

SELF-COUPPLINGS OF GAUGE BOSONS IN 3-3-1 MODELS

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In this work, we will present detailed calculations to derive the exact formulas of the triple and quartic self-couplings of all gauge bosons predicted by the general 3-3-1 model with an arbitrary parameter β used to classify different 3-3-1 models. We show that our result is consistent with all previous results calculated for particular β values. The respective Feynman rules are introduced for further studies on physical processes relating to these gauge bosons.

Keywords: Gauge boson coupling; gauge extension of the standard model; exotic gauge boson.

1. Introduction

The self-couplings of the gauge bosons are necessary for calculating many decay rates such as $e_b \rightarrow e_a \gamma$, $(g-2)$ anomalies of charged leptons, loop-induced decays of SM-like Higgs bosons $h \rightarrow \gamma\gamma, Z\gamma, f\bar{f}_y, \dots$. They are used to compare with experimental results or predict the possibilities of searching them by experiments. The gauge couplings in many particular 3-3-1 models were computed previously [1]-[4], or in the general value of parameter β used to define the electric charges of new particles with lepton anti-triplets [5] or triplets [6], [7]. But the small $Z - Z'$ mixing of the two massive neutral gauge bosons was still neglected. In this work, we will derive all triple and quartic couplings of gauge bosons in the exact formulas consisting of the small $Z - Z'$ mixing. Our results will be compared with some previous ones, which are still inconsistent with each other. The Feynman rules will be given in standard forms so that they can be used directly to calculate any relevant physical processes.

2 The 331β model

In this section, we focus on the model F_1 defined in Refs. [8, 9], where three lepton families are assigned as antitriplets. The model F_2 with lepton triplets was shown to be equivalent to the model F_1 by a transformation of $\beta \rightarrow -\beta$ as well as the vacuum expectation values (vev) of neutral Higgs that are not relevant here [9]. The electroweak group is $SU(3)_L \otimes U(1)_X$ with two gauge couplings g and g_X . Correspondingly, there are eight $SU(3)_L$ gauge bosons W_μ^a , $a=1,2,\dots,8$, and an $U(1)_X$ one X_μ . Left-handed leptons are assigned to antitriplets and right-handed leptons to singlets:

$$L_{aL} = \begin{pmatrix} e_a \\ -\nu_a \\ E_a \end{pmatrix}_L \sim \left(3^*, -\frac{1}{2} + \frac{\beta}{2\sqrt{3}} \right), \quad a = 1, 2, 3,$$

$$e_{aR} \sim (1, -1), \quad \nu_{aR} \sim (1, 0), \quad E_{aR} \sim \left(1, -\frac{1}{2} + \frac{\sqrt{3}\beta}{2} \right). \quad (2.1)$$

Three scalar triplets are needed to generate masses for gauge bosons and fermions:

$$\chi = \begin{pmatrix} \chi^{+A} \\ \chi^{+B} \\ \chi^0 \end{pmatrix} \sim \left(3, \frac{\beta}{\sqrt{3}} \right), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{-B} \end{pmatrix} \sim \left(3, \frac{1}{2} - \frac{\beta}{2\sqrt{3}} \right),$$

$$\eta = \begin{pmatrix} \eta^0 \\ \eta^- \\ \eta^{-A} \end{pmatrix} \sim \left(3, -\frac{1}{2} - \frac{\beta}{2\sqrt{3}} \right), \quad (2.2)$$

where A, B denote electric charges defined as follows

$$A = \frac{1}{2} + \beta \frac{\sqrt{3}}{2}, \quad B = -\frac{1}{2} + \beta \frac{\sqrt{3}}{2}. \quad (2.3)$$

Nonzero vev of neutral Higgs components are

$$\langle \chi^0 \rangle = \frac{u}{\sqrt{2}}, \quad \langle \rho^0 \rangle = \frac{v}{\sqrt{2}}, \quad \langle \eta^0 \rangle = \frac{v'}{\sqrt{2}}. \quad (2.4)$$

The breaking pattern is $SU(3)_L \otimes U(1)_X \xrightarrow{u} SU(2)_L \otimes U(1)_Y \xrightarrow{v, v'} U(1)_Q$ with the condition $u > v, v'$. The bases of neutral gauge bosons transform as $X_\mu, W_\mu^8, W_\mu^3 \xrightarrow{\theta_{331}} Z'_\mu B_\mu, W_\mu^3 \xrightarrow{\theta_W} Z'_\mu, Z_\mu, A_\mu \xrightarrow{\xi} Z_{2\mu}, Z_{1\mu}, A_\mu$, where

$$\frac{g_X^2}{g^2} = \frac{6s_W^2}{1 - (1 + \beta^2)s_W^2}, \quad g_2 = g, \quad g_1 = g_X \frac{g}{\sqrt{6g^2 + \beta^2 g_X^2}}. \quad (2.5)$$

Here, g_2 and g_1 are the two $SU(2)_L$ and $U(1)_Y$ gauge couplings of the SM, respectively: $g_1/g_2 = t_W \equiv s_W/c_W$. We also have [9]

$$\begin{aligned} s_{331} &\equiv \sin \theta_{331} = \frac{\sqrt{6}g}{\sqrt{6g^2 + \beta^2 g_X^2}} = \sqrt{1 - \beta^2 t_W^2}, \\ c_{331} &\equiv \cos \theta_{331} = \frac{\beta g_X}{\sqrt{6g^2 + \beta^2 g_X^2}} = \beta t_W. \end{aligned} \quad (2.6)$$

The relations between mass eigenstates $Z_\mu^{1,2}$ and the intermediate states containing one SM gauge bosons (Z_μ, Z'_μ) are, see Eq. (2) of ref. [8]:

$$Z_\mu^1 = c_\xi Z_\mu + s_\xi Z'_\mu, \quad Z_\mu^2 = -s_\xi Z_\mu + c_\xi Z'_\mu, \quad (2.7)$$

where $s_\xi \equiv \sin \xi$ and $c_\xi \equiv \cos \xi$ relating with the tiny $Z - Z'$ mixing angle ξ [9].

The relations between original and physical states of neutral gauge bosons is:

$$\begin{pmatrix} X_\mu \\ W_\mu^8 \\ W_\mu^3 \end{pmatrix} = \begin{pmatrix} s_{331} c_W A_\mu + (-s_{331} s_W c_\xi + c_{331} s_\xi) Z_\mu^1 + (c_{331} c_\xi + s_{331} s_W s_\xi) Z_\mu^2 \\ c_W c_{331} A_\mu - (c_{331} s_W c_\xi + s_{331} s_\xi) Z_\mu^1 + (-s_{331} c_\xi + c_{331} s_W s_\xi) Z_\mu^2 \\ s_W A_\mu + c_W c_\xi Z_\mu^1 - c_W s_\xi Z_\mu^2 \end{pmatrix}. \quad (2.8)$$

The limit $u \gg v, v'$ gives $s_\xi \equiv \sin \xi \rightarrow 0$ and $c_\xi \equiv \cos \xi \rightarrow 1$. Consequently, $Z_{2\mu} \simeq Z'_\mu$ and $Z_{1\mu} \simeq Z_\mu$, which is identified with the neutral SM gauge boson. In this case the relations in Eq. (2.8) are the same as that shown in [7].

The mass eigenstates of the non-diagonal gauge bosons, including the SM one W^\pm , are

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2), \quad Y_\mu^{\pm A} = \frac{1}{\sqrt{2}} (W_\mu^4 \mp iW_\mu^5), \quad V_\mu^{\pm B} = \frac{1}{\sqrt{2}} (W_\mu^6 \mp iW_\mu^7).$$

As a result, every gauge boson pair W_μ^b and W_μ^c with $\{b, c\} = \{1, 2\}, \{4, 5\}, \{6, 7\}$ relates to the physical states $G_\mu^{\pm Q} = W_\mu^\pm, Y_\mu^{\pm A}, Y_\mu^{\pm B}$ by the same following formulas:

$$W_\mu^b = \frac{G_\mu^Q + G_\mu^{-Q}}{\sqrt{2}}, \quad W_\mu^c = \frac{i(G_\mu^{+Q} - G_\mu^{-Q})}{\sqrt{2}}, \quad \forall b < c = b + 1. \quad (2.9)$$

The above results are used to compute all self-couplings of gauge bosons in the following.

3 Self-couplings of the gauge bosons and Feynman rules

The kinetic Lagrangian consisting of all self-couplings of gauge bosons is:

$$\mathcal{L}_{\text{gauge}}^k = -\frac{1}{4} \sum_{a=1}^8 F_{\mu\nu}^a F^{a\mu\nu}, \quad F_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + gf^{abc} W_\mu^b W_\nu^c, \quad (3.1)$$

where f^{abc} are the structure constants defining the algebra of the $SU(3)$: $[T^a, T^b] = i f^{abc} T^c$. Choosing the $SU(3)_L$ generators being the Gell-Mann matrices $T^a = \lambda_a/2$ corresponding to the fundamental representation gives $\text{Tr}(T_a T_b) = \delta_{ab}/2$ [10]. As a result, $f^{abc} = -2i \text{Tr}([\lambda_a/2, \lambda_b/2](\lambda_c/2))$, which leads to the following non-zero values of f^{abc} with $1 \leq a < b < c \leq 8$:

$$\begin{aligned} f^{123} &= 1, \quad f^{147} = -f^{156} = f^{246} = f^{257} = f^{345} = -f^{367} = \frac{1}{2}, \\ f^{458} &= f^{678} = \frac{\sqrt{3}}{2}. \end{aligned} \tag{3.2}$$

All other permutations sets of (a, b, c) are derived easily following the total antisymmetric property that $f^{abc} = f^{bca} = f^{cba} = -f^{cba} = -f^{bac} = -f^{acb}$. The Lagrangian (3.1) reads

$$\mathcal{L}_{\text{gauge}}^k = -\frac{1}{4} (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a) (\partial^\mu W^{a\nu} - \partial^\nu W^{a\mu}) + \mathcal{L}_{3g} + \mathcal{L}_{4g}, \tag{3.3}$$

where the first term is the free gauge boson kinetic term, while the two remaining ones consist of the respective triple and quartic couplings. They are derived as follows

$$\mathcal{L}_{3g} = -g f^{abc} (\partial_\mu W_\nu^a) W^{b\mu} W^{c\nu}, \tag{3.4}$$

$$\mathcal{L}_{4g} = -\frac{1}{4} (f^{abc} W_\mu^b W_\nu^c) (f^{ade} W^{d\mu} W^{e\nu}). \tag{3.5}$$

In Lagrangian (3.4), every structure constant f^{abc} implies a triple coupling $W^a W^b W^c$. Therefore, we know immediately that $f^{123} \sim W^1 W^2 W^3 \rightarrow G^0 W^+ W^-$; $f^{345}, f^{458} \sim G^0 Y^+ A Y^-$; $f^{367}, f^{678} \sim G^0 V^+ B V^-$, where $G^0 = A, Z_1, Z_2$ are the real neutral gauge bosons. The couplings of three non-hermitian gauge bosons are always $W^\pm Y^\pm B V^\mp A$, because the electric charge conservation of the vertex $A = B + 1$ must be satisfied. This is consistent with four remaining values of $f^{abc} = f^{147}, f^{156}, f^{246}$, and f^{257} given in Eq. (3.2).

Now we pay attention to the couplings of neutral gauge bosons G^0 involving one of the two indices $a = 3, 8$. These couplings come from one of the values $f^{abc} = f^{312} = f^{123}, f^{345}, f^{345}, f^{367}, f^{345}, f^{845} = f^{867}$, where $b < c$ defined in Eq. (2.9). Namely, consider a such set (a, b, c) with its permutations without any sum over the indices a, b, c , we get

$$\begin{aligned} -\mathcal{L}_{G^0 G^+ G^-}^{(abc)} &= g f^{abc} \left\{ (\partial_\mu W_\nu^a) (W^{b\mu} W^{c\nu} - W^{c\mu} W^{b\nu}) \right. \\ &\quad + W^{a\nu} [(\partial_\mu W_\nu^b) W^{c\mu} - (\partial_\mu W_\nu^c) W^{b\mu}] \\ &\quad \left. - W^{a\mu} [(\partial_\mu W_\nu^b) W^{c\nu} - (\partial_\mu W_\nu^c) W^{b\nu}] \right\}. \end{aligned} \tag{3.6}$$

The relations given in Eq. (2.9) for $b < c = b + 1$ give

$$\begin{aligned}
 W^{b\mu}W^{c\nu} - W^{c\mu}W^{b\nu} &= -i (G^{+Q\mu}G^{-Q\nu} - G^{-Q\mu}G^{+Q\nu}), \\
 (\partial_\mu W_\nu^b)W^{c\mu} - (\partial_\mu W_\nu^c)W^{b\mu} &= -i [(\partial_\mu G_\nu^{+Q})G^{-Q\mu} - (\partial_\mu G_\nu^{-Q})G^{+Q\mu}], \\
 &= - [G_\nu^{+Q}(p_2 \cdot G^{-Q}) - G_\nu^{-Q}(p_3 \cdot G^{+Q})], \\
 (\partial_\mu W_\nu^b)W^{c\nu} - (\partial_\mu W_\nu^c)W^{b\nu} &= -(G^{+Q} \cdot G^{-Q})(p_{2\mu} - p_{3\mu}), \tag{3.7}
 \end{aligned}$$

where $\partial_\mu = -ip_{\pm\mu}$ is the momenta of the charged gauge boson $G_\mu^{\pm Q}$ in the momenta space. In the same way for the neutral state, for example, $\partial_\mu W_\nu^a = -ip_{0\mu}W_\nu^a \rightarrow -ip_{0\mu}V_\nu^0$ after transformations into a physical state V_ν^0 . This transformation implies that all momenta are incoming at the vertex of all triple gauge bosons. The Lagrangian (3.6) is written in the scalar form after contracting all Lorentz indices as follows

$$\begin{aligned}
 \mathcal{L}_{G^0G^+G^-}^{(abc)} &= gf^{abc} \times \left\{ [(p_1 \cdot G^{+Q})(W^a \cdot G^{-Q}) - (p_1 \cdot G^{-Q})(W^a \cdot G^{+Q})] \right. \\
 &\quad + [(W^a \cdot G^{+Q})(p_2 \cdot G^{-Q}) - (W^a \cdot G^{-Q})(p_3 \cdot G^{+Q})] \\
 &\quad \left. - (G^{+Q} \cdot G^{-Q}) [(p_2 - p_3) \cdot W^a] \right\}. \tag{3.8}
 \end{aligned}$$

Hence, a triple vertex $G^{0\alpha}G^{+\beta}G^{-\lambda}$ with $G^{0\alpha} \sim W^{a\alpha}$ is derived as follows:

$$\begin{aligned}
 \mathcal{L}_{G^0G^+G^-}^{(abc)} &= gf^{abc}W^{a\alpha}G^{+\beta}G^{-\lambda} \\
 &\quad \times \left\{ (p_{1\beta}g_{\alpha\lambda} - p_{1\lambda}g_{\alpha\beta}) + (p_{2\lambda}g_{\alpha\beta} - p_{3\beta}g_{\alpha\lambda}) - g_{\beta\lambda}(p_2 - p_3)_\alpha \right\} \\
 &\equiv -gf^{abc}W^{a\alpha}G^{+\beta}G^{-\lambda}\Gamma_{\alpha\beta\lambda}(p_1, p_2, p_3), \tag{3.9}
 \end{aligned}$$

where

$$\Gamma_{\alpha\beta\lambda}(p_1, p_2, p_3) \equiv (p_1 - p_2)_\lambda g_{\alpha\beta} + (p_2 - p_3)_\alpha g_{\beta\lambda} + (p_3 - p_1)_\beta g_{\alpha\lambda}. \tag{3.10}$$

Now, the Lagrangian for the triple self-couplings of neutral gauge bosons is written in a very simple form:

$$\begin{aligned}
 \mathcal{L}_{G^0G^+G^-} &= -g\Gamma_{\alpha\beta\lambda}(p_1, p_2, p_3) \\
 &\quad \times [f^{312}W^{3\alpha}W^{+\beta}W^{-\lambda} + (f^{345}W^{3\alpha} + f^{845}W^{8\alpha})Y^{+A\beta}Y^{-A\lambda} \\
 &\quad + (f^{367}W^{3\alpha} + f^{867}W^{8\alpha})V^{+B\beta}V^{-B\lambda}], \tag{3.11}
 \end{aligned}$$

where the physical states of neutral gauge bosons are determined as follows

$$\begin{aligned}
f^{312}W^{3\alpha} &= s_W A^\alpha + c_W c_\xi Z^{1\alpha} - c_W s_\xi Z^{2\alpha}, \\
f^{345}W^{3\alpha} + f^{845}W^{8\alpha} &= A s_W A^\alpha + \frac{c_\xi c_W (1 - \sqrt{3}\beta t_W^2) - \sqrt{3}s_{331}s_\xi}{2} Z^{1\alpha} \\
&\quad - \frac{\sqrt{3}s_{331}c_\xi + s_\xi c_W (1 - \sqrt{3}\beta t_W^2)}{2} Z^{2\alpha}, \\
f^{367}W^{3\alpha} + f^{867}W^{8\alpha} &= B s_W A^\alpha - \frac{c_\xi c_W (1 + \sqrt{3}\beta t_W^2) + \sqrt{3}s_{331}s_\xi}{2} Z^{1\alpha} \\
&\quad + \frac{-\sqrt{3}s_{331}c_\xi + s_\xi c_W (1 + \sqrt{3}\beta t_W^2)}{2} Z^{2\alpha}. \tag{3.12}
\end{aligned}$$

For triple couplings of three charged Higgs bosons $W^\pm Y^\mp A V^{\pm B}$ corresponding to $f^{abc} = f^{147} = -f^{156} = f^{246} = f^{257}$ given in Eq. (3.2), we collect all relevant terms as follows:

$$\begin{aligned}
-\mathcal{L}_{WYV} &= g f^{147} \{ (\partial_\mu W_\nu^1) [(W^{4\mu} W^{7\nu} - W^{5\mu} W^{6\nu}) \\
&\quad - (W^{7\mu} W^{4\nu} - W^{6\mu} W^{5\nu})] + [\text{permutation}] \} \\
&\quad + g f^{147} \{ (\partial_\mu W_\nu^2) [(W^{4\mu} W^{6\nu} + W^{5\mu} W^{7\nu}) \\
&\quad - (W^{6\mu} W^{4\nu} + W^{7\mu} W^{5\nu})] + [\text{permutation}] \} \\
&= g f^{147} \{ (i\partial_\mu W_\nu^1) [(Y^{A\mu} V^{-B\nu} - Y^{-A\mu} V^{+B\nu}) - \text{h.c.}] + [\text{permutation}] \} \\
&\quad + g f^{147} \{ (\partial_\mu W_\nu^2) [(Y^{A\mu} V^{-B\nu} - V^{-B\mu} Y^{+A\nu}) + \text{h.c.}] + [\text{permutation}] \}, \tag{3.13}
\end{aligned}$$

where the "permutation" implies all similar terms appearing as those in calculating the cubic couplings relating to neutral gauge bosons. We make a further intermediate step for Lagrangian (3.13) as follows

$$\begin{aligned}
-\mathcal{L}_{WYV} &= g f^{147} \{ (i\partial_\mu W_\nu^1 + \partial_\mu W_\nu^2) [(Y^{A\mu} V^{-B\nu} - V^{-B\mu} Y^{A\nu})] + [\text{permutation}] \} \\
&\quad + g f^{147} \{ (-i\partial_\mu W_\nu^1 + \partial_\mu W_\nu^2) [(Y^{-A\mu} V^{B\nu} - V^{B\mu} Y^{-A\nu})] + [\text{permutation}] \} \\
&= \frac{g}{\sqrt{2}} \{ (i\partial_\mu W_\nu^-) [(Y^{A\mu} V^{-B\nu} - V^{-B\mu} Y^{A\nu})] + [\text{permutation}] \} \\
&\quad + \frac{g}{\sqrt{2}} \{ (-i\partial_\mu W_\nu^+) [(Y^{-A\mu} V^{B\nu} - V^{B\mu} Y^{-A\nu})] + [\text{permutation}] \}. \tag{3.14}
\end{aligned}$$

Because the structures of all Lorentz indices for all couplings in the two Lagrangian parts (3.14) and (3.8) are the same, the vertex couplings of three charged gauge bosons are derived similarly. By replacing $\partial_\mu = -ip_\mu$ as the incoming momentum, we have:

$$\mathcal{L}_{WYV} = -\frac{g}{\sqrt{2}} \Gamma_{\alpha\beta\lambda}(p_1, p_2, p_3) \left[W^{-\alpha} Y^{A\beta} V^{-B\lambda} - W^{+\alpha} Y^{-A\beta} V^{B\lambda} \right], \tag{3.15}$$

where $p_{1,2,3}$ are incoming momenta of $W^{\pm\alpha}$, $Y^{\pm A\beta}$, and $V^{\pm B\lambda}$, respectively. These couplings were only mentioned previously in Ref. [2] for two particular cases of $\beta = \pm\sqrt{3}, \pm 1/\sqrt{3}$, but they were not expressed precisely in the standard form in the momentum space.

To summarize the triple gauge boson couplings, the two Lagrangian parts (3.11) and (3.15) are written in the following general form:

$$\mathcal{L}_{3g} = \sum_{G_1, G_2, G_3} g_{G_{123}} \Gamma_{\alpha\beta\lambda}(p_1, p_2, p_3) \left[G_1^\alpha(p_1) G_2^\beta(p_2) G_3^\lambda(p_3) \right]. \quad (3.16)$$

Then, applying the Feynman rule for a coupling with three different gauge bosons $G_1^\alpha(p_1) G_2^\beta(p_2) G_3^\lambda(p_3)$ gives the vertex factor $ig_{G_{123}}$. All of them predicted by the 331 β model are listed in the Table 1,

where $e = g_{SW}$ is the electric charge constant, and $a_{1,2}$ relate to the $Z_{1,2}$ couplings as follows

$$a_1 = c_W(1 - \sqrt{3}\beta t_W^2), \quad a_2 = c_W(1 + \sqrt{3}\beta t_W^2). \quad (3.17)$$

Table 1: Feynman rules for triple gauge couplings in the 331 β model

$G_1^\alpha G_2^\beta G_3^\lambda$	$ig_{G_{123}}$	$G_1^\alpha G_2^\beta G_3^\lambda$	$ig_{G_{123}}$	$G_1^\alpha G_2^\beta G_3^\lambda$	$ig_{G_{123}}$
AW^+W^-	$-ie$	AY^AY^{-A}	$-ieA$	AV^BV^{-B}	$-ieB$
$Z^1W^+W^-$	$-igc_Wc_\xi$	$Z^1Y^AY^{-A}$	$\frac{igc_\xi}{2}$	$Z^1V^BV^{-B}$	$\frac{igc_\xi [a_2 + \sqrt{3}s_{331}t_\xi]}{2}$
$Z^2W^+W^-$	igc_Ws_ξ	$Z^2Y^AY^{-A}$	$\frac{igc_\xi [\sqrt{3}s_{331} + t_\xi a_1]}{2}$	$Z^2V^BV^{-B}$	$\frac{igc_\xi [\sqrt{3}s_{331} - t_\xi a_2]}{2}$
		$W^-Y^AV^{-B}$	$-\frac{ig}{\sqrt{2}}$	$W^+Y^{-A}V^B$	$\frac{ig}{\sqrt{2}}$

In the limit of $\xi = 0$, our results are consistent with those given in Ref [2] with two values $\beta = \sqrt{3}, -1/\sqrt{3}$ for the minimal 3-3-1 model with lepton antitriplets (M331 F_1) and the 3-3-1 model with right-handed neutrinos in lepton triplets (331RHNF $_2$), respectively. But our Feynman rules were expressed in the momentum space, instead of those containing derivatives in Ref. [2]. Our results are completely consistent with those given in Ref. [11] for two couplings AW^+W^- and $Z^1W^+W^-$, but have opposite signs with ref. [10], even in the same convention of the sign in the covariant derivative given in Eq. (3.1). The results of the couplings AY^AY^{-A} and AV^BV^{-B} coincide with ref. [12]. Ref. [6] did not mention the couplings $W \pm Y^{\mp A}V^{\pm B}$, while the remaining ones have opposite signs with our results. This is explained by the different choices of the signs of g defining $F_{\mu\nu}^a$ in Eq. (3.1). We emphasize that this sign convention must be consistent with the other covariant derivatives appearing in kinetic terms of fermions and Higgs bosons.

The Lorentz contraction form of Eq. (3.5) consisting of quartic gauge couplings is

$$\begin{aligned}
 \mathcal{L}_{4g} &= -\frac{1}{4} \sum_{b,c,b',c'=1}^8 \left[(W^b.W^{b'}) (W^c.W^{c'}) \times \sum_{a=1}^8 (f^{abc} f^{ab'c'}) \right] \\
 &= -\frac{1}{2} \sum_{b<c=1}^8 \left[(W^b.W^b) (W^c.W^c) \times \sum_{a=1}^8 (f^{abc} f^{abc}) \right] \\
 &\quad - \sum_{b=1}^8 \sum_{c<c'=1}^8 \left[(W^b.W^b) (W^c.W^{c'}) \times \sum_{a=1}^8 (f^{abc} f^{ab'c'}) \right] \\
 &\quad - \frac{1}{2} \sum_{b<b',c<c'=1}^8 \left[(W^b.W^{b'}) (W^c.W^{c'}) \times \sum_{a=1}^8 (f^{abc} f^{ab'c'} + f^{ab'c} f^{abc'}) \right]. \quad (3.18)
 \end{aligned}$$

The second form of the Lagrangian in Eq. (3.18) is used in order to reduce many repeated terms in the sum, because of the equivalence of the two sets $\{(b, b'), (c, c')\} \rightarrow \{(b, b) + (b \neq b'), (c, c) + (c, c')\} \rightarrow 2\{(b, b), (c, c) : b < c\} + \{(b, b), (c \neq c')\} + \{(b \neq b'), (c, c)\} + \{(b, b), (c \neq c')\} + \{(b \neq b'), (c \neq c')\} \rightarrow 2\{(b, b), (c, c) : b < c\} + 4\{(b, b), (c < c')\} + 2\{(b < b'), (c < c')\} \times (f^{abc} f^{ab'c'} \text{ permutation})$.

As mentioned above, the models consist of three real neutral gauge bosons $(A_\mu, Z_\mu^1, Z_\mu^2) \in (X_\mu, W_\mu^3, W_\mu^8)$, and three pairs of non-hermitian charged ones $W^\pm \sim W^{1,2}, Y^{\pm A} \sim W^{4,5}$, and $V^{\pm B} \sim W^{6,7}$. All of the quartic coupling types are listed as follows:

1. The couplings of four charged gauge bosons are found to be the same results given in Ref. [1] except an overall minus sign, namely

$$\begin{aligned}
 \mathcal{L}_{4g}^{G^\pm} &= -\frac{g^2}{2} \sum_G [(G^{-q}.G^q)^2 - (G^{-q}.G^{-q})(G^q.G^q)] \\
 &\quad - \frac{g^2}{2} \sum_{G_{1,2}} [(G^{-q_1}.G^{q_2})(G^{q_1}.G^{-q_2}) + (G^{-q_1}.G^{q_1})(G^{-q_2}.G^{q_2}) \\
 &\quad \quad - 2(G^{-q_1}.G^{-q_2})(G^{q_1}.G^{q_2})], \quad (3.19)
 \end{aligned}$$

where $G = W, Y, V$ and $q_1 \neq q_2$. Any vertex factors corresponding to the Feynman rules of the vertex $G_{1\mu}G_{2\nu}G_{3\alpha}G_{4\beta}$ is determined from the following formula:

$$\text{factor} = i \times \frac{\partial^4 \mathcal{L}_{4g}}{\partial G_{1\mu} \partial G_{2\nu} \partial G_{3\alpha} \partial G_{4\beta}}. \quad (3.20)$$

For example the Feynman rule for a vertex $G_\mu^q G_\nu^{-q} G_\alpha^q G_\beta^{-q}$ given in the first line of La-

grangian (3.19) is:

$$\begin{aligned}
 \text{factor} &= -\frac{ig^2}{2} \times \frac{\partial^4}{(\partial G_\mu^q)(\partial G_\nu^{-q})(\partial G_\alpha^q)(\partial G_\beta^{-q})} [(G^{-q}.G^q)^2 - (G^{-q}.G^{-q})(G^q.G^q)] \\
 &= -\frac{ig^2}{2} \frac{\partial^3}{(\partial G_\mu^q)(\partial G_\nu^{-q})(\partial G_\alpha^q)} [2(G^{-q}.G^q)G^{q\beta} - 2G^{-q\beta}(G^q.G^q)] \\
 &= -ig^2 \frac{\partial^2}{(\partial G_\mu^q)(\partial G_\nu^{-q})} [(G^{-q}.G^q)g^{\alpha\beta} + G^{-q\alpha}G^{q\beta} - 2G^{-q\beta}G^{q\alpha}] \\
 &= -ig^2 [g^{\mu\nu}g^{\alpha\beta} + g^{\mu\beta}g^{\nu\alpha} - 2g^{\mu\alpha}g^{\nu\beta}] \equiv ig^2 S^{\mu\alpha,\nu\beta}, \tag{3.21}
 \end{aligned}$$

with $S^{\mu\alpha,\nu\beta} \equiv 2g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha}$ being completely consistent with the result given in Ref. [11] for the W self-coupling in the SM. But this has a minus sign with that shown in Ref. [1], in which the original Lagrangian did not include a minus sign that always appears in the standard Lagrangian part given in Eq. (3.5), see also in Refs. [6, 10, 11].

2. Four real neutral bosons in the couplings: $G_1^0 G_2^0 G_3^0 G_4^0 \sim f^{a38} = 0 \ \forall a = 1, 2, \dots, 8$. There are not any couplings consisting of three real neutral gauge bosons because of the electric charge conversation.

3. Couplings with two real neutral gauge bosons: $G_1^0 G_2^0 G^{+Q} G^{-Q} \sim \{W^3, W^8\} \times \{W^+W^-, Y^+Y^-, V^+V^-\}$. Keeping the neutral gauge bosons as $W^{3,8}$ we have:

$$\begin{aligned}
 \mathcal{L}_{4g}^{G^{02}G^\pm} &= g^2 [(W^-.W^3)(W^+.W^3) - (W^+.W^-)(W^3)^2] \\
 &\quad + \frac{g^2}{4} [(Y^{-A}.W^{38A})(Y^A.W^{38A}) - (Y^{-A}.Y^A)(W^{38A})^2] \\
 &\quad + \frac{g^2}{4} [(V^{-B}.W^{38B})(V^B.W^{38B}) - (V^{-B}.V^B)(W^{38B})^2], \tag{3.22}
 \end{aligned}$$

where the physical states are derived in the following relations:

$$\begin{aligned}
 W_\mu^{38A} &\equiv W_\mu^3 + \sqrt{3}W_\mu^8 = 2A s_W A_\mu + \left[-\sqrt{3}s_{331}t_\xi + c_W \left(1 - \sqrt{3}\beta t_W^2 \right) \right] c_\xi Z_\mu^1 \\
 &\quad - \left[\sqrt{3}s_{331}c_\xi + s_\xi c_W \left(1 - \sqrt{3}\beta t_W^2 \right) \right] Z_\mu^2, \\
 W_\mu^{38B} &\equiv -W_\mu^3 + \sqrt{3}W_\mu^8 = 2B s_W A_\mu - \left[\sqrt{3}s_{331}s_\xi + c_\xi c_W \left(1 + \sqrt{3}\beta t_W^2 \right) \right] Z_\mu^1 \\
 &\quad + \left[-\sqrt{3}s_{331}c_\xi + s_\xi c_W \left(1 + \sqrt{3}\beta t_W^2 \right) \right] Z_\mu^2. \tag{3.23}
 \end{aligned}$$

4. Regarding to the couplings to one neutral gauge bosons, the electric charge conversations allows only the couplings $G^0 W^\pm Y^\mp A V^{\pm B}$:

$$\mathcal{L}_{4g}^{G^0 3G^\pm} = \frac{g^2}{2\sqrt{2}} \left\{ (W^- \cdot Y^A)(V^{-B} \cdot W'^{38A}) + (W^- \cdot V^{-B})(Y^A \cdot W'^{38B}) - 2\sqrt{3}(Y^A \cdot V^{-B})(W^- \cdot W^8) \right\} + \text{h.c.}, \quad (3.24)$$

where

$$\begin{aligned} W_\mu'^{38A} &\equiv 3W_\mu^3 + \sqrt{3}W_\mu^8 = 2(A+1)s_W A_\mu \\ &\quad + \sqrt{3} \left[-s_{331}t_\xi + c_W \left(\sqrt{3} - \beta t_W^2 \right) \right] c_\xi Z_\mu^1 \\ &\quad - \sqrt{3} \left[s_{331}c_\xi + s_\xi c_W \left(\sqrt{3} - \beta t_W^2 \right) \right] Z_\mu^2, \\ W_\mu'^{38B} &\equiv -3W_\mu^3 + \sqrt{3}W_\mu^8 = 2(B-1)s_W A_\mu \\ &\quad - \sqrt{3} \left[s_{331}t_\xi + c_W \left(\sqrt{3} + \beta t_W^2 \right) \right] c_\xi Z_\mu^1 \\ &\quad - \sqrt{3} \left[s_{331}c_\xi - s_\xi c_W \left(\sqrt{3} + \beta t_W^2 \right) \right] Z_\mu^2. \end{aligned} \quad (3.25)$$

Finally, the Feynman rules for the quartic couplings are given in Table 2, where $a_{1,2}$ are given in Eq. (3.17), and

$$b_1 = \left(\beta t_W t_\xi + \frac{s_{331}}{s_W} \right), \quad b_2 = \frac{\sqrt{3} - \beta t_W^2}{2t_W}, \quad b_3 = \frac{(\sqrt{3} + \beta t_W^2)}{2t_W}. \quad (3.26)$$

In the limit $\xi = 0$, i.e. $c_\xi = 1$ and $t_\xi = 0$, the results given in Table 2 are in general consistent with previous works. The quartic couplings of four SM gauge bosons coincide with the results given in Refs. [6, 11], but have opposite signs with Ref. [1]. Except for this difference, all Lorentz structures in the results of ours and Ref. [1] are the same, but some of them are different from those given in Ref. [6].

4 Conclusions

In this work, we present the Feynman rules in the standard forms for all self-couplings of gauge couplings needed for studying many processes relating to gauge bosons. For example, the calculation for one-loop contributions to the $h \rightarrow Z\gamma$ amplitude in Ref. [13] assumes a relation that $-g_{AZ^1 g_1 g_2} = g_{Ag_1 g_1} g_{Z^1 g_1 g_2} = g_{Ag_2 g_2} g_{Z^1 g_1 g_2}$ in order to cancel divergences in contributions from gauge boson exchanges. The results mentioned here confirm this relation for couplings with $g_{Ag_1 g_1} = g_{Ag_2 g_2} = -eQ_{g_1} = -eQ_{g_2}$ for all $g_1, g_2 = W^+, Y^A, V^B$. We also confirm a property that all triple photon couplings always contain two identical particles. Finally, the detailed calculations presented in this work are useful for readers to cross-check our results. This can be applied to determine the gauge boson couplings of many models beyond the SM with large electroweak gauge groups that predict more complicated gauge boson spectra.

Table 2: Feynman rules for quartic gauge couplings in the 331_β model with $G, G_1, G_2 = W, Y, V$, including the case $q_1 = q_2$, equivalently $G^{q_1} \equiv G^{q_2}$

$G_1^\mu G_2^\nu G_3^\alpha G_4^\beta$	$ig_{G_{1234}}$
$G^{q_1} G^{q_2} G^{-q_1} G^{-q_2}$	$ig^2 S_{\mu\nu,\alpha\beta} \times \frac{1+\delta_{q_1 q_2}}{2}$
$AAG^q G^{-q}$	$-ie^2 q^2 S_{\mu\nu,\alpha\beta}$
$\{Z^1 Z^1, Z^2 Z^2\} W^+ W^-$	$\{-ig^2 c_W^2 c_\xi^2, -ig^2 c_W^2 s_\xi^2\} S_{\mu\nu,\alpha\beta}$
$\{AZ^1, AZ^2\} W^+ W^-$	$\{-igec_W c_\xi, igec_W s_\xi\} S_{\mu\nu,\alpha\beta}$
$Z^1 Z^2 W^+ W^-$	$ig^2 c_W^2 c_\xi s_\xi S_{\mu\nu,\alpha\beta}$
$\{Z^1 Z^1, Z^2 Z^2\} Y^A Y^{-A}$	$-\frac{ig^2 c_\xi^2}{4} S_{\mu\nu,\alpha\beta} \times \left\{ [-\sqrt{3}s_{331}t_\xi + a_1]^2, [\sqrt{3}s_{331} + t_\xi a_1]^2 \right\}$
$AZ^1 Y^A Y^{-A}$	$-\frac{iegAc_\xi}{2} [-\sqrt{3}s_{331}t_\xi + a_1] S_{\mu\nu,\alpha\beta}$
$AZ^2 Y^A Y^{-A}$	$\frac{iegBc_\xi}{2} [\sqrt{3}s_{331} + t_\xi a_1] S_{\mu\nu,\alpha\beta}$
$Z^1 Z^2 Y^A Y^{-A}$	$\frac{ig^2 c_\xi^2}{4} [-\sqrt{3}s_{331}t_\xi - a_1] \times [\sqrt{3}s_{331} - t_\xi a_1] S_{\mu\nu,\alpha\beta}$
$\{Z^1 Z^1, Z^2 Z^2\} V^B V^{-B}$	$-\frac{ig^2 c_\xi^2}{4} S_{\mu\nu,\alpha\beta} \times \left\{ [\sqrt{3}s_{331}t_\xi + a_2]^2, [-\sqrt{3}s_{331} + t_\xi a_2]^2 \right\}$
$AZ^1 V^B V^{-B}$	$\frac{igeAc_\xi}{2} [\sqrt{3}s_{331}t_\xi + a_2] S_{\mu\nu,\alpha\beta}$
$AZ^2 V^B V^{-B}$	$-\frac{igeBc_\xi}{2} [-\sqrt{3}s_{331} + t_\xi a_2] S_{\mu\nu,\alpha\beta}$
$Z^1 Z^2 V^B V^{-B}$	$\frac{ig^2 c_\xi}{4} [\sqrt{3}s_{331}t_\xi + a_2] [-\sqrt{3}s_{331} + t_\xi a_2] S_{\mu\nu,\alpha\beta}$
$AW^+ Y^{-A} V^B$	$\frac{ieg}{\sqrt{2}} [-g_{\mu\nu} g_{\alpha\beta} (A+B) + g_{\mu\beta} g_{\nu\alpha} (A+1) + g_{\mu\alpha} g_{\nu\beta} (B-1)]$
$Z^1 W^+ Y^{-A} V^B$	$\frac{ieg c_\xi \sqrt{3}}{\sqrt{2}} \left[g_{\mu\nu} g_{\alpha\beta} b_1 - g_{\mu\beta} g_{\nu\alpha} \left(\frac{s_{331} t_\xi}{2s_W} - b_2 \right) - g_{\mu\alpha} g_{\nu\beta} \left(\frac{s_{331} t_\xi}{2s_W} + b_3 \right) \right]$
$Z^2 W^+ Y^{-A} V^B$	$\frac{-ieg c_\xi \sqrt{3}}{\sqrt{2}} \left[g_{\mu\nu} g_{\alpha\beta} b_1 - g_{\mu\beta} g_{\nu\alpha} \sqrt{3} \left(\frac{s_{331}}{2s_W} + t_\xi b_2 \right) + g_{\mu\alpha} g_{\nu\beta} \left(\frac{s_{331}}{2s_W} - t_\xi b_3 \right) \right]$

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TÓM TẮT

HỆ SỐ TỰ TƯƠNG TÁC BOSON CHUẨN TRONG CÁC MÔ HÌNH 3-3-1

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Trong nghiên cứu này, chúng tôi trình bày các tính toán chi tiết để dẫn ra biểu thức giải tích cho các hệ số đỉnh tự tương tác bậc ba và bốn của tất cả các trường boson chuẩn được tiên đoán trong mô hình 3-3-1 tổng quát với tham số β tùy ý phân loại các mô hình 3-3-1 khác nhau. Chúng tôi chỉ ra rằng kết quả trong nghiên cứu này phù hợp với tất cả các kết quả đã được tính toán trước đây cho một số giá trị cụ thể của tham số β . Các quy tắc Feynman tương ứng được xác định, cần thiết cho các nghiên cứu tiếp theo cho các quá trình vật lý có liên quan đến các boson chuẩn này.

Từ khoá: Tương tác boson chuẩn; mở rộng chuẩn của các mô hình chuẩn; boson chuẩn ngoại lai.