \to X$ *defined by*

$$Tx = x + \frac{1}{2x}.$$

Since T *is continuous on* X, *it follows that* T *is orbitally continuous.*

Consider $f : \mathbb{R}_+^3 \longrightarrow \mathbb{R}_+$ *defined by* $f(t_1, t_2, t_3) = t_1$ *for all* $t_1, t_2, t_3 \in \mathbb{R}_+$. *It is straightforward to verify that* f *satisfies condition (i) but does not satisfy condition (ii) in Theorem 2.1.*

For all $x, y \in X$, $x \neq y$, *we have*

$$|Tx - Ty| = \left|x + \frac{1}{2x} - y - \frac{1}{2y}\right| = |x - y|\left|1 - \frac{1}{2xy}\right| < |x - y|.$$

It follows that

$$d(Tx, Ty) < f\left[d(x, y), d(x, Tx), d(y, Ty)\right].$$

Thus, condition (2.1) *and condition (i) in Theorem 2.1 are satisfied; however, we observe that* T *does not have any fixed points.*

This example demonstrates that the Edelstein-type contraction condition does not guarantee the existence of fixed points in boundedly compact metric spaces.

Theorem 2.4. *Let (X, d) be a T-orbitally compact metric space, where $T : X \to X$ is orbitally continuous mapping. Let $f : \mathbb{R}_+^3 \longrightarrow \mathbb{R}_+$ be a function satisfying* (i) *and* (ii) *in Theorem 2.1 such that:*

$$d(Tx, Ty) < f\left[d(x, y), d(x, Tx), d(y, Ty)\right] \tag{2.4}$$

for all $x, y \in X$ with $x \neq y$. Then, T has a unique fixed point z and for any $x \in X$, the sequence of iterates $(T^n x)$ converges to z.

Proof. Let $x_0 \in X$ be arbitrary but fixed and consider the sequence (x_n), where $x_n = T^n x_0$ for each $n \in \mathbb{N}$. Since X is T-orbitally compact, the sequence (x_n) has a convergent subsequence, say (x_{n_k}), and let (x_{n_k}) converging to z in X. By the orbital continuity of T, (Tx_{n_k}) converges to Tz.

Now, proceeding as in Theorem 2.1, we can prove that the sequence $(d(x_n, x_{n+1}))$ converges to 0 and that (x_n) is a Cauchy sequence and hence $x_n \to z \in X$ as $n \to \infty$. Therefore, z is the unique fixed point of T. □

Example 2.5. *Let $X = (0, 1] \cup \{-1, -2\}$ with the usual metric $d(x, y) = |x - y|$ for all $x, y \in X$. Then, (X, d) is incomplete, but T-orbitally compact with $T : X \to X$ defined by*

$$Tx = \begin{cases} -2 & \text{if} \;\; x = 1, \\ -1 & \text{if} \;\; x \neq 1. \end{cases}$$

We also consider $f : \mathbb{R}_+^3 \longrightarrow \mathbb{R}_+$ defined by $f(t_1, t_2, t_3) = \dfrac{1}{3}(t_1 + t_2 + t_3)$ for all $t_1, t_2, t_3 \in \mathbb{R}_+$. Let $x, y \in X$ be arbitrary with $x \neq y$. Then the following cases arise:

Case I: $x = 1$, $y \neq 1$. Then, we have

$$|Tx - Ty| = 1$$

and

$$\frac{1}{3}\left[|x - y| + |x - Tx| + |y - Ty|\right] = \frac{1}{3}\left[|1 - y| + 3 + |y + 1|\right] > 1.$$

Thus,

$$d(Tx, Ty) < f\left[d(x, y), d(x, Tx), d(y, Ty)\right].$$

Case II: $x \neq 1$, $y \neq 1$, $x \neq y$. Then, we have

$$|Tx - Ty| = 0$$

and

$$\frac{1}{3}\left[|x - y| + |x - Tx| + y - Ty|\right] = \frac{1}{3}\left[|x - y| + |x + 1| + |y + 1|\right] > 0.$$

Thus,

$$d(Tx, Ty) < f[d(x, y), d(x, Tx), d(y, Ty)].$$

Therefore, we have

$$d(Tx, Ty) < f[d(x, y), d(x, Tx), d(y, Ty)]$$

for all $x, y \in X$ *with* $x \neq y$. *By Theorem 2.4,* T *has a unique fixed point. Indeed* $x = -1$ *is the unique fixed point of* T.

If we take $f(t_1, t_2, t_3) = \frac{1}{2}(t_2 + t_3)$ in Theorem 2.1 and Theorem 2.4, then we get the following results which are mains results in [6].

Corollary 2.6. *Let* (X, d) *be a boundedly compact metric space and* T *be a orbitally continuous mapping on* X *such that*

$$d(Tx, Ty) < \frac{1}{2}[d(x, Tx) + d(y, Ty)]$$

for all $x, y \in X$ *with* $x \neq y$. *Then* T *has a unique fixed point.*

Corollary 2.7. *Let* (X, d) *be a* T*-orbitally compact metric space and* T *be a orbitally continuous mapping on* X *such that*

$$d(Tx, Ty) < \frac{1}{2}[d(x, Tx) + d(y, Ty)]$$

for all $x, y \in X$ *with* $x \neq y$. *Then* T *has a unique fixed point.*

REFERENCES

[1] G. V. R. Babu, "Generalization of fixed point theorems relating to the diameter of orbits by using a control function," *Tamkang J. Math.*, vol. 35, no. 2, pp. 159-168, 2004.

[2] L. B. Ćirić, "A generalization of Banach's contraction principle," *Proc. Am. Math. Soc.*, vol. 45, no. 2, pp. 267-273, 1974.

[3] M. Edelstein, "On fixed and periodic points under contractive mappings," *J. London Math. Soc.*, vol. 1, no. 1, pp. 74-79, 1962.

[4] R. E. Edwards, *Functional analysis: theory and applications.* Courier Corporation, 2012.

[5] H. Garai, L. K. Dey, and Y. J. Cho, "On contractive mappings and discontinuity at fixed points," *Appl. Anal. Discret. Math.*, vol. 14, no. 1, pp. 33–54, 2020.

[6] H. Garai, L. K. Dey, and T. Senapati, "On Kannan-type contractive mappings," *Numer. Funct. Anal. Optim.*, vol. 39, no. 13, pp. 1466–1476, 2018.

[7] J. Górnicki, "Fixed point theorems for Kannan type mappings," *J. fixed point theory Appl.*, vol. 19, pp. 2145–2152, 2017.

[8] R. Kannan, "Some results on fixed points," *Bull. Cal. Math. Soc.*, vol. 60, pp. 71–76, 1968.

[9] D. Roux, E. Maluta, and others, "Kannan maps in compact spaces," *Riv. di Mat. della Univ. di parma*, vol. 5, pp. 141–145, 1979.

TÓM TẮT

CÁC ĐỊNH LÝ ĐIỂM BẤT ĐỘNG CHO CÁC ÁNH XẠ $\mathcal{A}$-CO TRONG CÁC KHÔNG GIAN MÊTRIC COMPẮC BỊ CHẶN VÀ T-COMPẮC QUỸ ĐẠO

Nguyễn Hữu Học
Khoa Khoa học Tự nhiên, Trường Đại học Hồng Đức, Thanh Hóa, Việt Nam
Ngày nhận bài 22/10/2024, ngày nhận đăng 27/12/2024

Trong bài báo này, chúng tôi chứng minh một số định lý điểm bất động cho các ánh xạ thỏa mãn các điều kiện co ẩn trong các không gian mêtric compắc bị chặn và T-compắc quỹ đạo. Các kết quả của chúng tôi cải tiến và mở rộng một vài kết quả trong tài liệu [6]. Một số ví dụ cũng được đưa ra để minh họa cho các phát hiện của chúng tôi.

Từ khoá: Không gian compắc bị chặn; điểm bất động; không gian mêtric; không gian T-compắc quỹ đạo.