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DYNAMIC INTERACTION BETWEEN THE TWO-AXLE VEHICLE AND CONTINUOUS GIRDER BRIDGE WITH CONSIDERING VEHICLE BRAKING FORCE

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Abstract. Nowadays, the structures of continuous girder bridges are becoming more and more popular with the rapid development of highway networks in many nations around the world, including Vietnam. High strength materials are commonly used to construct the bridge structures, so they are very slender and sensitive to the effects of dynamic loads, especially in the cases that vehicles run with high speed or brake suddenly on the bridges. In this paper, the author would like to introduce the study of a model of dynamic interaction between two-axle vehicle and continuous girder bridge. The model of a two-axle vehicle consists of three masses, taking into account the inertia force and friction force between the tires and the bridge surface due to vehicle braking. Vertical reaction forces of axles which change with time make bending vibration of beam increase significantly. The results of the experiment on the Hoaxuan Bridge and the analysis of the computerized model indicate that dynamic factors are substantial when vehicle brakes suddenly on bridge.

Keywords. Dynamic interaction, braking force, two-axle vehicle, continuous girder, Hoaxuan bridge.

1. INTRODUCTION

The vibration of bridges subjected to moving loads or moving masses has been a topic of interest for over a century. Interest in this problem originated in civil engineering for the design of railway tracks and bridges and in mechanical engineering for the trolleys of overhead cranes that move on their girders, as well as in machining processes. The problem arose from observation as follows: bridge structure is subjected to moving vehicles or trains, the dynamic deflection as well as the stresses could become significantly higher than those for static loads. Two early interesting contributions in this area were made by Stokes [1] and Willis [2] for the cases of a mass passing over a beam and for the analysis of trains crossing a bridge. Timoshenko [3] presented the classical solution of a beam subjected to a constant moving load. Early models adopted to simulate bridge-vehicle interaction are normally considered simply supported beams with a single, lumped load moving at constant speed along its span. These models evolve from the original work by Fryba [4] and Timoshenko et al. [5]. Warburton [6] analytically investigated the same problem and found that the maximum dynamic factor of deflection was 1.743. Esmailzadeh and Ghorashi [7] have tackled the problem of transverse vibration of simply supported beams traversed by a partially distributed moving mass. In a later study, a comprehensive investigation involving the dynamic response of a Timoshenko beam subjected to a partially distributed moving mass, in its most general form, has been carried out by Esmailzadeh and Ghorashi [8] and Wang [9]. The relationship between the bridge vibration characteristics and the vehicle speed was established. Resulting in a search for a particular speed that determines the maximum values of dynamic deflection of the bridge has been carried out by Jalili and Esmailzadeh [10].

Law and Zhu [11] have recently studied the dynamic behavior of a multi-span non-uniform continuous bridge under a moving vehicle with considering the effect of interaction between the structure, the road surface roughness. Ju and Lin [12] applied the finite element method to analyze vehicle-bridge dynamic responses due to vehicle braking with simple model.

In Vietnam, the bending vibration of continuous beams under the effect of moving objects was calculated by Do Xuan Tho [13]. Bending vibration of beam element under moving loads has been carried out by Nguyen Xuan Toan when vehicle braking forces are considered [14].

In this paper, the author studied the dynamic interaction between a two-axle vehicle and a continuous girder bridge considering the braking force. The continuous girder bridge is divided into beam elements to apply the Finite Element Method (FEM) for the vibration analysis. The model of a two-axle vehicle consists of three masses, taking into account the inertia force and friction force between the tires and the bridge surface due to vehicle braking. This model is nearly similar to the model in the reference [14]. However, the vertical displacements of the masses m, m_1, m_2 have been taken into account in the torque balance of the whole system. Numerical analysis results have been compared with the experimental testing results carried out on Hoaxuan bridge in Danang city.

2. COMPUTATIONAL MODEL AND ASSUMPTIONS

The diagram of a two-axle vehicle on the Hoaxuan Bridge in Danang city, is shown in Fig. 1. The dynamic interaction model between a two-axle vehicle and a beam element, when we consider vehicle braking forces, is shown in Fig. 2.

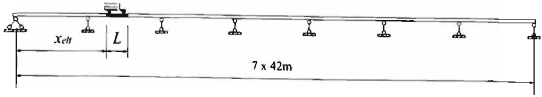

Fig. 1. Diagram of a two-axle vehicle on the Hoaxuan Bridge

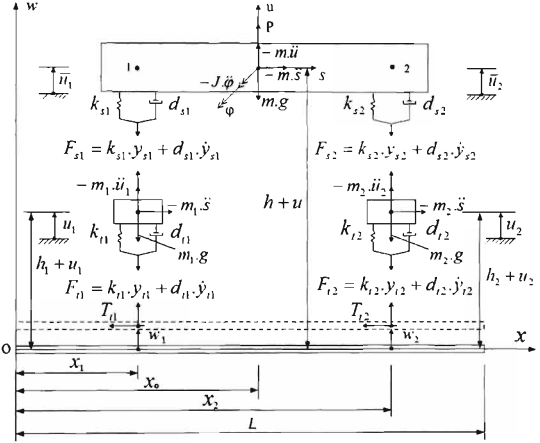

Fig. 2. Dynamic interaction model between a two-axle vehicle and a beam element

Where:

$$x_i = \begin{cases} v_i.(t - t_i) - x_{elf}; & \text{when } t_i \le t \le t_{bi} \\ v_i.(t_{bi} - t_i) + \left[\dfrac{a_i.(t - t_{bi})}{2} + v_i\right].(t - t_{bi}) - x_{elf}; & \text{when } t_{bi} < t \le t_{ei} \end{cases} \quad : 0 \le x_i \le L \tag{1}$$

L - length of the beam elements
t - time variation
x_0 - coordinate of the center of mass m at time t
x_i - coordinate of the i^{th} axes of the vehicle at time t
x_{elf} - distance between the left side of the bridge and the left side of the beam element
v_i - velocity of the i^{th} axle before a brake is used
a_i - acceleration of the i^{th} axle when a brake is used ($a_i < 0$)
t_i - time when the i^{th} axle begins entering the bridge
t_{bi} - time when a brake on the i^{th} axle is applied
t_{ei} - time when the i^{th} axle stops
$P = G.\sin(\Omega.t + \alpha)$ is the stimulation force caused by the eccentric mass of the engine
m - mass of the entire vehicle and goods, excluding the mass of the axles

m_1, m_2 - mass of the 1st, 2nd axles respectively
d_{s1}, d_{s2} - damping factors of the 1st, 2nd axle suspension respectively
d_{t1}, d_{t2} - damping factors of the 1st, 2nd tire respectively
k_{s1}, k_{s2} - spring stiffness of the 1st, 2nd axle suspension respectively
k_{t1}, k_{t2} - spring stiffness of the 1st, 2nd tire respectively
$\ddot{s}$ - acceleration of vehicle
φ - rotation angle of the chassis
u - vertical displacement of centre of the chassis
$\overline{u}_1, \overline{u}_2$ - vertical displacements of the chassis at the 1st, 2nd axles respectively
u_1, u_2 - vertical displacements of the 1st, 2nd axles respectively
y_{s1}, y_{s2} - relative displacements between the chassis and the 1st, 2nd axles respectively
y_{t1}, y_{t2} - relative displacements between the beam element and the 1st, 2nd axles respectively
h, h_1, h_2 - heights from the centre of beam to center of mass m, m_1, m_2 respectively
T_{t1}, T_{t2} - frictional forces between tire and bridge surface when the brake is used

Inertial forces, damping forces, elastic forces, stimulating forces and braking forces affecting on the system are shown in Fig. 2.

The following assumptions are adopted:

The mass of the whole vehicle, excluding the mass of the axles is transferred to the center of masses of the whole system. It is equivalent to the mass m and the rotational inertia J.

The mass of the 1st axle is m_1, which is regarded as a mass point at the center of the corresponding axle. This is the same case for the mass of 2nd axle, m_2.

The chassis is assumed to be absolutely rigid.

The materials of a beam are in the linear elastic stage. The bridge surface has the homogeneous friction coefficient over the entire bridge surface.

Brake forces of the axles of vehicle are assumed to occur simultaneously. The direction of the forces between bridge surface and tires are assumed to be in the opposite direction of the movement of a vehicle as shown in Fig. 2.

According to this assumption, the friction forces between bridge surface and tires, called T_{t1}, T_{t2}, will decelerate gradually the vehicle and produce inertia forces $-m_1 . \ddot{s}$, $-m_2 . \ddot{s}$, $-m . \ddot{s}$.

When the vehicle's brake is applied suddenly, the forces T_{t1}, T_{t2} are assumed to be directly proportional to loaded weight of vehicle

$$T_{t1} + T_{t2} = (m + m_1 + m_2).g.\tau \tag{2}$$

τ - the fiction factor between bridge surface and tires
g - the acceleration of gravity

3. BENDING VIBRATION OF BEAM ELEMENTS DUE TO BRAKING APPLIED ON A TWO-AXLE VEHICLE

Based on the calculation model and assumptions in Section 1, the system of masses m, m_1, m_2, inertial forces, damping forces, elastic forces, stimulating forces, and braking forces is taken into account. In this case, braking forces are converted to the bridge surface friction forces T_{t1}, T_{t2} as shown in Fig. 2.

Applying the principle of D'Alembert, and considering the balance of each mass m, m_1, m_2 on the vertical axis and that of the whole system on the horizontal axis, we have

$$\begin{aligned} &P - m.\ddot{u} - F_{s1} - F_{s2} - m.g = 0 \\ &F_{s1} - F_{t1} - m_1.\ddot{u}_1 - m_1.g = 0 \\ &F_{s2} - F_{t2} - m_2\,\ddot{u}_2 - m_2.g = 0 \\ &T_{t1} + T_{t2} + (m + m_1 + m_2)\,.\ddot{s} = 0 \end{aligned} \tag{3}$$

Similarly, considering the torque balance of the whole system with the 0 point, we have

$$\begin{aligned} &(P - m.\ddot{u} - m.g)\,.x_0 + m.\ddot{s}.(h+u) - J.\ddot{\varphi} - (m_1.\ddot{u}_1 + m_1.g).x_1 + m_1.\ddot{s}.(h_1 + u_1) \\ &- (m_2.\ddot{u}_2 + m_2.g).x_2 + m_2.\ddot{s}.(h_2 + u_2) + T_{t1}.w_1 + T_{t2}.w_2 - (F_{t1}.x_1 + F_{t2}.x_2) = 0 \end{aligned} \tag{4}$$

Combining (2) with (3) and (4), then having them transformed, we obtain a set of equations

$$\begin{aligned} &J.\ddot{\varphi} + [d_{s1}(x_1 - x_0)^2 + d_{s2}(x_2 - x_0)^2].\dot{\varphi} + [d_{s1}(x_1 - x_0) + d_{s2}(x_2 - x_0)].\dot{u} \\ &\quad - d_{s1}(x_1 - x_0).\dot{u}_1 - d_{s2}(x_2 - x_0).\dot{u}_2 + [k_{s1}(x_1 - x_0)^2 + k_{s2}(x_2 - x_0)^2].\varphi \\ &\quad + [k_{s1}(x_1 - x_0) + k_{s2}(x_2 - x_0) - m.\ddot{s}].u - [k_{s1}(x_1 - x_0) + m_1.\ddot{s}].u_1 \\ &\quad - [k_{s2}(x_2 - x_0) + m_2.\ddot{s}].u_2 - T_{t1}.w_1 - T_{t2}.w_2 - (m.h + m_1.h_1 + m_2.h_2).\ddot{s} = 0 \\ &m.\ddot{u} + [d_{s1}(x_1 - x_0) + d_{s2}(x_2 - x_0)].\dot{\varphi} + (d_{s1} + d_{s2}).\dot{u} - d_{s1}.\dot{u}_1 - d_{s2}.\dot{u}_2 \\ &\quad + [k_{s1}(x_1 - x_0) + k_{s2}(x_2 - x_0)].\varphi + (k_{s1} + k_{s2}).u - k_{s1}.u_1 - k_{s2}.u_2 - P + m.g = 0 \\ &m_1.\ddot{u}_1 - d_{s1}(x_1 - x_0).\dot{\varphi} - d_{s1}.\dot{u} + (d_{s1} + d_{t1}).\dot{u}_1 \\ &\quad - k_{s1}(x_1 - x_0).\varphi - k_{s1}.u + (k_{s1} + k_{t1}).u_1 - d_{t1}.\dot{w}_1 - k_{t1}.w_1 + m_1.g = 0 \\ &m_2.\ddot{u}_2 - d_{s2}(x_2 - x_0).\dot{\varphi} - d_{s2}.\dot{u} + (d_{s2} + d_{t2}).\dot{u}_2 \\ &\quad - k_{s2}(x_2 - x_0).\varphi - k_{s2}.u + (k_{s2} + k_{t2}).u_2 - d_{t2}.\dot{w}_2 - k_{t2}.w_2 + m_2.g = 0 \\ &\ddot{s} = -\,g.\tau \end{aligned} \tag{5}$$

The reactive forces F_{t1} and F_{t2} are as follows

$$\begin{aligned} F_{t1} &= -\,m_1.\ddot{u}_1 + d_{s1}(x_1 - x_0).\dot{\varphi} + d_{s1}.\dot{u} - d_{s1}.\dot{u}_1 + k_{s1}(x_1 - x_0).\varphi + k_{s1}.u - k_{s1}.u_1 - m_1.g \\ F_{t2} &= -\,m_2.\ddot{u}_2 + d_{s2}(x_2 - x_0).\dot{\varphi} + d_{s2}.\dot{u} - d_{s2}.\dot{u}_2 + k_{s2}(x_2 - x_0).\varphi + k_{s2}.u - k_{s2}.u_2 - m_2.g \end{aligned} \tag{6}$$

According to [15], the differential equations of beam element can be written as follows

$$EJ_d.\left(\frac{\partial^4 w}{\partial x^4} + \theta.\frac{\partial^5 w}{\partial x^4.\partial t}\right) + \rho F_d.\frac{\partial^2 w}{\partial t^2} + \beta.\frac{\partial w}{\partial t} = \xi(x_1).F_{t1}.\delta(x - x_1) + \xi(x_2).F_{t2}.\delta(x - x_2) \tag{7}$$

In which:

$\delta(x - x_i)$ is Dirac delta function

$$\xi(x_i) = \begin{cases} 1 & \text{when } 0 \le x_i \le L \\ 0 & \text{when } x_i < 0 \text{ or } x_i > L \end{cases}$$

is the logic control signal function

x_i - is determined by the formula (1).

w - deflection of the beam elements

EJ_d - bending stiffness of beam elements

ρF_d - mass of the beam element on a length unit

θ and β - coefficient of internal friction and coefficient of external friction

The Galerkin method and Green theory are applied to transform (5), (6) and (7) into a matrix form, and the differential equations of beam element can be written as follows

$$M_e.\ddot{q} + C_e.\dot{q} + K_e.q = f_e \tag{8}$$

$\ddot{q}, \dot{q}, q, f_e$ - mixed acceleration vector, mixed velocity vector, mixed displacement vector, mixed forces vector, respectively

$$\ddot{q} = \begin{Bmatrix} \ddot{w}_e \\ \ddot{z} \end{Bmatrix}; \dot{q} = \begin{Bmatrix} \dot{w}_e \\ \dot{z} \end{Bmatrix}; q = \begin{Bmatrix} w_e \\ z \end{Bmatrix}; f_e = \begin{Bmatrix} f_w \\ f_z \end{Bmatrix}; w_e = \begin{Bmatrix} w_1 \\ \varphi_1 \\ w_2 \\ \varphi_2 \end{Bmatrix}; z = \begin{Bmatrix} \varphi \\ u \\ u_1 \\ u_2 \end{Bmatrix}; \tag{9}$$

w_1, φ_1 - deflection and rotation angle of the left end of element

w_2, φ_2 - deflection and rotation angle of the right end of element

M_e, C_e, K_e - mass matrix, damper matrix, stiffness matrix, respectively

$$M_e = \begin{bmatrix} M_{ww} & M_{wz} \\ M_{zw} & M_{zz} \end{bmatrix}; C_e = \begin{bmatrix} C_{ww} & C_{wz} \\ C_{zw} & C_{zz} \end{bmatrix}; K_e = \begin{bmatrix} K_{ww} & K_{wz} \\ K_{zw} & K_{zz} \end{bmatrix}; \tag{10}$$

M_{ww}, C_{ww}, K_{ww} - mass matrix, damper matrix, stiffness matrix of beam elements [15].

$$M_{wz} = \begin{bmatrix} 0 & 0 & m_1.P_{11} & m_2.P_{12} \\ 0 & 0 & m_1.P_{21} & m_2.P_{22} \\ 0 & 0 & m_1.P_{31} & m_2.P_{32} \\ 0 & 0 & m_1.P_{41} & m_2.P_{42} \end{bmatrix}; f_w = -\begin{Bmatrix} g(m_1.P_{11} + m_2.P_{12}) \\ g(m_1.P_{21} + m_2.P_{22}) \\ g(m_1.P_{31} + m_2.P_{32}) \\ g(m_1.P_{41} + m_2.P_{42}) \end{Bmatrix}; \tag{11}$$

$$C_{wz} = -\begin{bmatrix} d_{s1}(x_1 - x_0)P_{11} + d_{s2}(x_2 - x_0)P_{12} & d_{s1}P_{11} + d_{s2}P_{12} & -d_{s1}P_{11} & -d_{s2}P_{12} \\ d_{s1}(x_1 - x_0)P_{21} + d_{s2}(x_2 - x_0)P_{22} & d_{s1}P_{21} + d_{s2}P_{22} & -d_{s1}P_{21} & -d_{s2}P_{22} \\ d_{s1}(x_1 - x_0)P_{31} + d_{s2}(x_2 - x_0)P_{32} & d_{s1}P_{31} + d_{s2}P_{32} & -d_{s1}P_{31} & -d_{s2}P_{32} \\ d_{s1}(x_1 - x_0)P_{41} + d_{s2}(x_2 - x_0)P_{42} & d_{s1}P_{41} + d_{s2}P_{42} & -d_{s1}P_{41} & -d_{s2}P_{42} \end{bmatrix} \tag{12}$$

$$K_{wz} = -\begin{bmatrix} k_{s1}(x_1 - x_0)P_{11} + k_{s2}(x_2 - x_0)P_{12} & k_{s1}P_{11} + k_{s2}P_{12} & -k_{s1}P_{11} & -k_{s2}P_{12} \\ k_{s1}(x_1 - x_0)P_{21} + k_{s2}(x_2 - x_0)P_{22} & k_{s1}P_{21} + k_{s2}P_{22} & -k_{s1}P_{21} & -k_{s2}P_{22} \\ k_{s1}(x_1 - x_0)P_{31} + k_{s2}(x_2 - x_0)P_{32} & k_{s1}P_{31} + k_{s2}P_{32} & -k_{s1}P_{31} & -k_{s2}P_{32} \\ k_{s1}(x_1 - x_0)P_{41} + k_{s2}(x_2 - x_0)P_{42} & k_{s1}P_{41} + k_{s2}P_{42} & -k_{s1}P_{41} & -k_{s2}P_{42} \end{bmatrix} \tag{13}$$

$$M_{zz} = \begin{bmatrix} J & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix}; \quad f_z = \begin{Bmatrix} (m.h + m_1.h_1 + m_2.h_2).\ddot{s} \\ P - m.g \\ -m_1.g \\ -m_2.g \end{Bmatrix}; \tag{14}$$

$$C_{zz} = \begin{bmatrix} d_{s1}(x_1 - x_0)^2 + d_{s2}(x_2 - x_0)^2 & d_{s1}(x_1 - x_0) + d_{s2}(x_2 - x_0) & -d_{s1}(x_1 - x_0) & -d_{s2}(x_2 - x_0) \\ d_{s1}(x_1 - x_0) + d_{s2}(x_2 - x_0) & d_{s1} + d_{s2} & -d_{s1} & -d_{s2} \\ -d_{s1}(x_1 - x_0) & -d_{s1} & d_{s1} + d_{t1} & 0 \\ -d_{s2}(x_2 - x_0) & -d_{s2} & 0 & d_{s2} + d_{t2} \end{bmatrix} \tag{15}$$

$$K_{zz} = \begin{bmatrix} k_{s1}(x_1 - x_0)^2 + k_{s2}(x_2 - x_0)^2 & k_{s1}(x_1 - x_0) + k_{s2}(x_2 - x_0) - m\ddot{s} & -k_{s1}(x_1 - x_0) - m_1\ddot{s} & -k_{s2}(x_2 - x_0) - m_2\ddot{s} \\ k_{s1}(x_1 - x_0) + k_{s2}(x_2 - x_0) & k_{s1} + k_{s2} & -k_{s1} & -k_{s2} \\ -k_{s1}(x_1 - x_0) & -k_{s1} & k_{s1} + k_{t1} & 0 \\ -k_{s2}(x_2 - x_0) & -k_{s2} & 0 & k_{s2} + k_{t2} \end{bmatrix} \tag{16}$$

$$M_{zw} = 0; \quad C_{zw} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -d_{t1}.P_{11} & -d_{t1}.P_{21} & -d_{t1}.P_{31} & -d_{t1}.P_{41} \\ -d_{t2}.P_{12} & -d_{t2}.P_{22} & -d_{t2}.P_{32} & -d_{t2}.P_{42} \end{bmatrix} \tag{17}$$

$$K_{zu} = -\begin{bmatrix} T_{t1}.P_{11} + T_{t2}.P_{12} & T_{t1}.P_{21} + T_{t2}.P_{22} & T_{t1}.P_{31} + T_{t2}.P_{32} & T_{t1}.P_{41} + T_{t2}.P_{42} \\ 0 & 0 & 0 & 0 \\ d_{t1}.\dot{P}_{11} + k_{t1}.P_{11} & d_{t1}.\dot{P}_{21} + k_{t1}.P_{21} & d_{t1}.\dot{P}_{31} + k_{t1}.P_{31} & d_{t1}.\dot{P}_{41} + k_{t1}.P_{41} \\ d_{t2}.\dot{P}_{12} + k_{t2}.P_{12} & d_{t2}.\dot{P}_{22} + k_{t2}.P_{22} & d_{t2}.\dot{P}_{32} + k_{t2}.P_{32} & d_{t2}.\dot{P}_{42} + k_{t2}.P_{42} \end{bmatrix} \tag{18}$$

In which

$$\begin{aligned} P_{1i} &= \frac{\xi(x_i)}{L^3}.(L + 2x_i)(L - x_i)^2; & P_{2i} &= \frac{\xi(x_i)}{L^2}.x_i(L - x_i)^2; \\ P_{3i} &= \frac{\xi(x_i)}{L^3}.x_i^2(3L - 2x_i); & P_{4i} &= \frac{\xi(x_i)}{L^2}.x_i^2(x_i - L); \end{aligned} \tag{19}$$

4. APPLYING FEM TO ANALYSE THE VIBRATION OF THE HOAXUAN BRIDGE IN DANANG CITY

The Hoaxuan Bridge is a continuous girder bridge which has 7 spans shown in Fig. 1. It is divided into many beam elements shown in Fig. 2. Applying the FEM and the algorithm of the FEM [16], we have the vibration differential equation for the whole system as in (20)

$$M.\ddot{Q} + C.\dot{Q} + K.Q = F \tag{20}$$

In which:

M, C, K - are the mass matrix, the damper matrix, and the stiffness matrix of the whole system.

$\ddot{Q}, \dot{Q}, Q, F$ - are the acceleration vector, the velocity vector, the deflection vector, and the force vector of the whole system.

After imposing boundary and initial conditions on (20), we can solve this equation by the Runge-Kutta-Merson method. The numerical values of the parameters were used in the computer simulation and the field test as follows:

Hoaxuan Bridge

$L_b = 7 \times 42$ m, $E = 3230769.23$ T/m^2, $J_d = 0.6879$ m^4, $F_d = 1.3776$ m^2, $\rho F_d = 3.8$ T/m, $\theta = 0.027$, $\beta = 0.01$, $\tau = 0.25$, $g = 9.81$ m/s^2.

Two-axle vehicle

$J = 13.8$ Tm^2, $m = 10.5$ T, $m_1 = 0.055$ T, $m_2 = 0.107$ T, $P = 0$, $b_1 = x_1 - x_0 = 2.3$ m, $b_2 = x_0 - x_2 = 1.21$ m, $h = 1.1$ m, $h_1 = h_2 = 0.5$ m, $k_{1s} = 115$ T/m, $k_{1t} = 140$ T/m, $k_{2s} = 220$ T/m, $k_{2t} = 280$ T/m, $d_{1s} = 0.73$ Ts/m, $d_{1t} = 0.4$ Ts/m, $d_{2s} = 0.4$ Ts/m, $d_{2t} = 0.8$ Ts/m.

According to the FEM results, the deflections of the Hoaxuan Bridge caused by a two-axle vehicle which is running at 30 km/h on the first span and its brakes are suddenly used are shown in Figs. 3-6. According to the experimental results, the first span deflections of the Hoaxuan Bridge are shown in Figs. 7-10.

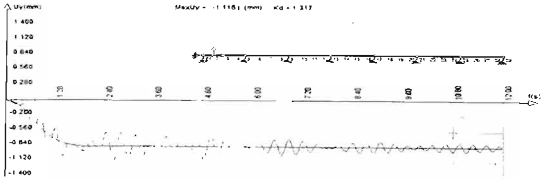

Fig. 3. The quarter-span deflection caused by the braking of the vehicle at the quarter-span position

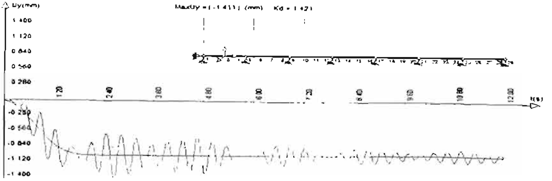

Fig. 4. The mid-span deflection caused by the braking of the vehicle at the quarter-span position

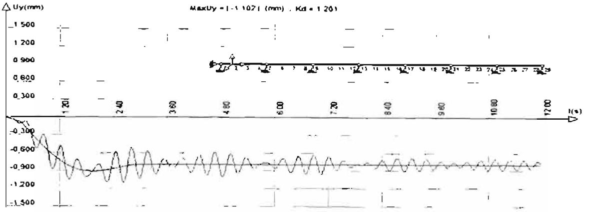

Fig. 5. The quarter-span deflection caused by the braking of the vehicle at the mid-span position

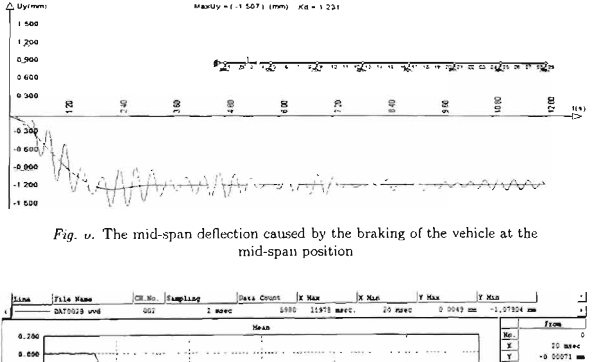

Fig. 6. The mid-span deflection caused by the braking of the vehicle at the mid-span position

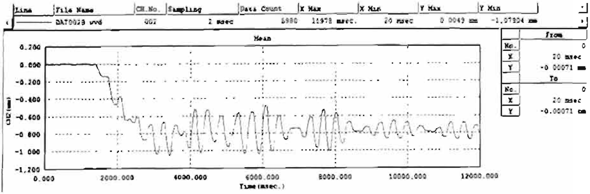

Fig. 7. The quarter-span deflection caused by the braking of the vehicle at the quarter-span position

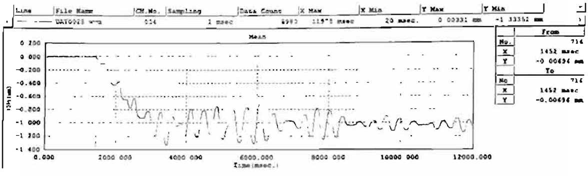

Fig. 8. The mid-span deflection caused by the braking of the vehicle at the quarter-span position

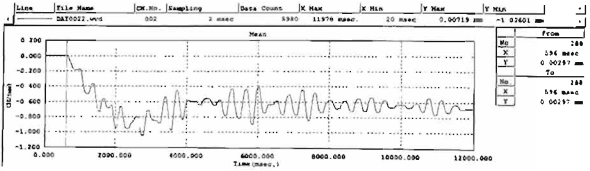

Fig. 9 The quarter-span deflection caused by the braking of the vehicle at the mid-span position

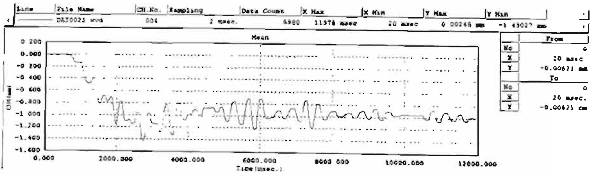

Fig. 10. The mid-span deflection caused by the braking of the vehicle at the mid-span position

The results of the maximum dynamic deflections and the dynamic factor of the Hoaxuan Bridge caused by a two-axle vehicle which is running at 30 km/h on the first span and its brakes are suddenly used are shown in Tab. 1.

In which: $K_D = \dfrac{U_D}{U_S}$

K_D - dynamic factor of deflection

U_D - maximum dynamic deflection of beam

U_S - maximum static deflection of beam
ΔU_D - difference of U_D between from FEM and experiment
ΔK_D - difference of K_D between from FEM and experiment

Table 1. The results of the deflections of the Hoaxuan Bridge caused by a two-axle vehicle

Positions of deflections	Braking positions	The FEM results		The experimental results		The difference between from FEM and experiment	
		U_D	K_D	U_D	K_D	ΔU_D	ΔK_D
Quarter-span	*Quarter-span*	1.118 mm	1.317	1.079 mm	1.348	3.6 %	-2.3%
Mid-span	*Quarter-span*	1.411 mm	1.421	1.333 mm	1.334	5.8 %	6.5%
Quarter-span	*Mid-span*	1.102 mm	1.201	1.026 mm	1.140	7.4 %	5.4%
Mid-span	*Mid-span*	1.507 mm	1.231	1.430 mm	1.197	5.4 %	2.8%

In the scope of the study, the FEM results were compared with the experiment ones in Tab. 1. As for the maximum dynamic deflection of beam, the difference between the FEM results and the experimental results are $3.6 \div 7.4\%$ while as for the dynamic factor of deflection, it is $2.3 \div 6.5\%$. The FEM results are suitable when compared with the experimental results. The maximum dynamic factor of deflection is 1.421 according to the FEM results and it is 1.348 according to the experimental results. The FEM and experimental results of the maximum dynamic factor of deflection are significant.

5. CONCLUSIONS

This paper introduces the results of research on a dynamic interaction model between a two-axle vehicle and a continuous girder bridge when braking forces are taken into account. The FEM has been applied to analyse the vibration of Hoaxuan bridge caused by a brake suddenly used on a two-axle vehicle. The analysis results were tested by the experiments. The FEM results are suitable when compared with the experimental results. The FEM and experimental results of the dynamic factor of the Hoaxuan Bridge are significantly. Therefore, the author recommends that when engineers design bridges, they should take into account the dynamic interaction caused by the fact that the brake of the vehicle is suddenly applied on the bridge.

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