Vietnam Journal of Mechanics, VAST, Vol. 37, No. 1 (2015), pp. 17 – 27
DOI:10.15625/0866-7136/37/1/3923

A FUZZY FINITE ELEMENT ALGORITHM BASED ON RESPONSE SURFACE METHOD FOR FREE VIBRATION ANALYSIS OF STRUCTURE

Nguyen Hung Tuan[1,*], **Le Xuan Huynh**[2], **Pham Hoang Anh**[2]
[1]*Hanoi Community College, Vietnam*
[2]*National University of Civil Engineering, Hanoi, Vietnam*
*E-mail: hungtuan161@gmail.com
Received April 22, 2014

Abstract. This paper introduces an improved response surface-based fuzzy finite element analysis of structural dynamics. The free vibration of structure is established using superposition method, so that fuzzy displacement responses can be presented as functions of fuzzy mode shapes and fuzzy natural frequencies Instead of direct determination of these fuzzy quantities by modal analysis which will involve the calculation of the whole finite element model, the paper proposes a felicitous approach to design the response surface as surrogate model for the problem. In the design of the surrogate model, complete quadratic polynomials are selected with all fuzzy variables are transformed to standardized fuzzy variables. This methodology allows accurate determination of the fuzzy dynamic outputs, which is the major issue in response surface based techniques. The effectiveness of the proposed fuzzy finite element algorithm is illustrated through a numerical analysis of a linear two-storey shear frame structure.

Keywords: Fuzzy finite element method, response surface method, surrogate model, free vibration analysis.

1. INTRODUCTION

Fuzzy finite element method (FFEM) is an extension of finite element method (FEM) in the case that the input quantities in the finite element model such as load, material properties, stiffness of supports, connections, contain incomplete, none-clear parameters, which are described in the form of fuzzy numbers. In recent years, there is an increasing focus on the development of FFEM, first for static analysis and then extended in dynamics [1–9]. Fundamental strategies for FFEM can be categorized into two main groups: the *interval arithmetic approach* and the *optimization strategy*. An overview of these methodologies can be found in [10].

In the first approach applied for dynamic analysis of structures, [1–3] extended the frequency-response method to determine fuzzy frequency response function FRF with uncertain input parameters described in the form of fuzzy numbers. Firstly, an optimization was used to determine normalized modal stiffness and modal mass. Then, FRF was calculated from these modal parameters using interval arithmetic. To reduce the expansion of the fuzzy output FRF, the authors proposed the Modal Rectangle Method with Eigenvalue interval (MRE) instead of the Modal Rectangle Method (MR). As common problem in *interval arithmetic approach*, the uncertain parameter decoupling (here are the normalized modal stiffness and mass parameters) will take place in the calculation of the FRF. Hence, the obtained FRF can derivate from the expected realistic fuzzy FRF. In general, the interval arithmetic approach introduces a very high amount of conservatism into the analysis [10].

The optimization approach has overcome the shortcoming of the interval arithmetic approach by performing a search algorithm inside the domain defined by the interval inputs to determine lower and upper bounds of the outputs. This means that it does not guarantee conservatism, unless the actual bounds on the goal function are found [10]. Because of this characteristic, the optimization strategy is gradually acknowledged as the standard procedure for FFEM. In optimization strategy, the search process is performed on the input domain to seek the exact bounds of the objective function by iteratively evaluating the objective function at designated points [7–9]. Often, the search process is very time consuming because finite element analysis has to be carried out for every evaluation. To overcome this limitation researchers have tried to develop other methods which can yield faster results. One of the most common methods is the *Response Surface Method* (RSM). In this method, the goal function of the optimization problem is approximated by a known function chosen appropriately. The optimization is then performed on this surrogate response function. The advantage of this method is the reduction of time in each iterative evaluation. The most importance aspect is how appropriate the response surface is, since the accuracy of the method depends on the exactness of the approximate response function. RSM has been developed for structural dynamics in some researches [5, 6]. In those works, all original fuzzy variables are presented in the surrogate model. This can lead to errors due to round-off in calculating regression coefficients when the fuzzy variables have very different domains. Besides that, co-linearity can occur as the fuzzy variables are correlated to each other. With additional experiments to test the bias error between surrogate model outputs and the exact outputs, [5] has used split sample model to calculate regression coefficients. It can be seen that the accuracy of a surrogate response function relies on factors such as selection of experiments and regression model, the variables in the regression model, estimation of errors and determination of regression coefficients from experiment designs.

The present work deals with the usage of RSM for fuzzy free vibration analysis of linear elastic structure. The free vibration of structure is established using superposition method, so that displacement responses can be presented as function of modal quantities (mode shapes and natural frequencies). Response surfaces (surrogate functions) in terms of complete quadratic polynomials are presented for modal quantities, in which all fuzzy variables are standardized. With suitable design of experiments and selection of coefficients based on least error estimation, accurate surrogate regression model can be obtained. This introduced method is tested with a two-storey frame structure.

2. FREE VIBRATION OF STRUCTURE

Consider the free vibration of a linear structure governed by the equation of motion

$$[M]\ \{\ddot{u}\} + [C]\ \{\dot{u}\} + [K]\ \{u\} = \{0\}, \tag{1}$$

in which $[M], [K]$ denote the mass matrix and stiffness matrix, respectively; $[C]$ is the damping matrix, assumed with proportional damping model

$$[C] = \alpha[M] + \beta[K], \tag{2}$$

and $\{u\}$ is the displacement response. Initial conditions: at time $t = 0$

$$\{u\} = \{u_0\} \quad \text{and} \quad \{\dot{u}\} = \{\dot{u}_0\}. \tag{3}$$

Basically, solving the free vibration involves two steps

- The first step: Determine modal frequencies ω_i, and modal mode shape $[\Phi]$ through modal analysis.

- The second step: use modal superposition to calculate the displacement solution of Eq. (1).

Set $\{u\} = [\Phi]\{v\}$ and apply the orthogonality of normal modes, Eq. (1) is transformed into

$$\{\ddot{v}\} + \left(\alpha\,[I] + \beta\left[\omega^2\right]\right)\ \{\dot{v}\} + \left[\omega^2\right]\ \{v\} = \{0\}, \tag{4}$$

where $\left[\omega^2\right]$ is the diagonal matrix with the elements at diagonal are ω_i^2.

The i^{th} solution of Eq. (4) takes the form

$$v_i = e^{-\xi_i\omega_i t}\left(A_i \sin\left(\sqrt{1-\xi_i^2}\,\omega_i t\right) + B_i \cos\left(\sqrt{1-\xi_i^2}\,\omega_i t\right)\right), \tag{5}$$

where ξ_i is damping ratio of i-th normal mode; A_i, B_i values are determined by the initial conditions (3).

Rewriting Eq. (5) in a matrix form

$$\{v\} = [A][E_{st}] + [B][E_{ct}], \tag{6}$$

where

$$[A] = \begin{bmatrix} A_1 & 0 & \dots & 0 \\ 0 & A_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & A_n \end{bmatrix}, \quad [B] = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & B_2 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & B_n \end{bmatrix}, \tag{7}$$

$$[E_{st}] = \begin{bmatrix} e^{-\xi_1\omega_1 t}\sin\left(\sqrt{1-\xi_1^2}\,\omega_1 t\right) \\ e^{-\xi_2\omega_2 t}\sin\left(\sqrt{1-\xi_2^2}\,\omega_2 t\right) \\ \dots \\ e^{-\xi_n\omega_n t}\sin\left(\sqrt{1-\xi_n^2}\,\omega_n t\right) \end{bmatrix}, \quad [E_{ct}] = \begin{bmatrix} e^{-\xi_1\omega_1 t}\cos\left(\sqrt{1-\xi_1^2}\,\omega_1 t\right) \\ e^{-\xi_2\omega_2 t}\cos\left(\sqrt{1-\xi_2^2}\,\omega_2 t\right) \\ \dots \\ e^{-\xi_n\omega_n t}\cos\left(\sqrt{1-\xi_n^2}\,\omega_n t\right) \end{bmatrix}. \tag{8}$$

Hence, solution of (1) is

$$\{u\} = [\Phi]\{v\} = [A_s][E_{st}] + [A_c][E_{ct}], \tag{9}$$

where $[A_s] = [\Phi][A], [A_c] = [\Phi][B]$.

When uncertainty is present in structural parameters such as the material properties and dimensions and given by fuzzy numbers, the mode shape $[\Phi]$ and the natural (modal) frequencies ω_i are also fuzzy, thus displacement response is also fuzzy. To obtain these fuzzy quantities by optimization strategy, modal analysis needs to be performed at each iteration point in the search domain of the inputs. This process is usually very costly. The next sections present a felicitous approach to design suitable surrogate model for the solution of modal mode shapes and frequencies in order to reduce time consume in each evaluation.

3. PROPOSED RESPONSE SURFACE METHODOLOGY

3.1. Surrogate function with standardized fuzzy variables

In the statistical theory, surrogate models are often used including polynomial regression model, Kringing model, radial basis function [11]. The first and the second models are parametric model, based on assumed functional form of the response in terms of the design variables; the last model (radial basis function) is non-parametric that uses different types of local models in different regions of the data to build up an overall model. Among these models, polynomial regression model is often used to build a response surface function due to its calculation simplicity. In this paper, *a complete quadratic polynomial regression model* is used as surrogate model, in which all variables are standardized and assumed to be uncorrelated

$$y(X) = a_0 + \sum_{i=1}^{n} a_i X_i + \sum_{i=1, i<j}^{n-1} a_{ij} X_i X_j + \sum_{i=1}^{n} a_{ii} X_i^2, \tag{10}$$

with X_i are the standardized fuzzy variables, $a_0 = y(\mathbf{X} = 0)$, and a_i, a_{ij} are the unknown coefficients which will be determined by the method of least squares. In the proposed algorithm, $y(\mathbf{X})$ represents the surrogate functions of modal frequencies ω_i and the terms of the matrices $[A_s]$ and $[A_c]$.

The original fuzzy variables may have very different domains, which can lead to round-off errors in calculating regression coefficients. Using standardized variables instead of original fuzzy variables in the surrogate model will ensure the accuracy in calculating the regression coefficients [12]. For the present analysis, we assume uncertain structural parameters as symmetric triangular fuzzy numbers, $x_i = (a, l, l)_{LR}$. The standardized fuzzy variables X_i is defined as

$$X_i = \frac{x_i - a}{l}. \tag{11}$$

With the above definition, we transformed from original fuzzy variables $x_i = (a, l, l)_{LR}$ to standardized fuzzy variables $X_i = (0, 1, 1)_{LR}$.

3.2. Design for response experiments

To complete the surrogate polynomial functions of Eq. (10), all coefficients a_i, a_{ij} shall be determined by a fitting procedure, which minimizes the difference (error) between the outputs of surrogate function and the outputs of actual finite element model. Normally, some experiments with deterministic input data are carried out and the *best-fitting* function can be obtained by minimize the sum of the square errors from the given output data.

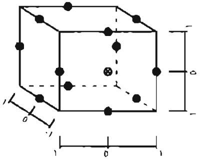

Fig. 1. The Box-Behnken design with three variables [11]

In RSM, three designs for response experiments are often applied in practice: (1) Latin hypercube sampling, (2) the face-centered cube design, (3) the Box-Behnken design. With the number of experiments not too large, and in fact, maximum, minimum responses usually occur on the surface of the cube, the face-centered cube design and the Box-Behnken designs are often used. According to [11], both designs have good results for actual problems. However, with the same number of input variables, the Box-Behnken design usually requires fewer response points than the face-centered cube design. Therefore, the Box-Behnken design is proposed for the present analysis. In the Box-Behnken design, the design points are at the center or the midpoints of the edge of the cube. Illustration of the Box-Behnken design with three input variables is shown in Fig. 1.

3.3. Error estimation

Error estimation assesses the quality of the response surface and is used to select the suitable design. The most prominent methods are split sample, cross-validation and bootstrapping, in which the split sample and the cross-validation appear easy to use. The drawbacks of the split sample method are that the error estimation can have a high variance and the amount of data used to construct the surrogate model is limited (because of separating between the training and the test sets). The cross-validation method on the other hand calculates and evaluates the errors of several models in the pre-determined data sets without distinction between the training and the test sets. The advantage of this method according to [12] is that it provides nearly unbiased estimation of the generalization error and the corresponding variance is reduced when compared to the split sample method. The disadvantage of this method is the requirement of many calculations of surrogate models. One can use automatic programming to select the combinations of experiments from pre-determining experiments to build the regression model. In this study, we apply the leave-one-out cross-validation, where each response point is tested once and trained $k-2$ times, since the center point has been used to determine a_0. The error estimation of j^{th} design (using $\mathbf{X}^{(j)}$ as the test set) is determined by the formula

$$GSE_j = \left(y_j - \hat{y}_j^{(-j)}\right)^2 \to \min \tag{12}$$

where GSE_j - the square error of j^{th} design; y_j - output value at $\mathbf{X}^{(j)}$, determined by classical FEM; $\hat{y}_j^{(-j)}$ - estimated value at $\mathbf{X}^{(j)}$ of j^{th} design.

4. A NUMERICAL ILLUSTRATION

Consider a two-storey shear frame structural system in Fig. 2. The elastic modulus $\tilde{E}$, the inertia moment of column $\tilde{I}_c$, storey height $\tilde{l}$, the mass $\tilde{M}_1 = \tilde{M}_2 = \tilde{M}$, the initial velocity at $t_k = 0$, are assumed as symmetric triangular fuzzy numbers given as following:

$\tilde{E} = (3, 0.3, 0.3)10^7\,\text{kN/m}^2$; $\tilde{I}_c = (0.665, 0.0665, 0.0665)10^{-4}\,\text{m}^2$; $\tilde{l} = (2.8, 0.2, 0.2)$ m; $\tilde{M} = (2, 0.2, 0.2)$ T; $\tilde{\dot{u}}_1(0) = (1, 0.2, 0.2)$ m/s ; $\tilde{\dot{u}}_2(0) = (1.5, 0.3, 0.3)$ m/s

It is clear that the fuzzy parameters span in very different domains.

The proportional damping is set as $[\tilde{M}] = \alpha[\tilde{M}] + \beta[\tilde{K}], \alpha = 0.01, \beta = 0.005$. The initial displacements are $u_1(0) = u_2(0) = 0$.

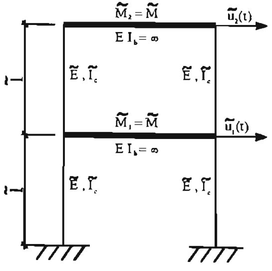

Fig. 2. The two-storey structure layout

4.1. Determination of fuzzy displacement outputs

In fuzzy free vibration analysis, it is desirable to determine the envelopes of fuzzy displacement response including:

- Upper bounds of the fuzzy displacements $u_{i,0\max}$: is the set of largest displacement values at membership level $\alpha = 0$.

- Lower bounds of fuzzy displacements $u_{i,0\min}$: is the sets of minimum displacement values at membership level $\alpha = 0$.

The upper and lower bound of the fuzzy displacement response can be determined by applying an optimization technique to the displacements given by Eqs. (8) and (9). For the studied problem, Genetic algorithm (GA) [13–15] is applied using the built-in functions in MATLAB.

Besides defining the envelopes, membership function of fuzzy displacement outputs at a predefined time t_k can be approximated by evaluating the displacement bounds at time t_k for different membership level α of the fuzzy inputs. In this procedure, GA is also applied in searching the output bounds for each α-cut of all input quantities.

4.2. Analysis results

The structure has two mode shapes and two frequencies. The response surface functions for the modal quantities are determined by the procedure presented in section 3. Tab. 1 shows the results of the coefficients of the surrogate functions for the natural frequencies.

Table 1. Coefficients of surrogate functions for the natural frequencies

Coefficients	1st natural frequency	2nd natural frequency
a_0	20.4086	53.4323
a_1	0.34193	0.89524
a_2	0.33424	0.87507
a_3	−0.73466	-1.92343
a_4	−0.34311	−0.89829
a_{12}	0.00558	0.01459
a_{13}	−0.01226	−0.03208
a_{14}	−0.00571	−0.01495
a_{23}	−0.00137	−0.00359
a_{24}	−0.0056	−0.0146
a_{34}	0.0123	0.0322
a_{11}	−0.00287	−0.00751
a_{22}	−0.00137	−0.00359
a_{33}	0.02186	0.05721
a_{44}	0.00860	0.02251

On the other hand, the exact modal frequencies and mode shapes obtained by symbolic simplification solution are

$$\omega_1 = 0.618\sqrt{\frac{k}{M}},\quad \omega_2 = 1.618\sqrt{\frac{k}{M}},\quad [\Phi] = \frac{1}{\sqrt{M}}\begin{bmatrix} 0.526 & 0.85 \\ 0.85 & -0.526 \end{bmatrix},\quad k = \frac{24EI_c}{l^3}. \tag{13}$$

To assess the accuracy of the proposed response surface, differences between the approximated results and the results from symbolic simplification (exact analysis) are evaluated, including:

- *Interval width error:*

$$I_e = \left| \frac{w_{th}\left(\tilde{u}_i^{RSM}(t)\right) - w_{th}\left(\tilde{u}_i^{a}(t)\right)}{w_{th}\left(\tilde{u}_i^{a}(t)\right)} \right| .100\%, \tag{14}$$

where I_e - the error of the width of an interval; $w_{th}\left(\tilde{u}_i^{RSM}(t)\right)$ - interval width of fuzzy displacement u_i at time t determined by the proposed RSM at $\alpha = 0$; $w_{th}\left(\tilde{u}_i^{a}(t)\right)$ - interval width of fuzzy displacement u_i at time t determined by symbolic simplification at $\alpha = 0$.

- *Bound error:*

$$A_{e,\max(\min)} = \left| \frac{u^{RSM}_{i,\alpha\max(\min)} - u^{a}_{i,\alpha\max(\min)}}{u^{a}_{i,\alpha\max(\min)}} \right| .100\%, \tag{15}$$

where $A_{e,\max(\min)}$ - the error of upper bound (lower bound) of fuzzy displacement; $u^{RSM}_{i,\alpha\max(\min)}$ - upper bound (lower bound) of fuzzy displacement u_i determined by the proposed RSM at α-cut; $u^{a}_{i,\alpha\max(\min)}$ - upper bound (lower bound) of fuzzy displacement u_i determined by symbolic simplification at α-cut.

Error of time corresponded to $\inf(u_{i,0\min})$, $\sup(u_{i,0\max})$

$$T_e = \left| \frac{t^{RSM} - t^{a}}{t^{a}} \right| .100\%, \tag{16}$$

where T_e - the error of the time corresponded to $\inf(u_{i,0\min})$, $\sup(u_{i,0\max})$; t^{RSM} - the time reaches $\inf(u_{i,0\min})$, $\sup(u_{i,0\max})$ according to the proposed RSM; t^a - the time reaches $\inf(u_{i,0\min})$, $\sup(u_{i,0\max})$ according to the symbolic simplification.

For both cases of analysis, the envelopes of displacement responses as well as membership functions of $u_1(t)$ and $u_2(t)$ are obtained using GA in MATLAB 7.12 with pop-size = 50, probability of crossover $p_c = 0.9$, probability of mutation $p_m = 0.05$. The lower and upper bounds of fuzzy displacement outputs $u_1(t)$ and $u_2(t)$ are determined at different times in the interval [0, 0.5s] with a time step $\Delta t = 0.02$s. Fig. 3 shows the comparison of displacement bounds for $u_1(t)$ and $u_2(t)$. Fig. 4 plots the membership functions of u_1 at t=0.095s and u_2 at $t = 0.097$s. In addition, the minimum values of lower bounds $\inf(u_{i,0\min})$ and the maximum values of upper bounds $\sup(u_{i,0\max})$ with the corresponding times are shown in Tab. 2 and Tab. 3.

Table 2. The values of $\sup(u_{1,0\max})$, $\inf(u_{1,0\min})$ and the corresponding times

No	The proposed algorithm			The symbolic simplification			T_e (%)	$A_{e,\min}$ (%)	$A_{e,\max}$ (%)
	Time t(s)	inf $(u_{1,0\min})$	sup $(u_{1,0\max})$	Time t(s)	Inf $(u_{1,0\min})$	Sup $(u_{1,0\max})$			
1	0.295	−0.0582		0.295	−0.0598		0.00	2.67	2.84
2	0.095		0.0650	0.095		0.0669	0.00		

Table 3. The values of $\sup(u_{2,0\max})$, $\inf(u_{2,0\min})$ and the corresponding times

No	The proposed algorithm			The symbolic simplification			T_e (%)	$A_{e,\min}$ (%)	$A_{e,\max}$ (%)
	Time t(s)	inf $(u_{1,0\min})$	sup $(u_{1,0\max})$	Time t(s)	Inf $(u_{1,0\min})$	Sup $(u_{1,0\max})$			
1	0.297	−0.0932		0.296	−0.0963		0.28	3.24	3.19
2	0.098		0.1062	0.097		0.1097	0.32		

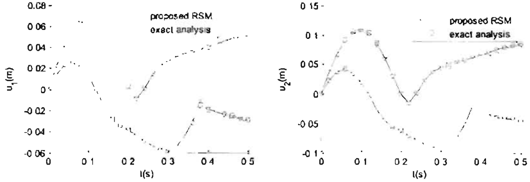

Fig. 3. Envelopes of fuzzy displacement $u_1(t)$ and $u_2(t)$

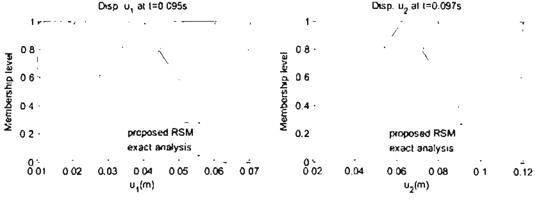

Fig. 4. Fuzzy displacements u_1 at $t = 0.095$s and u_2 at $t = 0.097$s

It is found from the analysis results that:

- The proposed RSM produces results closed to the results of exact analysis for both the displacement bounds and the time reaching extreme values (see Tabs. 2 and 3, Figs. 3 and 4), and has only small error in comparison with exact solution (the average values of $A_{e,\max(\min)}$ and I_e are less than 5%).

- The largest error I_e is 22.37% (at $t = 0.02$ s for u_2). Error I_e in this case is large because the interval width is small, so only small error in $u_{2,0\min}$ and $u_{2,0\max}$ can leads to large error I_e. Nevertheless, the *Bound error* is still small (A_e values are 2.51% and 6.56%, for $u_{2,0\min}$ and $u_{2,0\max}$, respectively), thus ensuring the accuracy of the calculated quantities.

- The largest error $A_{e,\min}$ is 34.46% (at $t = 0.12$ s for free modal u_1), but the absolute difference in $u_{1,0\min}$ is small (the difference is 5.10^{-4}).

These results imply that the complete quadratic polynomials with the standardized fuzzy variables are accurate approximation models for the response surface of the modal quantities of the presented problem.

In contrast, the approximation models using quadratic polynomials of the original fuzzy variables result much larger error, as shown in Tab. 4 and Tab. 5.

Table 4. The values $\sup(u_{1,0\max})$, $\inf(u_{1,0\min})$ using the original fuzzy variables

No	The proposed algorithm			The symbolic simplification			T_e (%)	$A_{e,\min}$ (%)	$A_{e,\max}$ (%)
	Time t(s)	inf ($u_{1,0\min}$)	sup ($u_{1,0\max}$)	Time t(s)	Inf ($u_{1,0\min}$)	Sup ($u_{1,0\max}$)			
1	0.351	−0.1450		0.295	−0.0598		19.06	142.35	136.51
2	0.063		0.1582	0.095		0.0669	34.05		

Table 5. The values $\sup(u_{2,0\max})$, $\inf(u_{2,0\min})$ using the original fuzzy variables

No	The proposed algorithm			The symbolic simplification			T_e (%)	$A_{e,\min}$ (%)	$A_{e,\max}$ (%)
	Time t(s)	inf ($u_{1,0\min}$)	sup ($u_{1,0\max}$)	Time t(s)	Inf ($u_{1,0\min}$)	Sup ($u_{1,0\max}$)			
1	0.352	−0.1682		0.296	−0.0963		18.84	74.59	61.62
2	0.066		0.1773	0.097		0.1097	32.25		

5. CONCLUSION

This paper introduces a response surface-based algorithm which is applicable for fuzzy finite element dynamic analysis of linear structure. By using the complete quadratic polynomials with the standardized fuzzy variables and the suitable experimental design, accurate surrogate functions can be obtained for the modal quantities. The results from a simple numerical example suggest that the proposed response surface methodology is suitable for free vibration analysis of structure with fuzzy parameters.

REFERENCES

[1] H. De Gersem, D. Moens, W. Desmet, and D. Vandepitte. Interval and fuzzy finite element analysis of mechanical structures with uncertain parameters. In *Proceedings of ISMA*. Citeseer, (2004), pp. 3009–3022.

[2] D. Moens and D. Vandepitte. A fuzzy finite element procedure for the calculation of uncertain frequency-response functions of damped structures: Part 1-procedure. *Journal of Sound and Vibration*, **288**, (3), (2005), pp. 431–462.

[3] H. De Gersem, D. Moens, W. Desmet, and D. Vandepitte. A fuzzy finite element procedure for the calculation of uncertain frequency response functions of damped structures: Part 2-numerical case studies. *Journal of Sound and Vibration*, **288**, (3), (2005), pp. 463–486

[4] S. Donders, D. Vandepitte, J. Van de Peer, and W. Desmet. Assessment of uncertainty on structural dynamic responses with the short transformation method. *Journal of Sound and Vibration*, **288**, (3), (2005), pp. 523–549.

[5] M. De Munck, D. Moens, W. Desmet, and D. Vandepitte. A response surface based optimisation algorithm for the calculation of fuzzy envelope FRFs of models with uncertain properties. *Computers & Structures*, **86**, (10), (2008), pp. 1080–1092.

[6] U. O. Akpan, T. S. Koko, I. R. Orisamolu, and B. K. Gallant. Practical fuzzy finite element analysis of structures. *Finite Elements in Analysis and Design*, **38**, (2), (2001), pp. 93–111.

[7] B. Möller, W. Graf, and M. Beer. Fuzzy structural analysis using α-level optimization. *Computational Mechanics*, **26**, (6), (2000), pp. 547–565.

[8] D. Degrauwe, G. De Roeck, and G Lombaert. Fuzzy frequency response function of a composite floor subject to uncertainty by application of the gad algorithm. In *III European Conference on Computational Mechanics*. Springer, (2006), pp. 290–290.

[9] L. Farkas, D. Moens, D. Vandepitte, and W. Desmet. Application of fuzzy numerical techniques for product performance analysis in the conceptual and preliminary design stage. *Computers & Structures*, **86**, (10), (2008), pp. 1061–1079.

[10] D. Moens and M. Hanss. Non-probabilistic finite element analysis for parametric uncertainty treatment in applied mechanics: Recent advances. *Finite Elements in Analysis and Design*, **47**, (1), (2011), pp. 4–16.

[11] R. L. Mason, R. F. Gunst, and J. L. Hess. *Statistical design and analysis of experiments: With applications to engineering and science*, Vol. 474. John Wiley & Sons, (2003).

[12] N. V. Queipo, R. T. Haftka, W. Shyy, T. Goel, R. Vaidyanathan, and P. K. Tucker. Surrogate-based analysis and optimization. *Progress in Aerospace Sciences*, **41**, (1), (2005), pp. 1–28.

[13] Z. Michalewicz. *Genetic algorithms + data structures = evolution programs*. Springer Science & Business Media, (1996).

[14] L. X. Huynh. *Design of structures by optimisation theory*. Science and Technics Publishing House, (2006). (in Vietnammese).

[15] C. B. Lucasius and G. Kateman. Understanding and using genetic algorithms Part 1. concepts, properties and context. *Chemometrics and Intelligent Laboratory Systems*, **19**, (1), (1993), pp. 1–33.